

# 4.2 Transversals and Parallel Lines

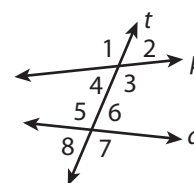


Resource Locker

**Essential Question:** How can you prove and use theorems about angles formed by transversals that intersect parallel lines?

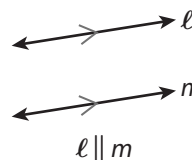
## Explore Exploring Parallel Lines and Transversals

A **transversal** is a line that intersects two coplanar lines at two different points. In the figure, line  $t$  is a transversal. The table summarizes the names of angle pairs formed by a transversal.



Angle Pair	Example
<b>Corresponding angles</b> lie on the same side of the transversal and on the same sides of the intersected lines.	$\angle 1$ and $\angle 5$
<b>Same-side interior angles</b> lie on the same side of the transversal and between the intersected lines.	$\angle 3$ and $\angle 6$
<b>Alternate interior angles</b> are nonadjacent angles that lie on opposite sides of the transversal between the intersected lines.	$\angle 3$ and $\angle 5$
<b>Alternate exterior angles</b> lie on opposite sides of the transversal and outside the intersected lines.	$\angle 1$ and $\angle 7$

Recall that parallel lines lie in the same plane and never intersect. In the figure, line  $\ell$  is parallel to line  $m$ , written  $\ell \parallel m$ . The arrows on the lines also indicate that they are parallel.



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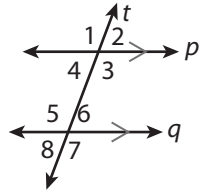
When parallel lines are cut by a transversal, the angle pairs formed are either congruent or supplementary. The following postulate is the starting point for proving theorems about parallel lines that are intersected by a transversal.

**Same-Side Interior Angles Postulate**

If two parallel lines are cut by a transversal, then the pairs of same-side interior angles are supplementary.

Follow the steps to illustrate the postulate and use it to find angle measures.

- (A) Draw two parallel lines and a transversal, and number the angles formed from 1 to 8.



- (B) Identify the pairs of same-side interior angles.

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- (C) What does the postulate tell you about these same-side interior angle pairs?

\_\_\_\_\_

- (D) If  $m\angle 4 = 70^\circ$ , what is  $m\angle 5$ ? Explain.

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**Reflect**

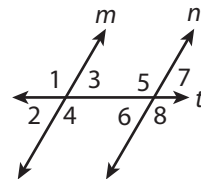
1. Explain how you can find  $m\angle 3$  in the diagram if  $p \parallel q$  and  $m\angle 6 = 61^\circ$ .

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2. **What If?** If  $m \parallel n$ , how many pairs of same-side interior angles are shown in the figure? What are the pairs?

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## Explain 1

# Proving that Alternate Interior Angles are Congruent

Other pairs of angles formed by parallel lines cut by a transversal are alternate interior angles.

### Alternate Interior Angles Theorem

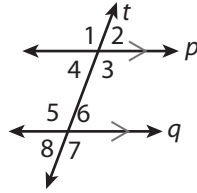
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles have the same measure.

To prove something to be true, you use definitions, properties, postulates, and theorems that you already know.

### Example 1 Prove the Alternate Interior Angles Theorem.

Given:  $p \parallel q$

Prove:  $m\angle 3 = m\angle 5$



Complete the proof by writing the missing reasons. Choose from the following reasons. You may use a reason more than once.

