Name ______________________ Class ____________ Date ________

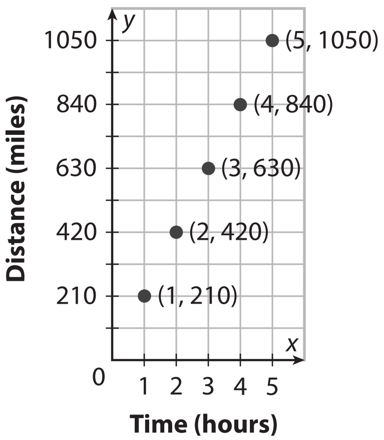
5.1 Understanding Linear Functions

Essential Question: What is a linear function?

Resource Locker

Explore 1 Recognizing Linear Functions

A race car can travel up to 210 mph. If the car could travel continuously at this speed, $y = 210x$ gives the number of miles y that the car would travel in x hours. Solutions are shown in the graph below.



The graph of the car's speed is a function because every x-value is paired with exactly one y-value. Because the graph is a non-vertical straight line, it is also a **linear function.**

(A) Fill in the table using the data points from the graph above.

x	y

(B) Using the table, check that x has a constant change between consecutive terms.

(C) Now check that y has a constant change between consecutive terms.

(D) Using the answers from before, what change in x corresponds to a change in y?

(E) All linear functions behave similarly to the one in this example. Based on this information, a generalization can be made that a ____________ change in x will correspond to a ____________ change in y.

Reflect

1. **Discussion** Will a non-linear function have a constant change in x that corresponds to a constant change in y?

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2. $y = x^2$ represents a typical non-linear function. Using the table of values, check whether a constant change in x corresponds to a constant change in y.

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x	$y = x^2$
1	1
2	4
3	9
4	16
5	25

Explore 2 Proving Linear Functions Grow by Equal Differences Over Equal Intervals

Linear functions change by a constant amount (change by equal differences) over equal intervals. Now you will explore the proofs of these statements. $x_2 - x_1$ and $x_4 - x_3$ represent two intervals in the x-values of a linear function.

It is also important to know that any linear function can be written in the form $f(x) = mx + b$, where m and b are constants.

Complete the proof that linear functions grow by equal differences over equal intervals.

Given: $x_2 - x_1 = x_4 - x_3$

f is a linear function of the form $f(x) = mx + b$.

Prove: $f(x_2) - f(x_1) = f(x_4) - f(x_3)$

Proof:

Statement	Reason
1. $x_2 - x_1 = x_4 - x_3$	Given.
2. $m(x_2 - x_1) = \boxed{}(x_4 - x_3)$	Mult. Property of Equality
3. $mx_2 - \boxed{} = mx_4 - \boxed{}$	______
4. $mx_2 + b - mx_1 - b = mx_4 + \boxed{} - mx_3 - \boxed{}$	______
5. $mx_2 + b - (mx_1 + b) = mx_4 + b - \boxed{}$	______
6. $f(x_2) - f(x_1) = \boxed{}$	Definition of $f(x)$

Reflect

3. **Discussion** Consider the function $y = x^3$. Use two equal intervals to determine if the function is linear. The table for $y = x^3$ is shown.

x	$y = x^3$
1	1
2	8
3	27
4	64
5	125

4. In the given of the proof it states that: f is a linear function of the form $f(x) = mx + b$. What is the name of the form for this linear function?

Explain 1 Graphing Linear Functions Given in Standard Form

Any linear function can be represented by a linear equation. A **linear equation** is any equation that can be written in the **standard form** expressed below.

Standard Form of a Linear Equation

$Ax + By = C$ where A, B, and C are real numbers and A and B are not both 0.

Any ordered pair that makes the linear equation true is a **solution of a linear equation in two variables**. The graph of a linear equation represents all the solutions of the equation.

Example 1 **Determine whether the equation is linear. If so, graph the function.**

(A) $5x + y = 10$

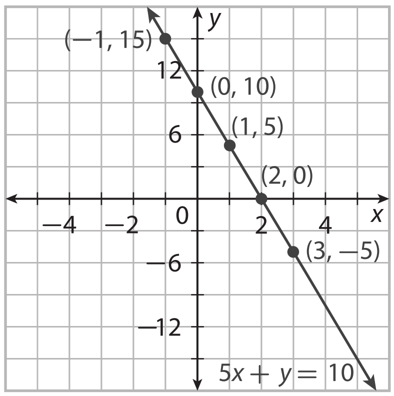
The equation is linear because it is in the standard form of a linear equation: $A = 5$, $B = 1$, and $C = 10$.

