Name ______________________ Class ____________ Date ________

22.4 Choosing a Method for Solving Quadratic Equations

Essential Question: How can you choose a method for solving a given quadratic equation?

Resource Locker

Explore Comparing Solution Methods for Quadratic Equations

$7x^2 - 3x - 5 = 0$

Try to solve the equation by factoring.

(A) Find the factors of 7 and −5 to complete the table:

Factors of 7	Factors of −5	Outer Product + Inner Product
1, 7	1, −5	2
1, 7	5, −1	
1, 7	−1, 5	
1, 7	−5, 1	

(B) None of the sums of the inner and outer products of the factor pairs of 7 and −5 equal −3.

Does this mean the equation cannot be solved? ______.

Now, try to solve the equation by completing the square.

(C) The leading coefficient is not a perfect square. Multiply both sides by a value that makes the coefficient a perfect square.

$$\boxed{}\left(7x^2 - 3x - 5\right) = (0)\boxed{}$$

$$\boxed{}x^2 - \boxed{}x - \boxed{} = \boxed{}$$

(D) Add or subtract to move the constant term to the other side of the equation.

$$\boxed{}x^2 - \boxed{}x = \boxed{}$$

(E) Find $\frac{b^2}{4a}$ and reduce to simplest form.

$$\frac{b^2}{4a} = \frac{\boxed{}^2}{4\left(\boxed{}\right)} = \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

(F) Add $\frac{b^2}{4a}$ to both sides of the equation,

$$\square x^2 - \square x + \frac{\square}{\square} = \square + \frac{\square}{\square}$$

$$\square x^2 - \square x + \frac{\square}{\square} = \frac{\square}{\square}$$

(G) Factor the perfect-square trinomial on the left side of the equation.

$$\left(\square x - \frac{\square}{\square}\right)^2 = \frac{\square}{\square}$$

(H) Take the square root of both sides. $\square x - \frac{\square}{\square} = \pm\sqrt{\frac{\square}{\square}}$

(I) Add the constant to both sides, and then divide by a. Find both solutions for x.

$$\square x - \frac{\square}{\square} + \frac{\square}{\square} = \pm\sqrt{\frac{\square}{\square}} + \frac{\square}{\square}$$

$$\frac{\square x}{\square} = \frac{\pm\sqrt{\frac{\square}{\square}} + \frac{\square}{\square}}{\square}$$

$$x = \frac{\pm\sqrt{\frac{\square}{\square}} + \frac{\square}{\square}}{\square}$$

$$x = \frac{\sqrt{\frac{\square}{\square}} + \frac{\square}{\square}}{\square} \quad \text{or} \quad x = -\frac{\sqrt{\frac{\square}{\square}} + \frac{\square}{\square}}{\square}$$

(J) Solve both equations to three decimal places using your calculator.

$x = \square$ or $x = \square$

Now use the quadratic formula to solve the same equation.

(K) Identify the values of a, b, and c. $a = \square$, $b = \square$, $c = \square$

(L) Substitute values into the quadratic formula.

$$x = \frac{-\square \pm \sqrt{\square^2 - 4(7)(\square)}}{2(\square)}$$

Ⓜ Simplify the discriminant and the denominator.

$$x = \frac{3 \pm \sqrt{\square}}{\square}$$

Ⓝ Use your calculator to finish simplifying the expression for x.

$x = \square$ or $x = \square$

Reflect

1. **Discussion** Another method you learned for solving quadratics is taking square roots. Why would that not work in this case?

Explain 1 Solving Quadratic Equations Using Different Methods

You have seen several ways to solve a quadratic equation, but there are reasons why you might choose one method over another.

Factoring is usually the fastest and easiest method. Try factoring first if it seems likely that the equation is factorable.

Both completing the square and using the quadratic formula are more general. Quadratic equations that are solvable can be solved using either method.

Example 1 **Speculate which method is the most appropriate for each equation and explain your answer. Then solve the equation using factoring (if possible), completing the square, and the quadratic formula.**

Ⓐ $x^2 + 7x + 6 = 0$

Factor the quadratic.

Set up a factor table adding factors of c.