- Same-Side Interior Angles Postulate
- Given
- Definition of supplementary angles
- Subtraction Property of Equality
- Substitution Property of Equality
- Linear Pair Theorem

Statements	Reasons
1. $p \parallel q$	
2. $\angle 3$ and $\angle 6$ are supplementary.	
3. $m\angle 3 + m\angle 6 = 180^\circ$	
4. $\angle 5$ and $\angle 6$ are a linear pair.	
5. $\angle 5$ and $\angle 6$ are supplementary.	
6. $m\angle 5 + m\angle 6 = 180^\circ$	
7. $m\angle 3 + m\angle 6 = m\angle 5 + m\angle 6$	
8. $m\angle 3 = m\angle 5$	

### Reflect

3. In the figure, explain why  $\angle 1$ ,  $\angle 3$ ,  $\angle 5$ , and  $\angle 7$  all have the same measure.

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4. Suppose  $m\angle 4 = 57^\circ$  in the figure shown. Describe two different ways to determine  $m\angle 6$ .

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## Explain 2 Proving that Corresponding Angles are Congruent

Two parallel lines cut by a transversal also form angle pairs called corresponding angles.

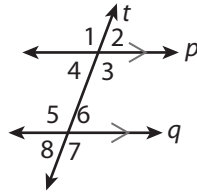
### Corresponding Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of corresponding angles have the same measure.

**Example 2** Complete a proof in paragraph form for the Corresponding Angles Theorem.

**Given:**  $p \parallel q$

**Prove:**  $m\angle 4 = m\angle 8$



By the given statement,  $p \parallel q$ .  $\angle 4$  and  $\angle 6$  form a pair of \_\_\_\_\_.

So, using the Alternate Interior Angles Theorem, \_\_\_\_\_.

$\angle 6$  and  $\angle 8$  form a pair of vertical angles. So, using the Vertical Angles Theorem,

\_\_\_\_\_. Using the \_\_\_\_\_

in  $m\angle 4 = m\angle 6$ , substitute \_\_\_\_\_ for  $m\angle 6$ . The result is \_\_\_\_\_.

### Reflect

5. Use the diagram in Example 2 to explain how you can prove the Corresponding Angles Theorem using the Same-Side Interior Angles Postulate and a linear pair of angles.

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6. Suppose  $m\angle 4 = 36^\circ$ . Find  $m\angle 5$ . Explain.

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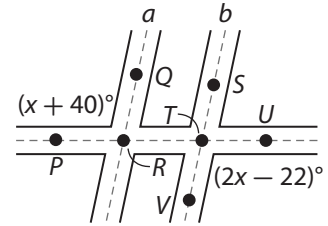
## Explain 3 Using Parallel Lines to Find Angle Pair Relationships

You can apply the theorems and postulates about parallel lines cut by a transversal to solve problems.

**Example 3** Find each value. Explain how to find the values using postulates, theorems, and algebraic reasoning.

- A** In the diagram, roads  $a$  and  $b$  are parallel. Explain how to find the measure of  $\angle VTU$ .

It is given that  $m\angle PRQ = (x + 40)^\circ$  and  $m\angle VTU = (2x - 22)^\circ$ .  
 $m\angle PRQ = m\angle RTS$  by the Corresponding Angles Theorem and  
 $m\angle RTS = m\angle VTU$  by the Vertical Angles Theorem.  
 So,  $m\angle PRQ = m\angle VTU$ , and  $x + 40 = 2x - 22$ . Solving for  $x$ ,  
 $x + 62 = 2x$ , and  $x = 62$ . Substitute the value of  $x$  to find  $m\angle VTU$ :  
 $m\angle VTU = (2(62) - 22)^\circ = 102^\circ$ .



- B** In the diagram, roads  $a$  and  $b$  are parallel. Explain how to find the measure of  $m\angle WUV$ .

It is given that  $m\angle PRS = (9x)^\circ$  and  $m\angle WUV = (22x + 25)^\circ$ .

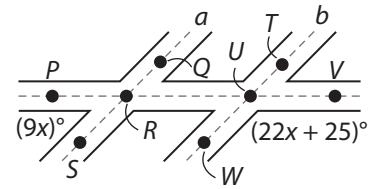
$m\angle PRS = m\angle RUW$  by the \_\_\_\_\_.

$\angle RUW$  and \_\_\_\_\_ are supplementary angles.