To graph the function, first solve the equation for y.

$$\begin{aligned} 5x + y &= 10 \\ y &= 10 - 5x \end{aligned}$$

Make a table and plot the points. Then connect the points.

x	−1	0	1	2	3
y	15	10	5	0	−5



Note that because the domain and range of functions of a non-horizontal line are all real numbers, the graph is continuous.

(B) $-4x + y = 11$

The equation is linear because it is in the ________ form of a linear equation:

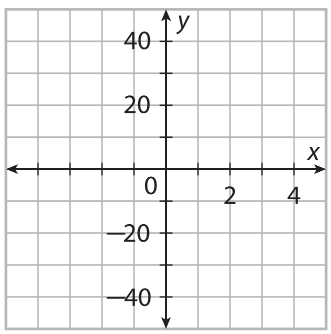
$A =$ ______, $B =$ ______, and $C =$ ______.

To graph the function, first solve the equation for ______.

$$\begin{aligned} -4x + y &= 11 \\ y &= 11 + \underline{\qquad}x \end{aligned}$$

Make a table and plot the points. Then connect the points.

x	−4	−2	0	2	4
y					



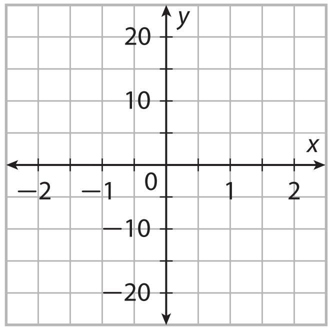
Reflect

5. Write an equation that is linear but is not in standard form.

6. If $A = 0$ in an equation written in standard form, how does the graph look?

Your Turn

7. Determine whether $6x + y = 12$ is linear. If so, graph the function.



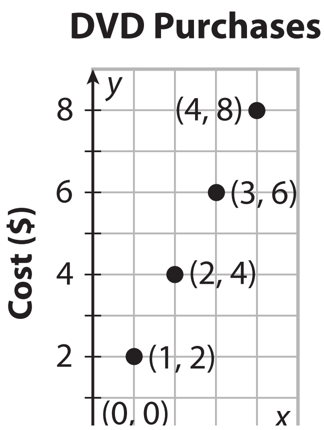
Explain 2 Modeling with Linear Functions

A **discrete function** is a function whose graph has unconnected points, while a **continuous function** is a function whose graph is an unbroken line or curve with no gaps or breaks. For example, a function representing the sale of individual apples is a discrete function because no fractional part of an apple will be represented in a table or a graph. A function representing the sale of apples by the pound is a continuous function because any fractional part of a pound of apples will be represented in a table or graph.

Example 2 **Graph each function and give its domain and range.**

(A) Sal opens a new video store and pays the film studios \$2.00 for each DVD he buys from them. The amount Sal pays is given by $f(x) = 2x$, where x is the number of DVDs purchased.

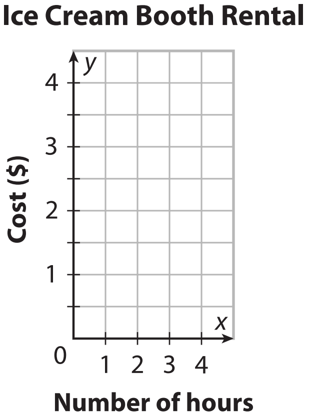
x	$f(x) = 2x$
0	$f(0) = 2(0) = 0$
1	$f(1) = 2(1) = 2$
2	$f(2) = 2(2) = 4$
3	$f(3) = 2(3) = 6$
4	$f(4) = 2(4) = 8$



This is a discrete function. Since the number of DVDs must be a whole number, the domain is $\{0, 1, 2, 3, \ldots\}$ and the range is $\{0, 2, 4, 6, 8 \ldots\}$.