Factors of c	Sum of Factors
1, 6	7
2, 3	5

Substitute in factors. $(x + 1)(x + 6) = 0$

Use the Zero Product Property $x + 1 = 0$ or $x + 6 = 0$

Solve both equations for x. $x = -1$ or $x = -6$

Complete the square.

Move the constant term to the right side. $x^2 + 7x = -6$

Add $\frac{b^2}{4a}$ to both sides. $x^2 + 7x + \frac{49}{4} = -6 + \frac{49}{4}$

Simplify. $x^2 + 7x + \frac{49}{4} = \frac{25}{4}$

Factor the perfect-square trimonial on the left. $\left(x + \frac{7}{2}\right)^2 = \frac{25}{4}$

Take the square root of both sides. $x + \frac{7}{2} = \pm\frac{5}{2}$

Write two equations. $x + \frac{7}{2} = \frac{5}{2}$ or $x + \frac{7}{2} = -\frac{5}{2}$

Solve both equations. $x = -1$ or $x = -6$

Apply the quadratic formula.

Identify the values of a, b, and c. $a = 1, b = 7, c = 6$

Substitute values into the quadratic formula. $x = \frac{-7 \pm \sqrt{7^2 - 4(1)(6)}}{2(1)}$

Simplify the discriminant and denominator. $x = \frac{-7 \pm \sqrt{25}}{2}$

Evaluate the square root and write as two equations. $x = \frac{-7 + 5}{2}$ or $x = \frac{-7 - 5}{2}$

Simplify. $x = -1$ or $x = -6$

Because the list of possible factors that needed to be checked was short, it makes sense to try factoring $x^2 + 7x + 6$ first, even if you don't know if you will be able to factor it. Once factored, the remaining steps are fewer and simpler than either completing the square or using the quadratic formula.

(B) $2x^2 + 8x + 3 = 0$

Factor the quadratic.

Factors of c	Factors of c	Sum of Inner and Outer Products
1, 2	1, 3	
1, 2	3, 1	

Can the quadratic be factored? ________.

Complete the square.

Move the constant term to the right side. $2x^2 + 8x = -3$

Multiply both sides by $\boxed{}$ to make a perfect square. $\boxed{}\, x^2 + 16x = \boxed{}$

Add $\frac{b^2}{4a}$ to both sides. $4x^2 + 16x + \boxed{} = 10$

Factor the left side. $\left(\boxed{}\, x + \boxed{}\right)^2 = 10$

Take the square root of both sides. $2x + 4 = \boxed{}$

Write two equations. $2x + 4 = \boxed{}$ or $2x + 4 = -\sqrt{10}$

Solve both equations. $x = -2 \boxed{} \frac{\sqrt{10}}{2}$ or $x = -2 \boxed{} \frac{\sqrt{10}}{2}$

Apply the quadratic formula.

Identify the values of *a*, *b*, and *c*. $a = \boxed{}, b = \boxed{}, c = \boxed{}$

Substitute values into the quadratic formula. $x = \dfrac{\boxed{} \pm \sqrt{\boxed{}^2 - 4\left(\boxed{}\right)\left(\boxed{}\right)}}{2\left(\boxed{}\right)}$

Simplify the discriminant and denominator. $x = \dfrac{-8 \pm \sqrt{\boxed{}}}{4}$

Evaluate the square root and write as two equations. $x = \dfrac{-8 \pm \boxed{}\sqrt{\boxed{}}}{4}$

Simplify. $x = \boxed{} \pm \dfrac{\sqrt{10}}{\boxed{}}$

Reflect

2. What are the advantages and disadvantages of solving a quadratic equation by taking square roots?

3. What are the advantages and disadvantages of solving a quadratic equation by factoring?

4. What are the advantages and disadvantages of solving a quadratic equation by completing the square?

5. What are the advantages and disadvantages of solving a quadratic equation by using the quadratic formula?

Your Turn

Solve the quadratic equations by any method you chose. Identify the method and explain why you chose it.

6. $9x^2 - 100 = 0$

7. $x^2 + 4x - 7 = 0$

Explain 2 Choosing Solution Methods for Quadratic Equation Models

Recall that the formula for height, in feet, of a projectile under the influence of gravity is given by $h = -16t^2 + vt + s$, where t is the time in seconds, v is the upward initial velocity (at $t = 0$), and s is the starting height.