So,  $m\angle RUW + m\angle WUV =$  \_\_\_\_\_. Solving for  $x$ ,  $31x + 25 = 180$ ,

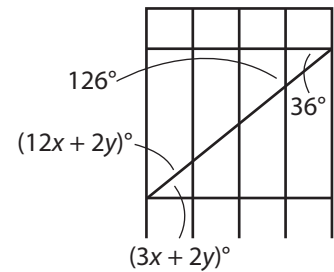
and \_\_\_\_\_. Substitute the value of  $x$  to find \_\_\_\_\_;

$m\angle WUV = (22(5) + 25)^\circ$  \_\_\_\_\_.



### Your Turn

- 7.** In the diagram of a gate, the horizontal bars are parallel and the vertical bars are parallel. Find  $x$  and  $y$ . Name the postulates and/or theorems that you used to find the values.




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## Elaborate

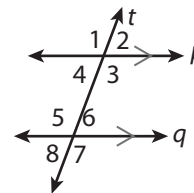
8. How is the Same-Side Interior Angles Postulate different from the two theorems in the lesson (Alternate Interior Angles Theorem and Corresponding Angles Theorem)?

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9. **Discussion** Look at the figure below. If you know that  $p$  and  $q$  are parallel, and are given one angle measure, can you find all the other angle measures? Explain.




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10. **Essential Question Check-In** Why is it important to establish the Same-Side Interior Angles Postulate before proving the other theorems?

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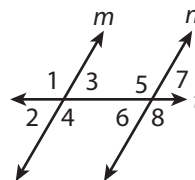
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## Evaluate: Homework and Practice

1. In the figure below,  $m \parallel n$ . Match the angle pairs with the correct label for the pairs. Indicate a match by writing the letter for the angle pairs on the line in front of the corresponding labels.



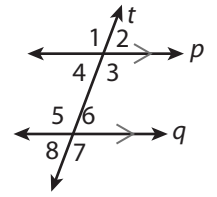
- A.  $\angle 4$  and  $\angle 6$  \_\_\_\_\_ Corresponding Angles  
 B.  $\angle 5$  and  $\angle 8$  \_\_\_\_\_ Same-Side Interior Angles  
 C.  $\angle 2$  and  $\angle 6$  \_\_\_\_\_ Alternate Interior Angles  
 D.  $\angle 4$  and  $\angle 5$  \_\_\_\_\_ Vertical Angles



- Online Homework
- Hints and Help
- Extra Practice

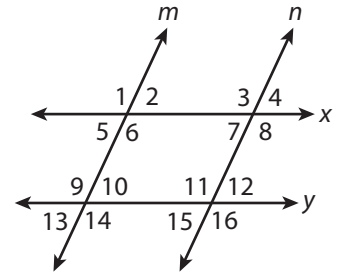
2. Complete the definition: A \_\_\_\_\_ is a line that intersects two coplanar lines at two different points.

Use the figure to find angle measures. In the figure,  $p \parallel q$ .



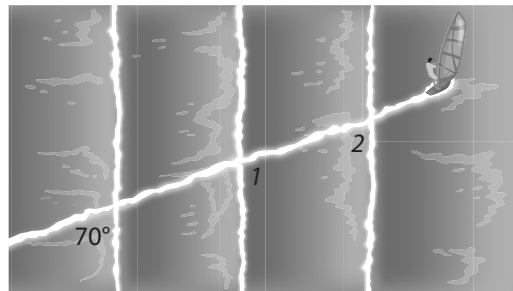
3. Suppose  $m\angle 4 = 82^\circ$ . Find  $m\angle 5$ .
4. Suppose  $m\angle 3 = 105^\circ$ . Find  $m\angle 6$ .
5. Suppose  $m\angle 3 = 122^\circ$ . Find  $m\angle 5$ .
6. Suppose  $m\angle 4 = 76^\circ$ . Find  $m\angle 6$ .
7. Suppose  $m\angle 5 = 109^\circ$ . Find  $m\angle 1$ .
8. Suppose  $m\angle 6 = 74^\circ$ . Find  $m\angle 2$ .