(B) Elsa rents a booth in her grandfather's mall to open an ice cream stand. She pays \$1 to her grandfather for each hour of operation. The amount Elsa pays each hour is given by $f(x) = x$, where x is the number of hours her booth is open.

x	$f(x) = x$
0	$f(0) = \square$
1	$f(1) = \square$
2	$f(2) = \square$
3	$f(3) = \square$
4	$f(4) = \square$



This is a __________ function. The domain is ______________________

and the range is ______________________.

Reflect

8. Why are the points on the graph in Example 2B connected?

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9. **Discussion** How is the graph of the function in Example 2A related to the graph of an arithmetic sequence?

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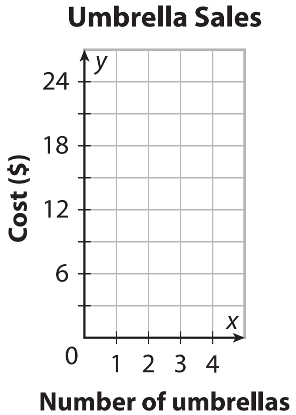
Your Turn

10. Kristoff rents a kiosk in the mall to open an umbrella stand. He pays \$6 to the mall owner for each umbrella he sells. The amount Kristoff pays is given by $f(x) = 6x$, where x is the number of umbrellas sold. Graph the function and give its domain and range.



Elaborate

11. What is a solution of a linear equation in two variables?

12. What type of function has a graph with a series of unconnected points?

13. **Essential Question Check-In** What is the standard form for a linear equation?

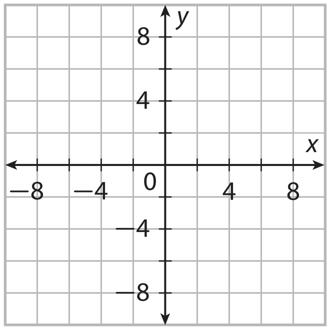
Evaluate: Homework and Practice

Personal Math Trainer

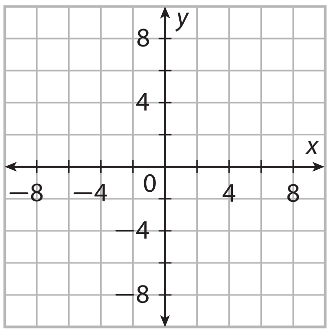
- Online Homework
- Hints and Help
- Extra Practice

Determine if the equation is linear. If so, graph the function.

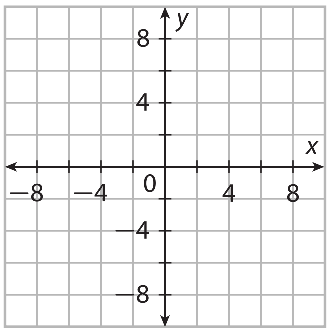
1. $2x + y = 4$



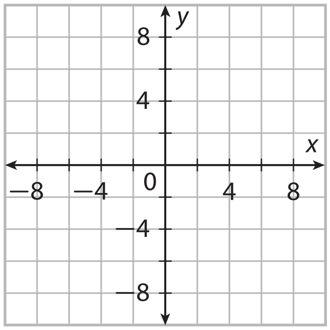
2. $2x^2 + y = 6$



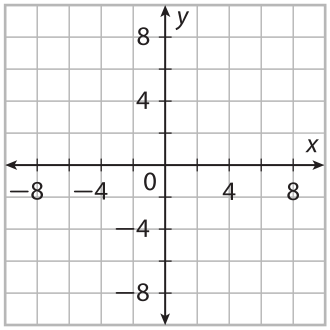
3. $\frac{2}{x} + \frac{y}{4} = \frac{3}{2}$



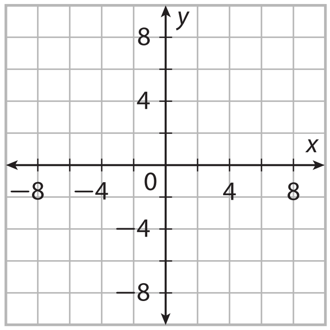
4. $3x + 4y = 8$



5. $x + y^2 = 1$



6. $x + y = 1$



State whether each function is discrete or continuous.

