Name ____________________ Class ____________ Date ________

22.1 Solving Equations by Taking Square Roots

Essential Question: How can you solve quadratic equations using square roots?

Resource Locker

Explore Exploring Square Roots

Recall that the **square root** of a nonnegative number a is the real number b such that $b^2 = a$. Since $4^2 = 16$ and $(-4)^2 = 16$, the square roots of 16 are 4 and -4. Thus, every positive real number has two square roots, one positive and one negative. The positive square root is given by $\sqrt{a}$ and the negative square root by $-\sqrt{a}$. These can be combined as $\pm\sqrt{a}$.

Properties of Radicals		
Property	**Symbols**	**Example**
Product Property of Radicals	For $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.	$\sqrt{36} = \sqrt{9 \cdot 4}$ $= \sqrt{9} \cdot \sqrt{4}$ $= 3 \cdot 2$ $= 6$
Quotient Property of Radicals	For $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.	$-\sqrt{0.16} = -\sqrt{\frac{16}{100}}$ $= -\frac{\sqrt{16}}{\sqrt{100}}$ $= -\frac{4}{10}$ $= -0.4$

Find each square root.

(A) $\pm\sqrt{49} = +\square$ and $-\square$

(B) $\pm\sqrt{25} = +\square$ and $-\square$

(C) $\pm\sqrt{12} = \pm\sqrt{\square \cdot 3} = \pm\sqrt{\square} \cdot \sqrt{\square}$

$= \pm\square \cdot \sqrt{\square}$

(D) $\pm\sqrt{\frac{16}{9}} = \pm\frac{\sqrt{\square}}{\sqrt{\square}}$

$= \pm\frac{\square}{\square}$

(E) $\pm\sqrt{0.27} = \pm\sqrt{\frac{\square}{100}} = \pm\frac{\sqrt{\square}}{\sqrt{100}} = \pm\frac{\sqrt{\square \cdot 3}}{\square} = \pm\frac{\sqrt{\square} \cdot \sqrt{3}}{\square} = \pm\frac{\square \cdot \sqrt{\square}}{\square}$

Reflect

1. **Discussion** Explain why $\sqrt{6^2}$ and $\sqrt{(-6)^2}$ have the same value.

2. **Discussion** Explain why a must be nonnegative when you find $\sqrt{a}$.

3. Does 0 have any square roots? Why or why not?

Explain 1 Solving $ax^2 - c = 0$ by Using Square Roots

Solving a quadratic equation by using square roots may involve either finding square roots of perfect squares or finding square roots of numbers that are not perfect squares. In the latter case, the solution is irrational and can be approximated.

Example 1 **Solve the equation. Give the answer in radical form, and then use a calculator to approximate the solution to two decimal places, if necessary. Use a graphing calculator to graph the related function and compare the roots of the equation to the zeros of the related function.**

(A) $4x^2 - 5 = 2$

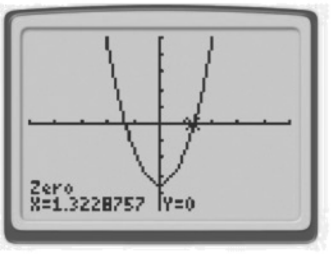
Solve the equation for x.

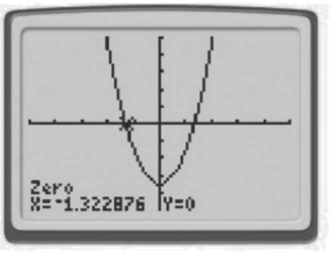
$4x^2 - 5 = 2$	Original equation
$4x^2 - 5 + 5 = 2 + 5$	Add 5 to both sides.
$4x^2 = 7$	Simplify.
$\frac{4x^2}{4} = \frac{7}{4}$	Divide both sides by 4.
$x^2 = 1.75$	Simplify.
$x = \pm\sqrt{1.75}$	Definition of a square root
$x \approx \pm 1.32$	Use a calculator to approximate the square roots.

The approximate solutions of the equation are $x \approx 1.32$ and $x \approx -1.32$.

Use a graphing calculator to graph the related function, $f(x) = 4x^2 - 7$, and find the zeros of the function.