Use the figure to find angle measures. In the figure,  $m \parallel n$  and  $x \parallel y$ .



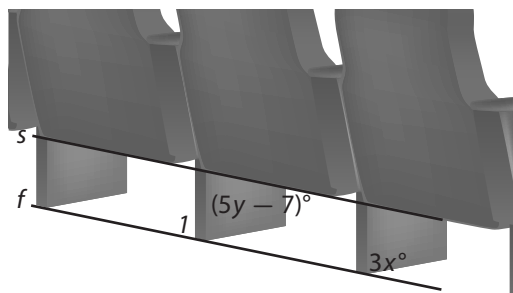
9. Suppose  $m\angle 5 = 69^\circ$ . Find  $m\angle 10$ .
10. Suppose  $m\angle 9 = 115^\circ$ . Find  $m\angle 6$ .
11. Suppose  $m\angle 12 = 118^\circ$ . Find  $m\angle 7$ .
12. Suppose  $m\angle 4 = 72^\circ$ . Find  $m\angle 11$ .
13. Suppose  $m\angle 4 = 114^\circ$ . Find  $m\angle 14$ .
14. Suppose  $m\angle 5 = 86^\circ$ . Find  $m\angle 12$ .

- 15.** Ocean waves move in parallel lines toward the shore. The figure shows the path that a windsurfer takes across several waves. For this exercise, think of the windsurfer's wake as a line. If  $m\angle 1 = (2x + 2y)^\circ$  and  $m\angle 2 = (2x + y)^\circ$ , find  $x$  and  $y$ . Explain your reasoning.



**In the diagram of movie theater seats, the incline of the floor,  $f$ , is parallel to the seats,  $s$ .**

- 16.** If  $m\angle 1 = 60^\circ$ , what is  $x$ ?

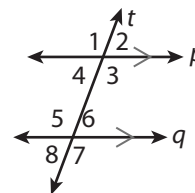


- 17.** If  $m\angle 1 = 68^\circ$ , what is  $y$ ?

- 18.** Complete a proof in paragraph form for the Alternate Interior Angles Theorem.

**Given:**  $p \parallel q$

**Prove:**  $m\angle 3 = m\angle 5$



It is given that  $p \parallel q$ , so using the Same-Side Interior Angles Postulate,  $\angle 3$  and  $\angle 6$  are \_\_\_\_\_. So, the sum of their measures is \_\_\_\_\_ and  $m\angle 3 + m\angle 6 = 180^\circ$ .

You can see from the diagram that  $\angle 5$  and  $\angle 6$  form a line, so they are a \_\_\_\_\_, which makes them \_\_\_\_\_. Then  $m\angle 5 + m\angle 6 = 180^\circ$ . Using the

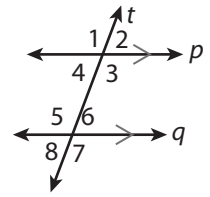
Substitution Property of Equality, you can substitute \_\_\_\_\_ in  $m\angle 3 + m\angle 6 = 180^\circ$  with  $m\angle 5 + m\angle 6$ . This results in  $m\angle 3 + m\angle 6 = m\angle 5 + m\angle 6$ . Using the Subtraction Property of Equality, you can subtract \_\_\_\_\_ from both sides. So, \_\_\_\_\_.



19. Write a proof in two-column form for the Corresponding Angles Theorem.

Given:  $p \parallel q$

Prove:  $m\angle 1 = m\angle 5$

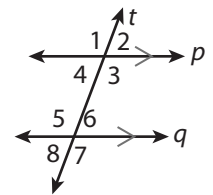


Statements	Reasons

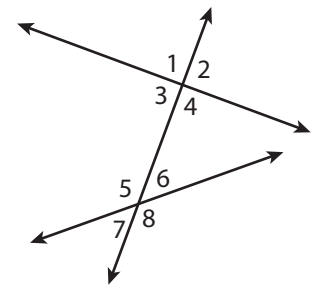
20. **Explain the Error** Angelina wrote a proof in paragraph form to prove that the measures of corresponding angles are congruent. Identify her error, and describe how to fix the error.