7. The number of basketballs manufactured per day

8. $x = \frac{y}{4}$, where x is the number of hours and y is the miles walked

9. The number of bulls eyes scored for each hour of practice

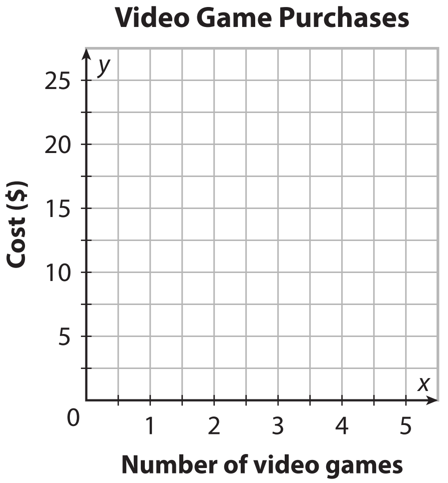
10. $y = 4^4x$, where x is the time and y is gallons of water

11. $y = 35x^1$, where x is distance and y is height

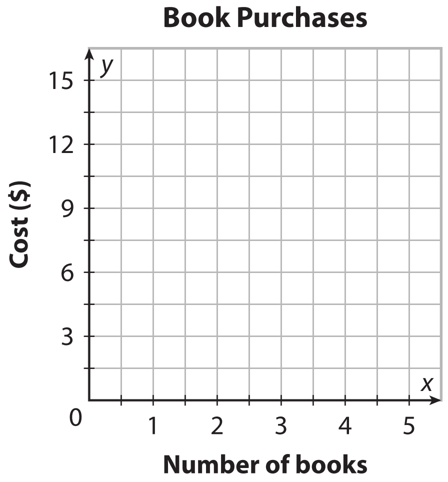
12. The amount of boxes shipped per shift

Graph each function and give its domain and range.

13. Hans opens a new video game store and pays the gaming companies \$5.00 for each video game he buys from them. The amount Hans pays is given by $f(x) = 5x$, where x is the number of video games purchased.

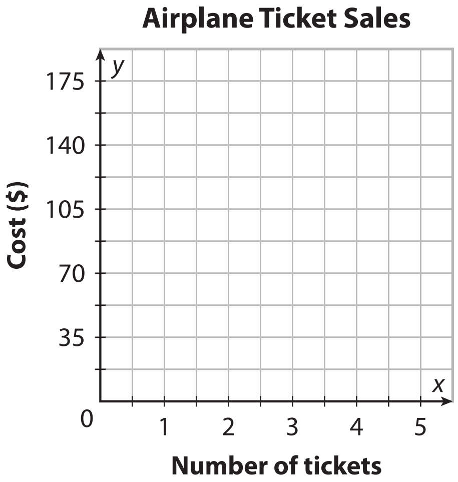


14. Peter opens a new bookstore and pays the book publisher \$3.00 for each book he buys from them. The amount Peter pays is given by $f(x) = 3x$, where x is the number of books purchased.



15. Steve opens a jewelry shop and makes \$15.00 profit for each piece of jewelry sold. The amount Steve makes is given by $f(x) = 15x$, where x is the number of pieces of jewelry sold.

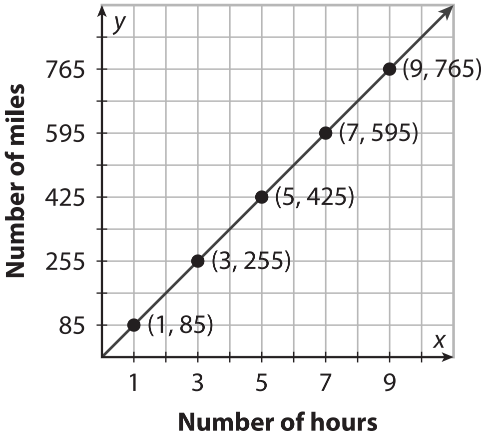
16. Anna owns an airline and pays the airport \$35.00 for each ticket sold. The amount Anna pays is given by $f(x) = 35x$, where x is the number of tickets sold.



17. A hot air balloon can travel up to 85 mph. If the balloon travels continuously at this speed, $y = 85x$ gives the number of miles y that the hot air balloon would travel in x hours.