The graph intersects the x-axis at approximately $(1.32, 0)$ and $(-1.32, 0)$. So, the roots of the equation are the zeros of the related function.





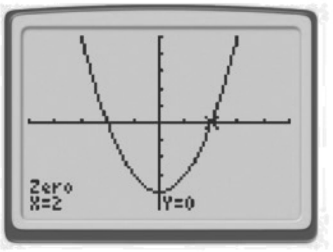
(B) $2x^2 - 8 = 0$

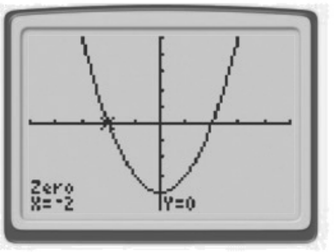
Solve the equation for x.

$2x^2 - 8 = 0$	Original equation
$2x^2 - 8 + \boxed{} = 0 + \boxed{}$	Add $\boxed{}$ to both sides.
$2x^2 = \boxed{}$	Simplify.
$\dfrac{2x^2}{\boxed{}} = \dfrac{8}{\boxed{}}$	Divide both sides by $\boxed{}$.
$x^2 = \boxed{}$	Simplify.
$x = \pm\sqrt{\boxed{}}$	Definition of a square root
$x = \pm\boxed{}$	Evaluate the square roots.

The solutions of the equation are $x = \boxed{}$ and $x = \boxed{}$.

Use a graphing calculator to graph the related function, $f(x) = 2x^2 - 8$, and find the zeros of the function.





The graph intersects the x-axis at $\left(\boxed{}, \boxed{}\right)$ and $\left(\boxed{}, \boxed{}\right)$. So,

the ________ of the equation are

the __________ of the related function.

Your Turn

Solve the equation. Give the answer in radical form, and then use a calculator to approximate the solution to two decimal places, if necessary. Use a graphing calculator to graph the related function to check your answer.

4. $3x^2 + 6 = 33$

5. $5x^2 - 9 = 2$

Explain 2 Solving $a(x+b)^2 = c$ by Using Square Roots

Solving a quadratic equation may involve isolating the squared part of a quadratic expression on one side of the equation first.

Example 2 **Solve the equation. Give the answer in radical form, and then use a calculator to approximate the solution to two decimal places, if necessary.**

(A) $(x+5)^2 = 36$

$(x+5)^2 = 36$	Original equation
$x+5 = \pm\sqrt{36}$	Take the square root of both sides.
$x+5 = \pm 6$	Simplify the square root.
$x = \pm 6 - 5$	Subtract 5 from both sides.
$x = -6 - 5$ or $x = 6 - 5$	Solve for both cases.
$x = -11$ $\quad$ $x = 1$	

The solutions are $x = -11$ and $x = 1$.

(B) $3(x-5)^2 = 18$

$3(x-5)^2 = 18$	Original equation
$(x-5)^2 = \boxed{}$	Divide both sides by $\boxed{}$.
$x-5 = \pm\sqrt{\boxed{}}$	Take the square roots of both sides.
$x = \pm\sqrt{\boxed{}} + \boxed{}$	Add $\boxed{}$ to both sides.
$x = \sqrt{\boxed{}} + 5$ or $x = -\sqrt{6} + \boxed{}$	Solve for both cases.
$x \approx \boxed{}$ or $x \approx \boxed{}$	

The approximate solutions are $x \approx \boxed{}$ and $x \approx \boxed{}$.

Reflect

6. Find the solution(s), if any, of $2(x-3)^2 = -32$. Explain your reasoning.

Your Turn

Solve the equation. Give the answer in radical form, and then use a calculator to approximate the solution to two decimal places, if necessary.

7. $4(x + 10)^2 = 24$

8. $(x - 9)^2 = 64$

Explain 3 Solving Equation Models by Using Square Roots

Real-world situations can sometimes be analyzed by solving a quadratic equation using square roots.

Example 3 **Solve the problem.**

(A) A contractor is building a fenced-in playground at a daycare. The playground will be rectangular with its width equal to half its length. The total area will be 5000 square feet. Determine how many feet of fencing the contractor will use.

