Name ________________ Class ________ Date ________

22.3 Using the Quadratic Formula to Solve Equations

Resource Locker

Essential Question: What is the quadratic formula, and how can you use it to solve quadratic equations?

Explore Deriving the Quadratic Formula

You can complete the square on the general form of a quadratic equation to derive a formula that can be used to solve any quadratic equation.

(A) Write the standard form of a quadratic equation.

$ax^2 + bx + c = \square$

(B) Subtract c from both sides.

$ax^2 + bx = \square$

(C) Multiply both sides by $4a$ to make the coefficient of x^2 a perfect square.

$4a^2x^2 + \square = \square$

(D) Add b^2 to both sides of the equation to complete the square.

$4a^2x^2 + 4abx + b^2 = -4ac + \square$

(E) Factor the left side to write the trinomial as the square of a binomial.

$\left(\square\right)^2 = b^2 - 4ac$

(F) Take the square roots of both sides.

$\square = \pm\sqrt{\square}$

(G) Subtract b from both sides.

$2ax = \square \pm \sqrt{\square}$

(H) Divide both sides by $2a$ to solve for x.

$$x = \frac{\square \pm \sqrt{\square}}{\square}$$

Ⓘ The formula you just derived, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, is called the **quadratic formula**. It gives you the values of x that solve any quadratic equation where $a \neq 0$.

Reflect

1. **What If?** If the derivation had begun by dividing each term by a, what would the resulting binomial of x have been after completing the square? Does one derivation method appear to be simpler than the other? Explain.

Explain 1 Using the Discriminant to Determine the Number of Real Solutions

Recall that a quadratic equation, $ax^2 + bx + c = 0$, can have two, one, or no real solutions. By evaluating the part of the quadratic formula under the radical sign, $b^2 - 4ac$, called the **discriminant**, you can determine the number of real solutions.

Example 1 **Determine how many real solutions each quadratic equation has.**

Ⓐ $x^2 - 4x + 3 = 0$

$a = 1, b = -4, c = 3$	Identify a, b, and c.
$b^2 - 4ac$	Use the discriminant.
$(-4)^2 - 4(1)(3)$	Substitute the identified values into the discriminant.
$16 - 12 = 4$	Simplify.

Since $b^2 - 4ac > 0$, the equation has two real solutions.

Ⓑ $x^2 - 2x + 2 = 0$

$a = \square, b = \square, c = \square$	Identify a, b, and c.
$b^2 - 4ac$	Use the discriminant.
$(\square)^2 - 4(\square)(\square)$	Substitute the identified values into the discriminant.
$\square - \square = \square$	Simplify.

Since $b^2 - 4ac$ $\square$ 0, the equation has ____ real solution(s).

Reflect

2. When the discriminant is positive, the quadratic equation has two real solutions. When the discriminant is negative, there are no real solutions. How many real solutions does a quadratic equation have if its discriminant equals 0? Explain.

Your Turn

Use the discriminant to determine the number of real solutions for each quadratic equation.

3. $x^2 + 4x + 1 = 0$

4. $2x^2 - 6x + 15 = 0$

5. $x^2 + 6x + 9 = 0$

Explain 2 Solving Equations by Using the Quadratic Formula

To use the quadratic formula to solve a quadratic equation, check that the equation is in standard form. If not, rewrite it in standard form. Then substitute the values of a, b, and c into the formula.

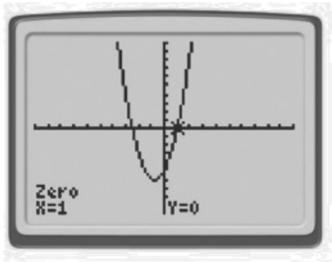
Example 2 **Solve using the quadratic formula.**

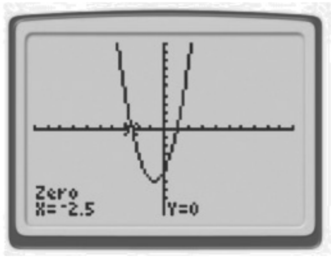
(A) $2x^2 + 3x - 5 = 0$

$a = 2, b = 3, c = -5$	Identify a, b, and c.
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Use the quadratic formula.
$x = \dfrac{-3 \pm \sqrt{(3)^2 - 4(2)(-5)}}{2(2)}$	Substitute the identified values into the quadratic formula.
$x = \dfrac{-3 \pm \sqrt{49}}{4}$	Simplify the radicand and the denominator.
$x = \dfrac{-3 \pm 7}{4}$	Evaluate the square root.
$x = \dfrac{-3 + 7}{4}$ or $x = \dfrac{-3 - 7}{4}$	Write as two equations.
$x = 1$ or $x = -\dfrac{5}{2}$	Simplify both equations.

The solutions are 1 and $-\dfrac{5}{2}$.

Graph $y = 2x^2 + 3x - 5$ to verify your answers.





The graph does verify the solutions.

Ⓑ $2x = x^2 - 4$

$x^2 - \square - 4 = 0$ — Write in standard form.