Angelina's proof:

I am given that  $p \parallel q$ .  $\angle 1$  and  $\angle 4$  are supplementary angles because they form a linear pair, so  $m\angle 1 + m\angle 4 = 180^\circ$ .  $\angle 4$  and  $\angle 8$  are also supplementary because of the Same-Side Interior Angles Postulate, so  $m\angle 4 + m\angle 8 = 180^\circ$ . You can substitute  $m\angle 4 + m\angle 8$  for  $180^\circ$  in the first equation above. The result is  $m\angle 1 + m\angle 4 = m\angle 4 + m\angle 8$ . After subtracting  $m\angle 4$  from each side, I see that  $\angle 1$  and  $\angle 8$  are corresponding angles and  $m\angle 1 = m\angle 8$ .

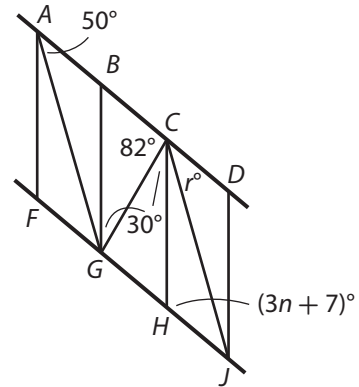


21. **Counterexample** Ellen thinks that when two lines that are not parallel are cut by a transversal, the measures of the alternate interior angles are the same. Write a proof to show that she is correct or use a counterexample to show that she is incorrect.



**H.O.T. Focus on Higher Order Thinking**

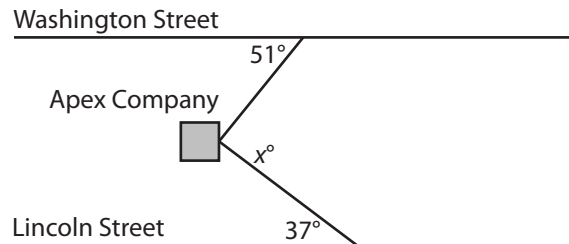
**Analyzing Mathematical Relationships** Use the diagram of a staircase railing for Exercises 22 and 23.  $\overline{AG} \parallel \overline{CJ}$  and  $\overline{AD} \parallel \overline{FJ}$ . Choose the best answer.



22. Which is a true statement about the measure of  $\angle DCJ$ ?
- A. It is  $30^\circ$ , by the Alternate Interior Angles Theorem.
  - B. It is  $30^\circ$ , by the Corresponding Angles Theorem.
  - C. It is  $50^\circ$ , by the Alternate Interior Angles Theorem.
  - D. It is  $50^\circ$ , by the Corresponding Angles Theorem.
23. Which is a true statement about the value of  $n$ ?
- A. It is 25, by the Alternate Interior Angles Theorem.
  - B. It is 25, by the Same-Side Interior Angles Postulate.
  - C. It is 35, by Alternate Interior Angles Theorem.
  - D. It is 35, by the Corresponding Angles Theorem.

## Lesson Performance Task

Washington Street is parallel to Lincoln Street. The Apex Company's headquarters is located between the streets. From headquarters, a straight road leads to Washington Street, intersecting it at a  $51^\circ$  angle. Another straight road leads to Lincoln Street, intersecting it at a  $37^\circ$  angle.



- a. Find  $x$ . Explain your method.
- b. Suppose that another straight road leads from the opposite side of headquarters to Washington Street, intersecting it at a  $y^\circ$  angle, and another straight road leads from headquarters to Lincoln Street, intersecting it at a  $z^\circ$  angle. Find the measure of the angle  $w$  formed by the two roads. Explain how you found  $w$ .