Name _____ Class _____ Date _____

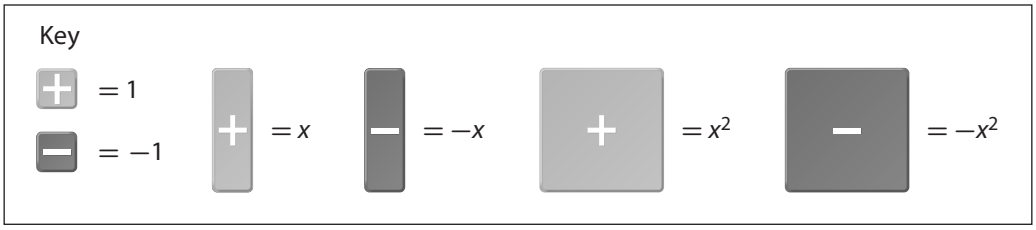
21.3 Using Special Factors to Solve Equations

Essential Question: How can you use special products to aid in solving quadratic equations by factoring?

Resource Locker

Explore Exploring Factors of Perfect Square Trinomials

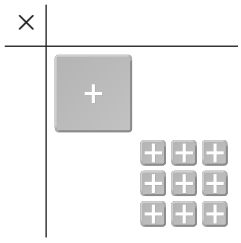
When you use algebra tiles to factor a polynomial, you must arrange the unit tiles on the grid in a rectangle. Sometimes, you can arrange the unit tiles to form a square. Trinomials of this type are called perfect-square trinomials.



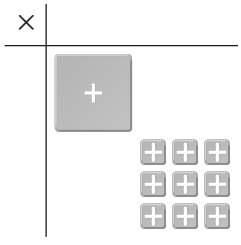
A Use algebra tiles to factor $x^2 + 6x + 9$.

Identify the number of tiles you need to model the expression. You need ____ x^2-tiles, ____ x-tiles, and ____ unit tiles.

B Arrange the algebra tiles on the grid. Place the ____ x^2-tile in the upper left corner, and arrange the ____ unit tiles in the lower right corner.



C Fill in the empty spaces on the grid with x-tiles.

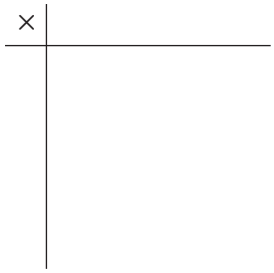


(D) All ☐ x-tiles were used, so all the tiles are accounted for and fit in the square with sides of length _____. Read the length and width of the square to get the factors of the trinomial $x^2 + 6x + 9 = \left(x + \square\right)\left(x + \square\right)$.

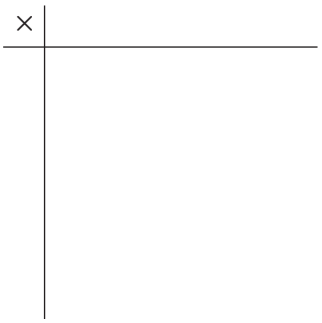
(E) Now, use algebra tiles to factor $x^2 - 8x + 16$.

You need ☐ x^2-tiles, ☐ $-x$ -tiles, and ☐ unit tiles to model the expression.

(F) Arrange the algebra tiles on the grid. Place the ☐ x^2-tile in the upper left corner, and arrange the ☐ unit tiles in the lower right corner.



(G) Fill in the empty spaces on the grid with $-x$-tiles.



(H) All ☐ $-x$ -tiles were used, so all the tiles are accounted for and fit in a square with sides of length _____. Read the length and width of the square to get the factors of the trinomial $x^2 - 8x + 16 = \left(\square\right)\left(\square\right)$.

Reflect

1. **What If?** Suppose that the middle term in $x^2 + 6x + 9$ was changed from $6x$ to $10x$. How would this affect the way you factor the polynomial?

_____

_____

_____

2. If the positive unit squares are arranged in a square of unit tiles when factoring with algebra tiles, what will be true about the binomial factors? (The coefficient of the x^2 term is 1 as in the previous problems.)