First, find the dimensions.
Let $A = 5000$, $\ell = x$, and $w = \frac{1}{2}x$.

$$A = \ell w$$

$$5000 = x \cdot \frac{1}{2}x$$

$$5000 = \frac{1}{2}x^2$$

$$10{,}000 = x^2$$

$\pm\sqrt{10{,}000} = x$ — Take the square root of both sides.

$\pm 100 = x$ — Evaluate the square root.

Since the width of a rectangle cannot be negative, the length of the playground is 100 feet. The width is half the length, or 50 feet.

Find the amount of fencing.

$P = 2\ell + 2w$

$= 2(100) + 2(50)$

$= 200 + 100$ — Multiply.

$= 300$ — Add.

So, the contractor will use 300 feet of fencing.

Ⓑ A person standing on a second-floor balcony drops keys to a friend standing below the balcony. The keys are dropped from a height of 10 feet. The height in feet of the keys as they fall is given by the function $h(t) = -16t^2 + 10$, where t is the time in seconds since the keys were dropped. The friend catches the keys at a height of 4 feet. Find the elapsed time before the keys are caught.

Let $h(t) = \boxed{}$. Substitute the value into the equation and solve for t.

$h(t) = -16t^2 + 10$	Original equation
$\boxed{} = -16t^2 + 10$	Substitute.
$4 - \boxed{} = -16t^2 + 10 - \boxed{}$	Subtract 10 from both sides.
$\boxed{} = -16t^2$	Simplify.
$\dfrac{-6}{\boxed{}} = \dfrac{-16t^2}{\boxed{}}$	Divide both sides by -16.
$\boxed{} = t^2$	Simplify.
$\pm\sqrt{\boxed{}} = t$	Take the square root of both sides.
$\pm\boxed{} \approx t$	Use a calculator to approximate the square roots.

Since time cannot be negative, the elapsed time before the keys are caught is approximately $\boxed{}$ second(s).

Your Turn

9. A zookeeper is buying fencing to enclose a pen at the zoo. The pen is an isosceles right triangle. There is already a fence along the hypotenuse, which borders a path. The area of the pen will be 4500 square feet. The zookeeper can buy the fencing in whole feet only. How many feet of fencing should he buy?

Elaborate

10. How many real solutions does $x^2 = -25$ have? Explain.

11. Suppose the function $h(t) = -16t^2 + 20$ models the height in feet of an object after t seconds. If the final height is given as 2 feet, explain why there is only one reasonable solution for the time it takes the object to fall.

12. **Essential Question Check-In** What steps would you take to solve $6x^2 - 54 = 42$?

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

Use the Product Property of Radicals, the Quotient Property of Radicals, or both to simplify each expression.

1. $\pm\sqrt{0.0081}$

2. $\pm\sqrt{\dfrac{8}{25}}$

3. $\pm\sqrt{96}$

Solve each equation. Give the answer in radical form, and then use a calculator to approximate the solution to two decimal places, if necessary. Use a graphing calculator to graph the related function to check your answer.

4. $5x^2 - 21 = 39$

5. $0.1x^2 - 1.2 = 8.8$

6. $6x^2 - 21 = 33$

7. $6 - \frac{1}{3}x^2 = -20$

8. $5 - 2x^2 = -3$

9. $7x^2 + 10 = 18$

Solve each equation. Give the answer in radical form, and then use a calculator to approximate the solution to two decimal places, if necessary.

10. $5(x - 9)^2 = 15$

11. $(x + 15)^2 = 81$

12. $3(x + 1)^2 = 27$

13. $\frac{2}{3}(x - 40)^2 = 24$

14. $(x - 12)^2 = 54$

15. $(x + 5.4)^2 = 1.75$

16. The area on a wall covered by a rectangular poster is 320 square inches. The length of the poster is 1.25 times longer than the width of the poster. What are the dimensions of the poster?