Example 2 **Marco is throwing a tennis ball at a kite that is stuck 42 feet up in a tree, trying to knock it loose. He can throw the ball at a velocity of 45 feet per second upward at a height of 4 feet. Will his throw reach the kite? How hard does Marco need to throw the ball to reach the kite?**

Analyze Information

The initial velocity is: ______

The starting height is: ______

The height of the kite is: ______

Formulate a Plan

Use the projectile motion formula to write an equation for the height of the ball t seconds after Marco throws it.

$h = \boxed{}\,t^2 + \boxed{}\,t + \boxed{}$

To determine if the ball can reach the height of the kite, set up the equation to find the time it takes the ball to reach the height of the kite.

$-16t^2 + 45t + 4 = \boxed{}$

Convert the equation to standard form.

$-16t^2 + 45t + \boxed{} = 0$

This problem will be easiest to solve by ______________________ To check if the ball reaches the kite, begin by calculating the ____________. To determine the velocity that Marco must throw the ball to reach the kite, we should find the velocity where the ____________ is equal to ____, which is the exact moment at which the ball changes direction and falls back to earth.

Solve

Identify values of a, b, and c. $a = \boxed{}$, $b = \boxed{}$, $c = \boxed{}$

Evaluate the discriminant first. $b^2 - 4ac = \boxed{}^2 - 4\left(\boxed{}\right)\left(\boxed{}\right)$

$= \boxed{}$

A negative discriminant means that there are ____ solutions to the equation. Marco's throw will/will not reach as high as the kite.

The velocity with which Marco needs to throw the ball to reach the kite is the coefficient b of the x-term of the quadratic equation.

$b = \boxed{}$

Substitute v into the discriminant and solve for a discriminant equal to 0 to find the velocity at which Marco needs to throw the ball.

Identify values of a, b, and c. $a = \boxed{}$, $b = \boxed{}$, $c = \boxed{}$

Evaluate the discriminant first. $\boxed{}^2 - 4\left(\boxed{}\right)\left(\boxed{}\right) = 0$

Simplify. $v^2 - \boxed{} = 0$

This quadratic equation should be solved by ________________ because it has no x-term.

Move the constant term to the right. $v^2 = \boxed{}$

Take square roots of both sides. Use your calculator. $v \approx \pm\boxed{}$

The negative velocity represents a downward throw and will not result in the ball hitting the kite. The tennis ball must have a velocity of at about ______ feet per second to reach the kite.

Justify and Evaluate

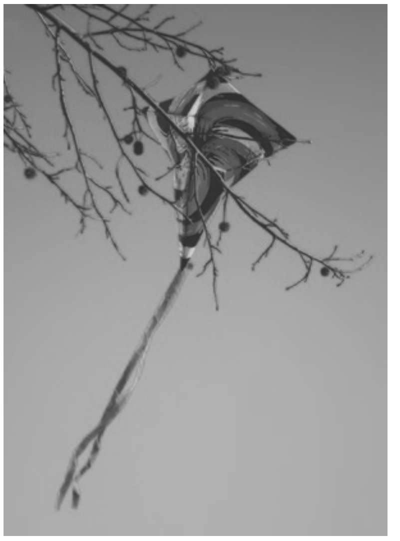
Plot the graph of Marco's throw on your graphing calculator to see that the conclusion you reached (no solution) makes sense because the graphs do not intersect. Sketch the graph.

Then plot the height of the ball when the discriminant is equal to zero. The graphs intersect in one point. Sketch the graph.

Your Turn

8. The wheel of a remote controlled airplane falls off while the airplane is climbing at 40 feet in the air. The wheel starts with an initial upward velocity of 24 feet per second. How long does it take to fall to the ground? Set up the equation to determine the time and pick one method to solve it. Explain why you chose that method.

9. Marco's brother, Jessie, is helping Marco knock a kite from the tree. He can throw the ball 50 feet per second upwards, from a height of 5 feet. Is he throwing the ball hard enough to reach the kite, and if so, how long does it take the ball to reach the kite?



Elaborate

10. Which method do you think is best if you are going to have to use a calculator?

11. Some factorable quadratic expressions are still quite difficult to solve by factoring rather than using another method. What makes an equation difficult to factor?