Explain 1 Factoring $a^2x^2 + 2abx + b^2$ and $a^2x^2 - 2abx + b^2$

Recall that a perfect-square trinomial can be represented algebraically in either the form $a^2 + 2ab + b^2$ or the form $a^2 - 2ab + b^2$.

Perfect-Square Trinomials

Perfect-Square Trinomials	
Perfect-Square Trinomial	**Examples**
$a^2 + 2ab + b^2 = (a + b)(a + b)$ $= (a + b)^2$	$x^2 + 6x + 9 = (x + 3)(x + 3)$ $= (x + 3)^2$
	$c^2x^2 + 2cdx + d^2 = (cx)^2 + 2cdx + d^2$ $= (cx + d)(cx + d)$ $= (cx + d)^2$
$a^2 - 2ab + b^2 = (a - b)(a - b)$ $= (a - b)^2$	$x^2 - 10x + 25 = (x - 5)(x - 5)$ $= (x - 5)^2$
	$c^2x^2 - 2cdx + d^2 = (cx)^2 - 2cdx + d^2$ $= (cx - d)(cx - d)$ $= (cx - d)^2$

Example 1 **Factor perfect-square trinomials.**

(A) $4x^3 - 24x^2 + 36x$

$4x^3 - 24x^2 + 36x = 4x(x^2 - 6x + 9)$ Factor out the common monomial factor $4x$.

$= 4x\,[x^2 - 2(1 \cdot 3)x + 3^2]$ Rewrite the perfect square trinomial in the form $a^2x^2 - 2abx + b^2$.

$= 4x(x - 3)(x - 3)$ Rewrite the perfect square trinomial in the form $(ax - b)(ax - b)$ to obtain factors.

The factored form of $4x^3 - 24x^2 + 36x$ is $4x(x - 3)(x - 3)$, or $4x(x - 3)^2$.

(B) $x^2 + 16x + 64$

$x^2 + 16x + 64 = x^2 + 2\left(\square \cdot \square\right)x + \square^2$ Rewrite in the form $a^2x^2 + 2abx + b^2$.

$= \left(x + \square\right)\left(x + \square\right)$ Rewrite in the form $(ax + b)(ax + b)$.

The factored form of $x^2 + 16x + 64$ is $\left(x + \square\right)\left(x + \square\right)$, or $\left(x + \square\right)^2$.

Your Turn

Factor perfect-square trinomials.

3. $2y^3 + 12y^2 + 18y$

4. $100z^2 - 20z + 1$

Explain 2 Factoring $a^2x^2 - b^2 = 0$

Recall that a difference of squares can be written algebraically as $a^2 - b^2$ and factored as $(a + b)(a - b)$.

Difference of Squares

Difference of Two Squares	
Perfect-Square Trinomial	**Examples**
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9 = (x + 3)(x - 3)$ $4x^2 - 9 = (2x + 3)(2x - 3)$ $9x^2 - 1 = (3x + 1)(3x - 1)$ $c^2x^2 - d^2 = (cx)^2 - d^2$ $= (cx + d)(cx - d)$

Example 2 **Factor each difference of squares.**

(A) $x^2 - 49$

$x^2 - 49 = x^2 - 7^2$ Rewrite in the form $a^2x^2 - b^2$.

$= (x + 7)(x - 7)$ Rewrite in the form $(ax + b)(ax - b)$.

The factored form of $x^2 - 49$ is $(x + 7)(x - 7)$.

(B) $49q^2 - 4p^2$

$49q^2 - 4p^2 = \square^2\ \square^2 - \left(\square\right)^2$ Rewrite in the form $a^2x^2 - b^2$.

$= \left(\square\right)\left(\square\right)$ Rewrite in the form $(ax + b)(ax - b)$.

The factored form of $49q^2 - 4p^2$ is $\left(\square\right)\left(\square\right)$.

Reflect

5. **Discussion** James was factoring a difference of squares but did not finish his work. What steps is he missing?