$a = \square,\ b = \square,\ c = \square$ — Identify a, b, and c.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Use the quadratic formula.

$$x = \frac{-(\square) \pm \sqrt{(\square)^2 - 4(\square)(\square)}}{2(\square)}$$ Substitute the identified values into the quadratic formula.

$$x = \frac{2 \pm \sqrt{\square}}{2}$$ Simplify the radicand and the denominator.

$$x = \frac{2 \pm \sqrt{\square \cdot 5}}{2} = \frac{2 \pm 2\square}{2} = 1 \pm \square$$ Simplify.

$x = \square$ or $x = \square$ — Write as two equations.

$x \approx \square$ or $x \approx \square$ — Use a calculator to find approximate solutions to three decimal places.

The exact solutions are $\square$ and $\square$. The approximate solutions are $\square$ and $\square$.

Graph $y = x^2 - \square - 4$ and find the zeros using the graphing calculator. The calculator will give approximate values.

The graph ________ confirm the solutions.

Reflect

6. Discussion How can you use substitution to check your solutions?

Your Turn

Solve using the quadratic formula.

7. $x^2 - 6x - 7 = 0$

8. $2x^2 = 8x - 7$

Explain 3 Using the Discriminant with Real-World Models

Given a real-world situation that can be modeled by a quadratic equation, you can find the number of real solutions to the problem using the discriminant, and then apply the quadratic formula to obtain the solutions. After finding the solutions, check to see if they make sense in the context of the problem.

In projectile motion problems the projectile height h is modeled by the equation $h = -16t^2 + vt + s$, where t is the time in seconds the object has been in the air, v is the initial vertical velocity in feet per second, and s is the initial height in feet. The -16 coefficient in front of the t^2 term refers to the effect of gravity on the object. This equation can be written using metric units as $h = -4.9t^2 + vt + s$, where the units are converted from feet to meters. Time remains in units of seconds.

Example 3 **For each problem, use the discriminant to determine the number of real solutions for the equation. Then, find the solutions and check to see if they make sense in the context of the problem.**

(A) A diver jumps from a platform 10 meters above the surface of the water. The diver's height is given by the equation $h = -4.9t^2 + 3.5t + 10$, where t is the time in seconds after the diver jumps. For what time t is the diver's height 1 meter?

Substitute $h = 1$ into the height equation. Then, write the resulting quadratic equation in standard form to solve for t.

$1 = -4.9t^2 + 3.5t + 10$ $\qquad$ $0 = -4.9t^2 + 3.5t + 9$

First, use the discriminant to find the number of real solutions of the equation.

$b^2 - 4ac$ $\qquad$ Use the discriminant.

$(3.5)^2 - 4(-4.9)(9) = 188.65$

Since $b^2 - 4ac > 0$, the equation has two real solutions.

Next, use the quadratic formula to find the real number solutions.

$a = -4.9$, $b = 3.5$, $c = 9$	Identify a, b, and c.
$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Use the quadratic formula.
$t = \dfrac{-3.5 \pm \sqrt{188.65}}{2(-4.9)}$	Substitute the identified values into the quadratic formula and the value of the discriminant.
$t \approx \dfrac{-3.5 \pm 13.73}{-9.8}$	Simplify.
$t \approx \dfrac{-3.5 + 13.73}{-9.8}$ or $t \approx \dfrac{-3.5 - 13.73}{-9.8}$	Write as two equations.
$t \approx -1.04$ or $t \approx 1.76$	Solutions

Disregard the negative solution because t represents the seconds after the diver jumps and a negative value has no meaning in this context. So, the diver is at height 1 meter after a time of $t \approx 1.76$ seconds.

B The height in meters of a model rocket on a particular launch can be modeled by the equation $h = -4.9t^2 + 102t + 100$, where t is the time in seconds after its engine burns out 100 meters above the ground. When will the rocket reach a height of 600 meters?

Substitute $h = \boxed{}$ into the height equation. Then, write the resulting quadratic equation in standard form to solve for t.

$$h = -4.9t^2 + 102t + 100$$
$$\boxed{} = -4.9t^2 + 102t + 100$$
$$0 = -4.9t^2 + 102t - \boxed{}$$

First, use the discriminant to find the number of real solutions of the equation.

$a = -4.9$, $b = \boxed{}$, $c = \boxed{}$	Identify a, b, and c.
$b^2 - 4ac$	Use the discriminant.
$\left(\boxed{}\right)^2 - 4(-4.9)\left(\boxed{}\right)$	Substitute the identified values into the discriminant.
$\boxed{} - \boxed{} = \boxed{}$	Simplify.

Since $b^2 - 4ac \boxed{} 0$, the equation has $\boxed{}$ real solutions.

Next, use the quadratic formula to find the real number solutions.

$$t = \frac{-\boxed{} \pm \sqrt{\boxed{}}}{2(-4.9)}$$

Substitute the identified values into the quadratic formula and the value of the discriminant.

$$t = \frac{\boxed{} \pm \boxed{}}{-9.8}$$

Simplify.

$$t \approx \frac{-102 + \boxed{}}{-9.8} \text{ or } t \approx \frac{-102 - \boxed{}}{-9.8}$$

Write as two equations.