12. You are taking a test on quadratic equations and you can't decide which method would be the fastest way to solve a particular problem. How could looking at a graph of the equation on a calculator help you decide which method to use?

13. **Essential Question Check-In** How should you determine a method for solving a quadratic equation?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

1. Look at this quadratic equation and explain what you think will be the best approach to solving it. Do not solve the equation.

$3.38x^2 + 2.72x - 9.31 = 0$

Solve the quadratic equation by any means. Identify the method and explain why you chose it. Irrational answers may be left in radical form or approximated with a calculator (round to two decimal places).

2. $x^2 - 7x + 12 = 0$

3. $36x^2 - 64 = 0$

4. $4x^2 - 4x - 3 = 2$

5. $8x^2 + 9x + 2 = 1$

6. $5x^2 + 0x - 13 = 0$

7. $7x^2 - 5x - 5 = 0$

8. $3x^2 - 6x = 0$

9. $2x^2 + 4x - 3 = 0$

10. $(x - 5)^2 = 16$

11. $(2x - 1)^2 = x$

12. $2(x+2)^2 - 5 = 3$

13. $(2x-3)^2 = 4x$

14. $6x^2 - 5x + 12 = 0$

15. $3x^2 + 6x + 2 = 0$

16. $\frac{1}{2}x^2 + 3x + \frac{5}{2} = 0$

17. $(6x+7)(x+1) = 26$

Use the projectile motion formula and solve the quadratic equation by any means. Identify the method and explain why you chose it. Irrational answers and fractions should be converted to decimal form and rounded to two places.

18. Gary drops a pair of gloves off of a balcony that is 64 feet high down to his friend on the ground. How long does it take the pair of gloves to hit the ground?

19. A soccer player jumps up and heads the ball while it is 7 feet above the ground. It bounces up at a velocity of 20 feet per second. How long will it take the ball to hit the ground?

20. A stomp rocket is a toy that is launched into the air from the ground by a sudden burst of pressure exerted by stomping on a pedal. If the rocket is launched at 24 feet per second, how long will it be in the air?

21. A dog leaps off of the patio from 2 feet off of the ground with an upward velocity of 15 feet per second. How long will the dog be in the air?

22. **Multipart Classification** Indicate whether the following statements about finding solutions to quadratic equations with integer coefficients are true or false.

a. Any quadratic equation with a real solution can be solved by using the quadratic formula. ○ True ○ False

b. Any quadratic equation with a real solution can be solved by completing the square. ○ True ○ False

c. Any quadratic equation with a real solution can be solved by factoring. ○ True ○ False

d. Any quadratic equation with a real solution can be solved by taking the square root of both sides of the equation. ○ True ○ False

e. If the equation can be factored, it has rational solutions. ○ True ○ False

f. If the equation has only one real solution, it cannot be factored. ○ True ○ False

H.O.T. Focus on Higher Order Thinking

23. **Justify Reasoning** Any quadratic equation with a real solution can be solved with the quadratic formula. Describe the kinds of equations where that would not be the best choice, and explain your reasoning.

24. **Critique Reasoning** Marisol decides to solve the quadratic equation by factoring $21x^2 + 47x - 24 = 0$. Do you think she chose the best method? How would you solve this equation?

25. **Communicate Mathematical Ideas** Explain the difference between the statements “The quadratic formula can be used to solve any quadratic equation with a real solution” and “Every quadratic equation has a real solution.”

Lesson Performance Task

A landscaper is designing a patio for a customer who has several different ideas about what to make.

Use the given information to set up a quadratic equation modeling the situation and solve it using the quadratic formula. Then determine if another method for solving quadratic equations would have been easier to use and explain why.

a. One of the customer's ideas is to buy bluestone tiles from a home improvement store using several gift cards he has received as presents over the past few years. If the total value of the gift cards is \$6500 and the bluestone costs \$9 per square foot, what are the dimensions of the largest patio that can be made that is 12 feet longer than it is wide?

b. Another of the customer's ideas is simply a quadratic equation scrawled on a napkin.

$x^2 - 54x + 720 = 0$

c. The third idea is also a somewhat random quadratic polynomial.

$x^2 - 40x + 397 = 0$