$16x^4 - 1 = (4x^2)^2 - 1$

$= (4x^2 + 1)(4x^2 - 1)$

Your Turn

Factor each difference of squares.

6. $x^2 - 144$

7. $81y^4 - 9y^2$

Explain 3 Solving Equations with Special Factors

Equations with special factors can be solved using the Zero Product Property. Remember, the Zero Product Property states that if the product of two factors is zero, then at least one of the factors must be zero. For example, if $(x + 1)(x + 9) = 0$ then $x + 1 = 0$ or $x + 9 = 0$. Consequently, the solutions for the equation are $x = -1$ or $x = -9$.

Example 3 **Solve the following equations with special factors.**

(A) $4x^2 + 12x + 9 = 0$

$4x^2 + 12x + 9 = 0$

$2^2x^2 + 2(2 \cdot 3)x + 3^2 = 0$ Rewrite in the form $a^2x^2 + 2abx + b^2$.

$(2x + 3)(2x + 3) = 0$ Rewrite in the form $(ax + b)(ax + b)$.

$2x + 3 = 0$ Set factors equal to 0 using Zero Product Property.

$x = -\frac{3}{2}$ Solve equation.

Ⓑ $25x^2 - 1 = 0$

$25x^2 - 1 = 0$

$\square^2 x^2 - \square^2 = 0$ Rewrite in the form $a^2x^2 - b^2$.

$\left(\square\right)\left(\square\right) = 0$ Rewrite in the form $(ax + b)(ax - b)$.

$\square = 0$ or $\square = 0$ Set factors equal to 0 using Zero Product Property.

$x = \square$ or $x = \square$ Solve equation.

Your Turn

Solve the following equations with special factors.

8. $25x^2 - 10x + 1 = 0$

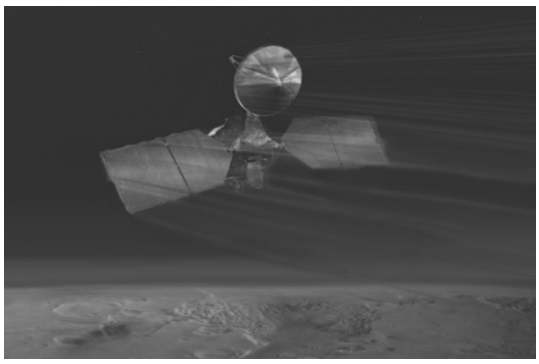
9. $8x^4 - 2x^2 = 0$

Explain 4 Solving Equation Models with Special Factors

For each real-world scenario, solve the model which involves an equation with special factors.

Example 4 **Write the given information and manipulate into a familiar form. Solve the equation to answer a question about the situation.**

As a satellite falls from outer space onto Mars, its distance in miles from the planet is given by the formula $d = -9t^2 + 776$, where t is the number of hours it has fallen. Find when the satellite will be 200 miles away from Mars.



Analyze Information

Identify the important information

- The satellite's distance in miles is given by the formula _____.
- The satellite distance at some time t is $d = \square$.

Formulate a Plan

Substituting the value of the constant $d =$ ☐ into the equation _____ you get the equation _____. Simplify the new equation into a familiar form and solve it.

Solve

Rewrite the equation to be equal to 0.

☐ $= -9t^2 + 776$ Subtract 200 from both sides.

$0 = -9t^2 +$ ☐ Divide both sides by -1.

$0 = 9t^2 -$ ☐ Factor out 9.

$0 =$ ☐ $\left(t^2 -$ ☐ $\right)$

The equation contains a _____ that you can factor.

$0 = 9\left($ ☐ $\right)\left($ ☐ $\right)$

Use the _____ Property to solve.

$0 = 9\left(t +$ ☐ $\right)\left(t -$ ☐ $\right)$

☐ $= 0$ or $t - 8=$ ☐

$t =$ ☐

The answer is $t =$ ☐ because time must be _____. So, the satellite has fallen for ☐ hours.

Justify and Evaluate

$t =$ ☐ makes sense because time must be _____. Check by substituting this value of t into the original equation.