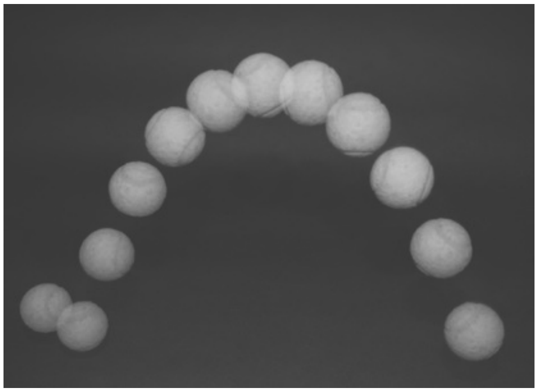
17. A circle is graphed with its center on the origin. The area of the circle is 144 square units. What are the x-intercepts of the graph? Round to the nearest tenth.

18. The equation $d = 16t^2$ gives the distance d in feet that a golf ball falls in t seconds. How many seconds will it take a golf ball to drop to the ground from a height of 4 feet? 64 feet?

19. **Entertainment** For a scene in a movie, a sack of money is dropped from the roof of a 600-foot skyscraper. The height of the sack above the ground in feet is given by $h = -16t^2 + 600$, where t is the time in seconds. How long will it take the sack to reach the ground? Round to the nearest tenth of a second.

20. A lot for sale is shaped like a trapezoid. The bases of the trapezoid represent the widths of the front and back yards. The width of the back yard is twice the width of the front yard. The distance from the front yard to the backyard, or the height of the trapezoid, is equal to the width of the back yard. Find the width of the front and back yards, given that the area is 6000 square feet. Round to the nearest foot.

21. To study how high a ball bounces, students drop the ball from various heights. The function $h(t) = -16t^2 + h$ gives the height (in feet) of the ball at time t measured in seconds since the ball was dropped from a height of h. If the ball is dropped from a height of 8 feet, find the elapsed time until the ball hits the floor. Round to the nearest tenth.



22. Match each equation with its solutions.

a. $2x^2 - 2 = 16$ ________ $= \pm \frac{2\sqrt{33}}{3}$

b. $2(x - 2)^2 = 16$ ________ $x = \pm 3$

c. $3x^2 + 4 = 48$ ________ $x = 2 \pm 2\sqrt{2}$

d. $3(x + 4)^2 = 48$ ________ $x = -8$ and $x = 0$

H.O.T. Focus on Higher Order Thinking

23. Explain the Error Trent and Lisa solve the same equation, but they disagree on the solution of the equation. Their work is shown. Which solution is correct? Explain.

Trent:	Lisa:
$5x^2 + 1000 = -125$	$5x^2 + 1000 = -125$
$5x^2 = -1125$	$5x^2 = -1125$
$x^2 = -225$	$x^2 = -225$
$x = \pm\sqrt{-225}$	no real solutions
$x = \pm 15$	

24. Multi-Step Construction workers are installing a rectangular, in-ground pool. To start, they dig a rectangular hole in the ground where the pool will be. The area of the ground that they will be digging up is 252 square feet. The length of the pool is twice the width of the pool.

a. What are the dimensions of the pool? Round to the nearest tenth.

b. Once the pool is installed, the workers will build a fence, that encloses a rectangular region, around the perimeter of it. The fence will be 10 feet from the edges of the pool, except at the corners. How many feet of fencing will the workers need?

25. Communicate Mathematical Ideas Explain why the quadratic equation $x^2 + b = 0$ where $b > 0$, has no real solutions, but the quadratic equation $x^2 - b = 0$ where $b > 0$, has two real solutions.

26. Justify Reasoning For the equation $x^2 = a$, describe the values of a that will result in two real solutions, one real solution, and no real solution. Explain your reasoning.

Lesson Performance Task

You have been asked to create a pendulum clock for your classroom. The clock will be placed on one wall of the classroom and go the entire height of the wall. Choose how large you want the face and hands on your clock to be and provide measurements for the body of the clock. The pendulum will start halfway between the center of the clock face and its bottom edge and will initially end 1 foot above the floor. Calculate the period of the pendulum using the formula $L = 9.78t^2$, where L is the length of the pendulum in inches and t is the length of the period in seconds.

Now, adjust the length of your pendulum so the number of periods in 1 minute or 60 seconds is an integer value. How long is your pendulum and how many periods equal one minute?