Fill in the table using the data points from the graph. Determine whether x and y have constant change between consecutive terms and whether they are in a linear function.

x					
y					



18. State whether each equation is in standard form.

a. $3x + y = 8$

b. $x - y = 15z$

c. $x^2 + y = 11$

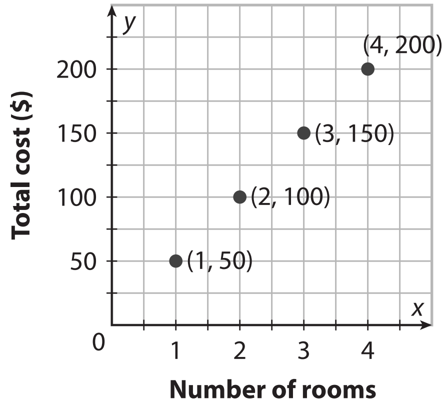
d. $3xy + y^2 = 4$

e. $x + 4y = 12$

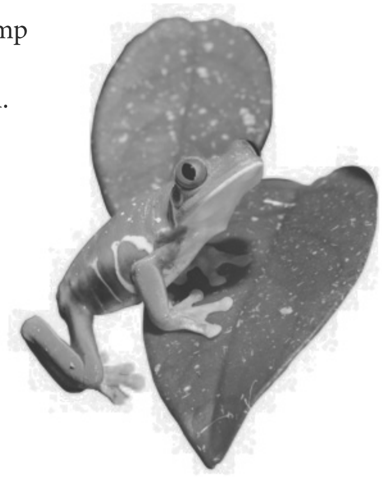
f. $5x + 24y = 544$

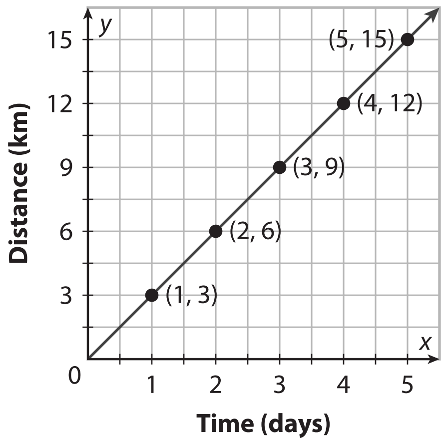
19. **Physics** A physicist working in a large laboratory has found that light particles traveling in a particle accelerator increase velocity in a manner that can be described by the linear function $-4x + 3y = 15$, where x is time and y is velocity in kilometers per hour. Use this function to determine when a certain particle will reach 30 km/hr.

20. Travel The graph shows the costs of a hotel for one night for a group traveling. The total cost depends on the number of hotel rooms the group needs. Does the plot follow a linear function? Is the graph discrete or continuous?



21. Biology The migration pattern of a species of tree frog to different swamp areas over the course of a year can be described using the graph below. Fill in the table and express whether this pattern follows a linear function. If the migration pattern is a linear function, express what constant change in y corresponds to a constant change in x.





x	y
1	
2	
3	
4	
5	

H.O.T. Focus on Higher Order Thinking

22. Representing Real-World Problems Write a real-world problem that is a discrete non-linear function.

23. Explain the Error A student used the following table of values and stated that the function described by the table was a linear function. Explain the student's error.

x	−1	0	2	3	4
y	−5	0	5	10	15

24. Communicate Mathematical Ideas Explain how graphs of the same function can look different.

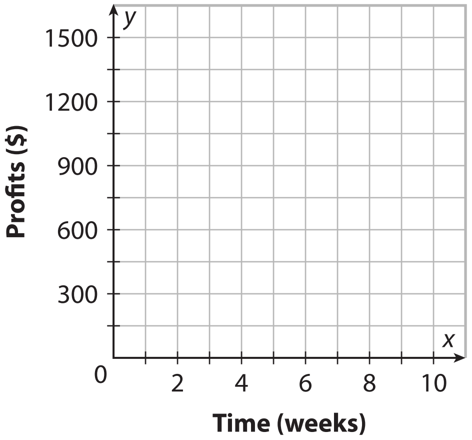
Lesson Performance Task

Jordan has started a new dog-walking service. His total profits over the first 4 weeks are expressed in this table.

Time (weeks)	Profits($)
1	150
2	300
3	450
4	600

a. Show that his profits can be described by a linear function.

b. Graph this function and use the graph to predict his business profit 9 weeks after he opens.



c. Explain why it is or is not a good idea to project his profits so far into the future. Give examples to support your answer.