$-9 \cdot$ ☐$^2 + 776 = -9 \cdot$ ☐ $+ 776$

$= 776 -$ ☐

$=$ ☐

This is what is expected from the given information.

Your Turn

Write the given information and manipulate it into a familiar form. Solve the equation to answer a question about the situation.

10. A volleyball player sets the ball in the air, and the height of the ball after t seconds is given in feet by $h = -16t^2 + 12t + 6$. A teammate wants to wait until the ball is 8 feet in the air before she spikes it. When should the teammate spike the ball? How many reasonable solutions are there to this problem? Explain.

11. The height of a model rocket is given (in centimeters) by the formula $h = -490t^2$, where t is measured in seconds and $h = 0$ refers to its original height at the top of a mountain. It begins to fly down from the mountain-top at time $t = 0$. When has the rocket descended 490 centimeters?

Elaborate

12. Are the perfect square trinomials $a^2 + 2ab + b^2$ and $a^2 - 2ab + b^2$ very different? How can you get one from the other?

13. How would you go about factoring $a^2 - 2ab + b^2 - 1$?

14. Setting a perfect-square trinomial equal to zero, $a^2x^2 + 2abx + b^2 = 0$, produces how many solutions? How many solutions are produced setting a difference of squares equal to zero, $a^2x^2 - b^2 = 0$?

15. Physical problems involving projectile motion can be modeled using the general equation $h = -16t^2 + v_0t$. Here, h refers to the relative height of the projectile from its initial position, v_0 is its initial vertical velocity, and t is time elapsed from launch. If you are measuring the height of the projectile as it descends from a high place, and it was launched with $v_0 = 0$ (which means it was thrown horizontally or dropped), how would you use special products to find the time at which it reaches a given height? (Assume that the height the projectile has descended is a square number in this question, although this is not a requirement in real life).

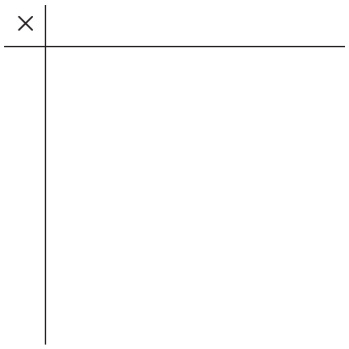
16. **Essential Question Check-In** How can you use special products to solve quadratic equations?

Evaluate: Homework and Practice

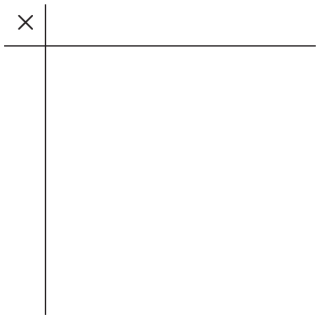
- Online Homework
- Hints and Help
- Extra Practice

For each trinomial, draw algebra tiles to show the factored form. Then, write the factored form.

1. $x^2 - 10x + 25$



2. $x^2 + 8x + 16$



Factor.

3. $4x^2 + 4x + 1$

4. $9x^2 - 18x + 9$

5. $16x^3 + 8x^2 + x$

6. $32x^3 - 16x^2 + 2x$

7. $x^2 - 169$

8. $4p^2 - 9q^4$

9. $32x^4 - 8x^2$

10. $2y^5 - 32z^4y$

Solve the following equations with special factors.

11. $25x^2 + 20x + 4 = 0$

12. $x^3 - 10x^2 + 25x = 0$

13. $4x^4 + 8x^3 + 4x^2 = 0$

14. $4x^2 - 8x + 4 = 0$

15. $x^2 - 81 = 0$

16. $2x^3 - 2x = 0$

17. $16q^2 - 81 = 0$

18. $4p^4 - 25p^2 = -16p^2$

Jivesh is analyzing the flight of a few of his model rockets with various equations. In each equation, h is the height of the rocket in centimeters, and the rocket was fired from the ground at time $t = 0$, where t is measured in seconds.

19. For Jivesh's Model A rocket, he uses the equation $h = -490t^2 + 1120t$. When is the height of the Model A rocket 640 centimeters?