$t \approx \boxed{}$ or $t \approx \boxed{}$

Solutions

Disregard the ____________ solution because t represents the seconds after the rocket has launched and a ____________ value has no meaning in this context. So, the rocket is at height 600 meters after a time of $t \approx \boxed{}$ seconds.

Your Turn

For each problem, use the discriminant to determine the number of real solutions for the equation. Then, find the solutions and check to see if they make sense in the context of the problem.

9. A soccer player uses her head to hit a ball up in the air from a height of 2 meters with an initial vertical velocity of 5 meters per second. The height h in meters of the ball is given by $h = -4.9t^2 + 5t + 2$, where t is the time elapsed in seconds. How long will it take the ball to hit the ground if no other players touch it?

10. The quarterback of a football team throws a pass to the team's receiver. The height h in meters of the football can be modeled by $h = -4.9t^2 + 3t + 1.75$, where t is the time elapsed in seconds. The receiver catches the football at a height of 0.25 meters. How long does the ball remain in the air until it is caught by the receiver?

Elaborate

11. How can the discriminant of a quadratic equation be used to determine the number of zeros (x-intercepts) that the graph of the equation will have?

12. What advantage does using the quadratic formula have over other methods of solving quadratic equations?

13. **Essential Question Check-In** How can you derive the quadratic formula?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Determine how many real solutions each quadratic equation has.

1. $4x^2 + 4x + 1 = 0$

2. $x^2 - x + 3 = 0$

3. $x^2 - 8x^2 - 9 = 0$

4. $2x^2 - x\sqrt{5} + 2 = 0$

5. $\frac{x^2}{2} - x + \frac{1}{4} = 0$

6. $\frac{x^2}{4} - x\sqrt{7} + 7 = 0$

7. $\frac{x^2}{2} - x\sqrt{2} + 1 = 0$

8. $x^2\sqrt{2} - x + \frac{1}{2} = 0$

Solve using the quadratic formula. Leave answers that are not perfect squares in radical form.

9. $10x + 4 = 6x^2$

10. $x^2 + x - 20 = 0$

11. $4x^2 = 4 - x$

12. $9x^2 + 3x - 2 = 0$

13. $14x + 3 = -8x^2$

14. $x^2 + 3x^2 + 1 = 0$

For each problem, use the discriminant to determine the number of real solutions for the equation. Then, find the solutions and check to see if they make sense in the context of the problem.

15. **Sports** A soccer player kicks the ball to a height of 1 meter inside the goal. The equation for the height h of the ball at time t is $h = -4.9t^2 - 5t + 2$. Find the time the ball reached the goal.

16. The length and width of a rectangular patio are, $(x + 8)$ feet and $(x + 6)$ feet, respectively. If the area of the patio is 160 square feet, what are the dimensions of the patio?

17. **Chemistry** A scientist is growing bacteria in a lab for study. One particular type of bacteria grows at a rate of $y = 2t^2 + 3t + 500$. A different bacteria grows at a rate of $y = 3t^2 + t + 300$. In both of these equations, y is the number of bacteria after t minutes. When is there an equal number of both types of bacteria?

Use this information for Exercises 18 and 19. A gymnast, who can stretch her arms up to reach 6 feet, jumps straight up on a trampoline. The height of her feet above the trampoline can be modeled by the equation $h = -16x^2 + 12x$, where x is the time in seconds after her jump.

18. Do the gymnast's hands reach a height of 10 feet above the trampoline? Use the discriminant to explain. (Hint: Since h = height of feet, you must use the difference between the heights of the hands and feet.)

19. Which of the following are possible heights she achieved? Select all that apply.

a. $h = \frac{9}{4}$

b. $h = 4$

c. $h = 3$

d. $h = 0.5$

e. $h = \frac{1}{4}$

20. **Explain the Error** Dan said that if a quadratic equation does not have any real solutions, then it does not represent a function. Explain Dan's error.

H.O.T. Focus on Higher Order Thinking

21. **Communicate Mathematical Ideas** Explain why a positive discriminant results in two real solutions.

22. **Multi-Step** A model rocket is launched from the top of a hill 10 meters above ground level. The rocket's initial speed is 10 meters per second. Its height h can be modeled by the equation $h = -4.9t^2 + 10t + 10$, where t is the time in seconds.

a. When does the rocket achieve a height of 100 meters?

b. How long does it take the rocket to reach ground level?

Lesson Performance Task

A baseball field is next to a building that is 130 feet tall. A series of batters hit pitched balls into the air with the given initial vertical velocities. (Assume each ball is hit from a height of 3 feet.) After the game, a fan reports that several hits resulted in the ball hitting the roof of the building. How can you use the discriminant to determine whether any of the hits described below could be among them? Explain. If any of the balls hit could have hit the roof, identify them. Can you tell if the ball actually did hit the roof?

Player	Initial Vertical Velocity (ft/s)
Janok	99
Jimenez	91
Serrano	88
Sei	89