20. Jivesh also has a more powerful Model B rocket. For this rocket, he uses the equation $h = -490t^2 + 1260t$. When is the height of the Model B rocket 810 centimeters?

21. Jivesh brought his Model B rocket on a camping trip near the top of a mountain. He wants to model how it descends down the mountain. Here, he uses the equation $h = -490t^2$. When has the rocket descended 1000 centimeters?

22. **Geometry** Claire is cutting a square out of a bigger square for an art project. She cuts out a square with an area of 9 cm^2. The leftover area is 16 cm^2. What is the length of one of the sides of the bigger square? The area of a square is $A = /^2$ where / is the length of one of its sides.

23. The height of a diver during a dive can be modeled by $h = -16t^2$, where h is height in feet relative to the diving platform and t is time in seconds. Find the time it takes for the diver to reach the water if the platform is 49 feet high.

24. **Physics** Consider a particular baseball player at bat. The height of the ball at time t can be modeled by $h = -16t^2 + v_0t + h_0$. Here, v_0 is the initial upward velocity of the ball, and h_0 is the height at which the ball is hit. If a ball is 4 feet off the ground when it is hit with a negligible upward velocity close to 0 feet per second, when will the ball hit the ground?

25. **Explain the Error** Jeremy factored $144x^2 - 100$ as follows:

$$144x^2 - 100 = (12x + 10)(12x - 10)$$
$$= 2(6x + 5)(6x - 5)$$

What was his error? Correct his work.

26. Which of the following are solutions to the equation $x^5 - 2x^3 + x = 0$? Select all that apply.

a. $x = -1$

b. $x = 2$

c. $x = 1$

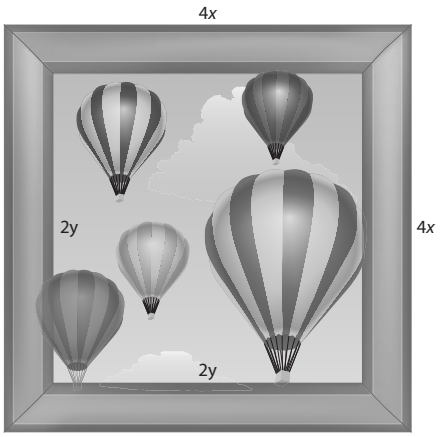
d. $x = 0.5$

e. $x = 0$

H.O.T. Focus on Higher Order Thinking

27. **Multi-Step** An artist framed a picture. The picture is a square with a side length of 2y. It is surrounded by a square frame with a side length of $4x$.

a. Find and completely factor the expression for the area of the frame.



b. The frame has an area of 11 square inches and the picture has an area of 25 square inches. Find the width of the frame.

28. **Critical Thinking** Sinea thinks that the fully factored form of the expression $x^4 - 1$ is $(x^2 - 1)(x^2 + 1)$. Is she correct? Explain.

29. **Persevere in Problem Solving** Samantha has the equation $x^3 + 2x^2 + x = x^3 - x$. Explain how she can find the solutions of the equation. Then solve the equation.

30. **Communicate Mathematical Ideas** Explain how to fully factor the expression $x^4 - 2x^2y^2 + y^4$.

Lesson Performance Task

A designer is planning to place a fountain in the lobby of an art museum. Four artists have each designed a fountain to fit the space. Some have designed rectangular fountains and the others designed square fountains. Given a quadratic equation representing the area of the fountain and the actual area of the fountain, find the dimensions of each fountain.

Artist	Artemis	Beatrice	Geoffrey	Daniel
Area equation	$\mathbf{A_A = 9x^2 - 25}$	$\mathbf{A_B = 4x^2 - 25}$	$\mathbf{A_G = 25x^2 + 80x + 64}$	$\mathbf{A_D = 81x^2 + 198x + 121}$
Fountain area	39 square feet	$28x - 74$ square feet	$160x$ square feet	$198x + 242$ square feet

Artemis:	Beatrice:

Geoffrey:	Daniel: