Name ______________________ Class ____________ Date ________

6.2 AAS Triangle Congruence

Essential Question: **What does the AAS Triangle Congruence Theorem tell you about two triangles?**

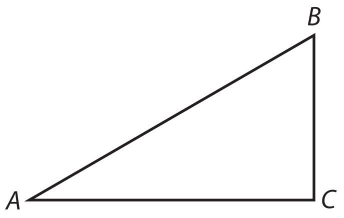
Resource Locker

Explore Exploring Angle-Angle-Side Congruence

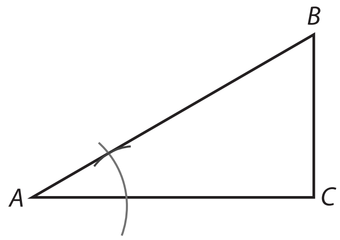
If two angles and a non-included side of one triangle are congruent to the corresponding angles and side of another triangle, are the triangles congruent?

In this activity you'll be copying a side and two angles from a triangle.

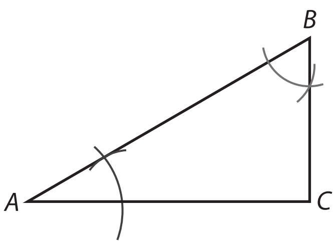
Ⓐ Use a compass and straightedge to copy segment *AC*. Label it as segment *EF*.



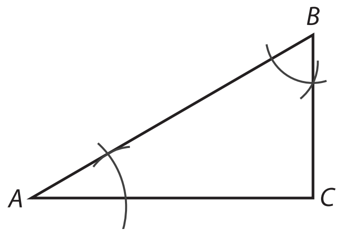
Ⓑ Copy $\angle A$ using $\overline{EF}$ as a side of the angle.



Ⓒ On a separate transparent sheet or a sheet of tracing paper, copy $\angle B$. Label its vertex *G*. Make the rays defining $\angle G$ longer than their corresponding sides on $\triangle ABC$.



Ⓓ Now overlay the ray from $\angle E$ with the ray from $\angle G$ to form a triangle. Make sure that side $\overline{EF}$ maintains the length you defined for it.



Ⓔ How many triangles can you construct?

Ⓕ Copy all of $\triangle EFG$ to the transparency. Then overlay it on $\triangle ABC$. Are the triangles congruent? How do you know?

Reflect

1. Suppose you had started this activity by copying segment BC and then angles A and C. Would your results have been the same? Why or why not?

2. Compare your results to those of your classmates. Does this procedure work with any triangle?

Explain 1 Justifying Angle-Angle-Side Congruence

The following theorem summarizes the previous activity.

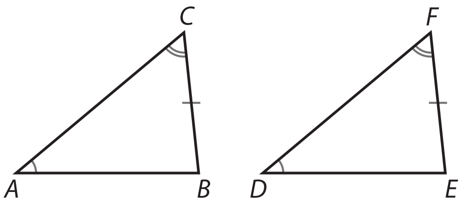
Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the triangles are congruent.

Prove the AAS Congruence Theorem.

Given: $\angle A \cong \angle D$, $\angle C \cong \angle F$, $\overline{BC} \cong \overline{EF}$

Prove: $\triangle ABC \cong \triangle DEF$



Statements	Reasons
1. $\angle A \cong \angle D$, $\angle C \cong \angle \square$, $\overline{BC} \cong \overline{EF}$	**1.** Given
2. $m\angle A + m\angle B + m\angle C = 180°$	**2.**
3. $m\angle B = 180° - m\angle A - m\angle \square$	**3.** Subtraction Property of Equality
4. $m\angle \square + m\angle E + m\angle F = 180°$	**4.** Triangle Sum Theorem
5. $m\angle E = 180° - m\angle D - m\angle \square$	**5.** Subtraction Property of Equality
6. $m\angle A = m\angle D$, $m\angle C = m\angle F$	**6.**
7. $m\angle E = 180° - m\angle A - m\angle C$	**7.**
8. $m\angle \square \cong m\angle B$	**8.** Transitive Property of Equality
9. $\angle B \cong m\angle E$	**9.**
10. $\triangle ABC \cong \triangle DEF$	**10.** Triangle Congruence Theorem

Reflect

3. **Discussion** The Third Angles Theorem says "If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent." How could using this theorem simplify the proof of the AAS Congruence Theorem?

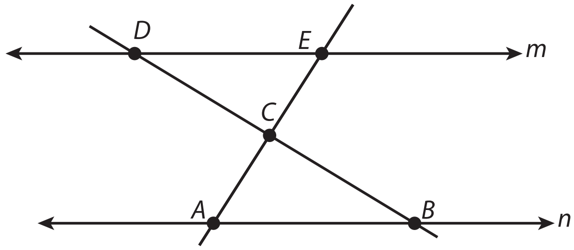
4. Could the AAS Congruence Theorem be used in the proof? Explain.

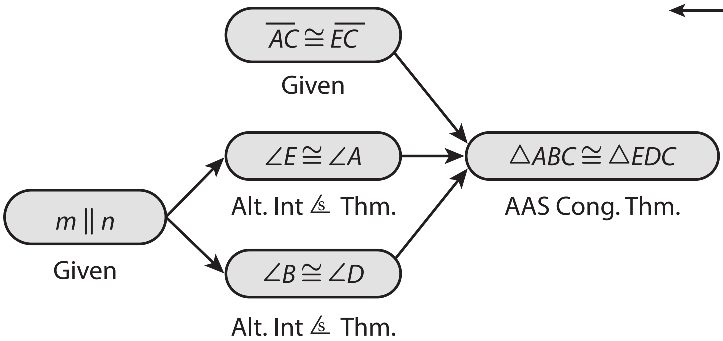
Explain 2 Using Angle-Angle-Side Congruence

Example 2 **Use the AAS Theorem to prove the given triangles are congruent.**

Ⓐ Given: $\overline{AC} \cong \overline{EC}$ and $m \| n$

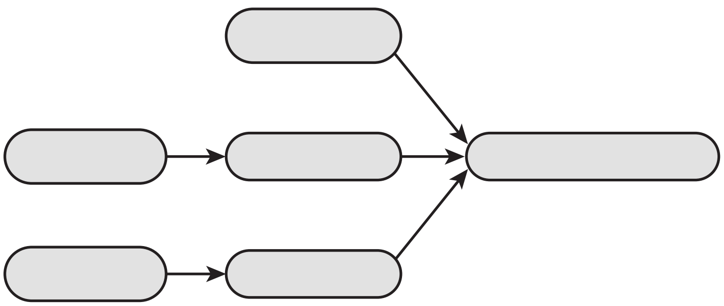
Prove: $\triangle ABC \cong \triangle EDC$

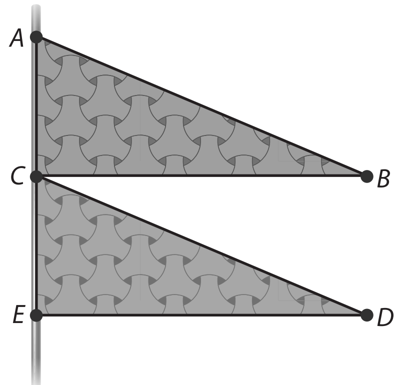




Ⓑ Given: $\overline{CB} \| \overline{ED}$, $\overline{AB} \| \overline{CD}$, and $\overline{CB} \cong \overline{ED}$.

Prove: $\triangle ABC \cong \triangle CDE$

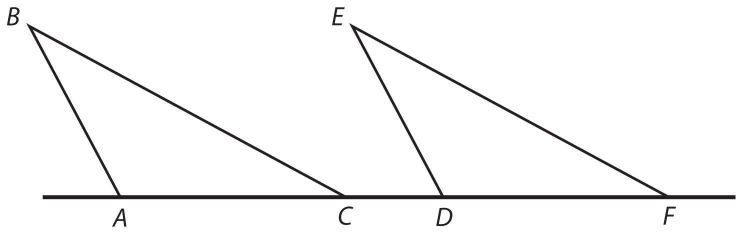




Your Turn

5. Given: $\angle ABC \cong \angle DEF$, $\overline{BC} \parallel \overline{EF}$, $\overline{AC} \cong \overline{DF}$. Use the AAS Theorem to prove the triangles are congruent.

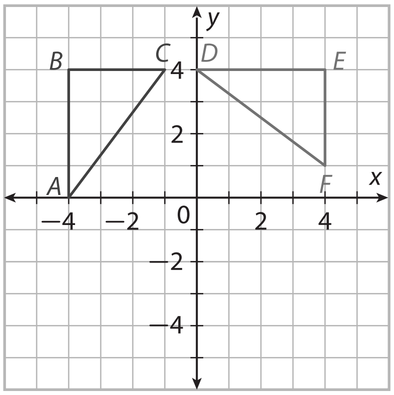
Write a paragraph proof.



Explain 3 Applying Angle-Angle-Side Congruence

Example 3 **The triangular regions represent plots of land. Use the AAS Theorem to explain why the same amount of fencing will surround either plot.**

Ⓐ Given: $\angle A \cong \angle D$



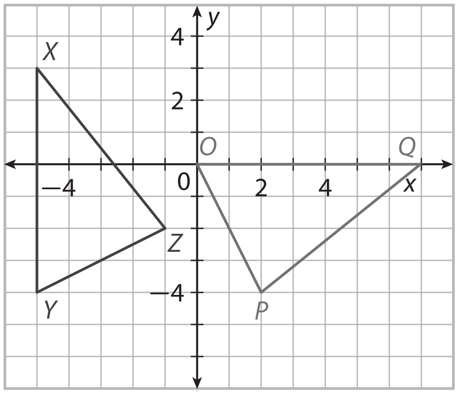
It is given that $\angle A \cong \angle D$. Also, $\angle B \cong \angle E$ because both are right angles. Compare AC and DF using the Distance Formula.

$$
\begin{aligned}
AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{\left(-1-(-4)\right)^2 + (4 - 0)^2} \\
&= \sqrt{3^2 + 4^2} \\
&= \sqrt{25} \\
&= 5
\end{aligned}
$$

$$
\begin{aligned}
DF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(4 - 0)^2 + (1 - 4)^2} \\
&= \sqrt{4^2 + (-3)^2} \\
&= \sqrt{25} \\
&= 5
\end{aligned}
$$

Because two pairs of angles and a pair of non-included sides are congruent, $\triangle ABC \cong \triangle DEF$ by AAS. Therefore the triangles have the same perimeter and the same amount of fencing is needed.

(B) Given: $\angle P \cong \angle Z$, $\angle Q \cong \angle X$



It is given that $\angle P \cong \angle Z$ and $\angle Q \cong \angle X$.

Compare YZ and ____ using the distance formula.

$$YZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left((-1) - \square\right)^2 + \left((-2) - \square\right)^2}$$

$$= \sqrt{\left(\square\right)^2 + \left(\square\right)^2}$$

$$= \sqrt{\square + \square}$$

$$= \sqrt{\square}$$

$$\_\_\_\_ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(\square - 0\right)^2 + \left(\square - 0\right)^2}$$

$$= \sqrt{\left(\square\right)^2 + \left(\square\right)^2}$$

$$= \sqrt{\square + \square}$$

$$= \sqrt{\square}$$

Because two pairs of angles and a pair of non-included sides are congruent,

$\triangle XYZ \cong \triangle$ $\square$ by AAS. Therefore the triangles have the same perimeter and the same amount of fencing is needed.

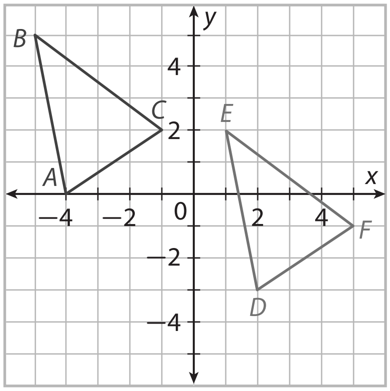
Reflect

6. Explain how you could have avoided using the distance formula in Example 2B.

Your Turn

Refer to the diagram to answer the questions.

Given: $\angle A \cong \angle D$ and $\angle B \cong \angle E$

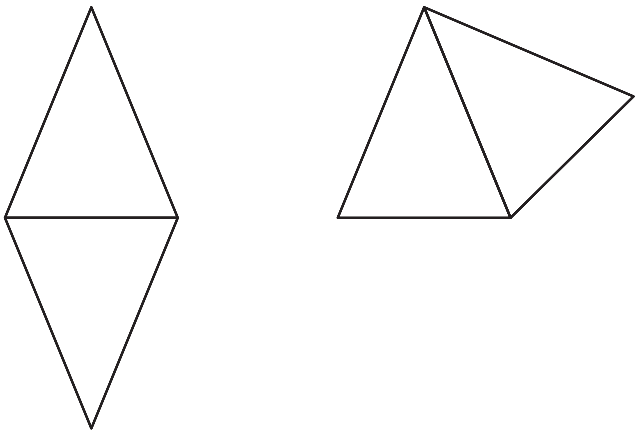


7. Show that the two triangles are congruent using the AAS Theorem. Use the distance formula to compare *BC* and *EF*.

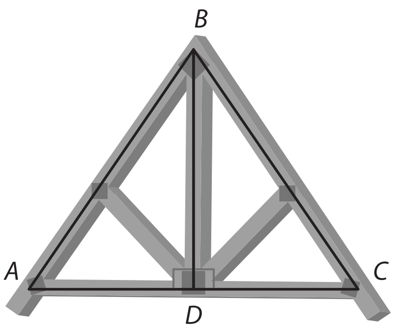
8. Show that the two triangles are congruent using the AAS Theorem. Use the distance formula to compare *AC* and *DF*.

Elaborate

9. Two isosceles triangles share a side. With which diagram can the AAS Theorem be used to show the triangles are congruent? Explain.



10. What must be true of the right triangles in the roof truss to use the AAS Congruence Theorem to prove the two triangles are congruent? Explain.



11. **Essential Question Check-In** You know that a pair of triangles has two pairs of congruent corresponding angles. What other information do you need to show that the triangles are congruent?

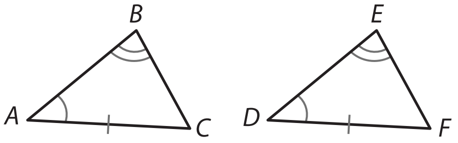
Evaluate: Homework and Practice

Personal Math Trainer

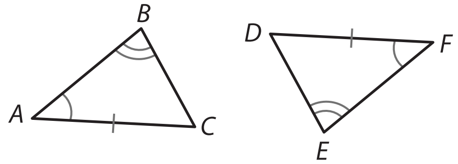
- Online Homework
- Hints and Help
- Extra Practice

Decide whether you have enough information to determine that the triangles are congruent. If they are congruent, explain why.

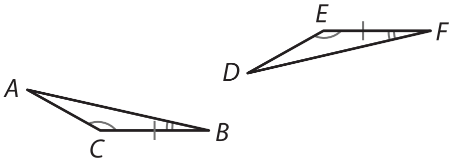
1.



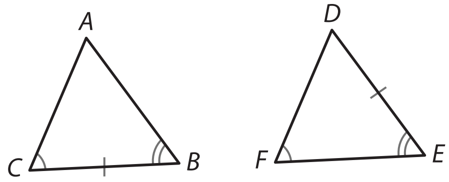
2.



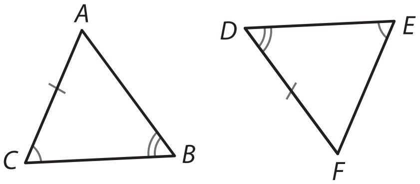
3.



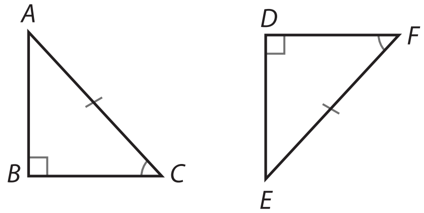
4.



5.

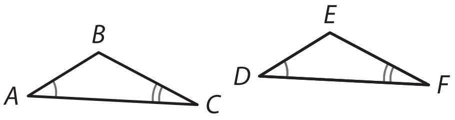


6.

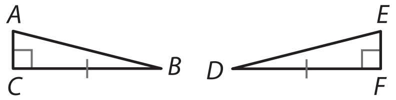


Each diagram shows two triangles with two congruent angles or sides. Identify one additional pair of corresponding angles or sides such that, if the pair were congruent, the two triangles could be proved congruent by AAS.

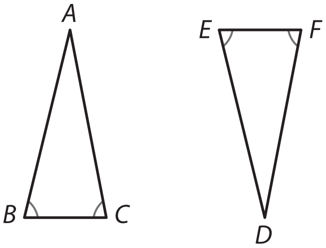
7.



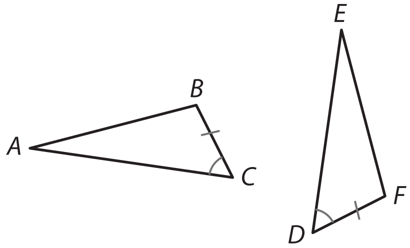
8.



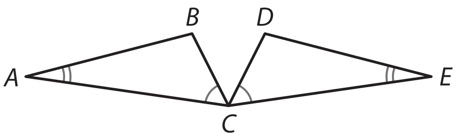
9.



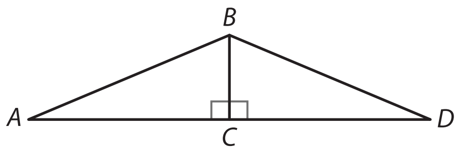
10.



11.



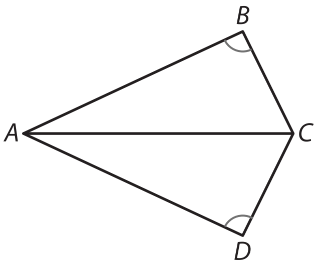
12.



13. Complete the proof.

Given: $\angle B \cong \angle D$, $\overleftrightarrow{AC}$ bisects $\angle BCD$.

Prove: $\triangle ABC \cong \triangle ADC$

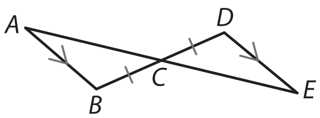


Statements	Reasons
1. $\overline{AC} \cong \overline{AC}$	**1.**
2. $\overleftrightarrow{AC}$ bisects $\angle BCD$.	**2.** Given
3.	**3.** Definition of angle bisector
4.	**4.** Given
5. $\triangle ABC \cong \triangle ADC$	**5.**

14. Write a two-column proof or a paragraph proof.

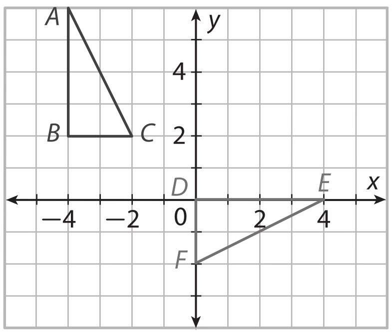
Given: $\overline{AB} \parallel \overline{DE}$, $\overline{CB} \cong \overline{CD}$.

Prove: $\triangle ABC \cong \triangle EDC$

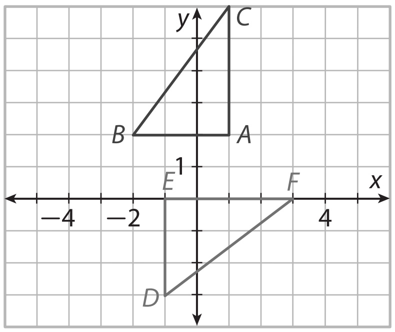


Each diagram shows $\triangle ABC$ and $\triangle DEF$ on the coordinate plane, with $\angle A \cong \angle E$, and $\angle C \cong \angle F$. Identify whether the two triangles are congruent. If they are not congruent, explain how you know. If they are congruent, find the length of each side of each triangle.

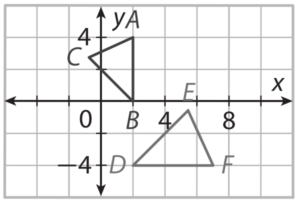
15.



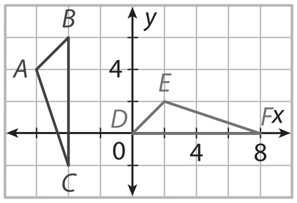
16.



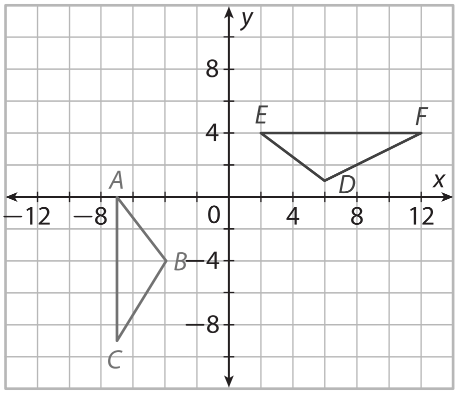
17.



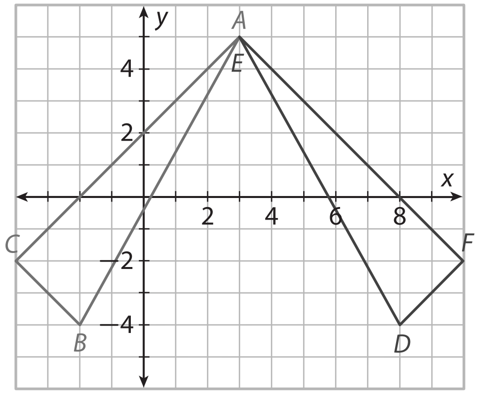
18.



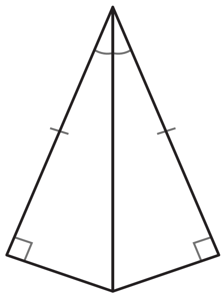
19.



20.



21. Which theorem or postulate can be used to prove that the triangles are congruent? Select all that apply.



A. ASA **B.** SAS **C.** SSS **D.** AAS

H.O.T. Focus on Higher Order Thinking

22. Analyze Relationships $\triangle XYZ$ and $\triangle KLM$ have two congruent angles: $\angle X \cong \angle K$ and $\angle Y \cong \angle L$. Can it be concluded that $\angle Z \cong \angle M$? Can it be concluded that the triangles are congruent? Explain.

23. Communicate Mathematical Ideas $\triangle GHJ$ and $\triangle PQR$ have two congruent angles: $\angle G \cong \angle P$ and $\angle H \cong \angle Q$. If $\overline{HJ}$ is congruent to one of the sides of $\triangle PQR$, are the two triangles congruent? Explain.

24. **Make a Conjecture** Combine the theorems of ASA Congruence and AAS Congruence into a single statement that describes a condition for congruency between triangles.

25. **Justify Reasoning** Triangles *ABC* and *DEF* are constructed with the following angles: $m\angle A = 35°$, $m\angle B = 45°$, $m\angle D = 65°$, $m\angle E = 45°$. Also, $AC = DF = 12$ units. Are the two triangles congruent? Explain.

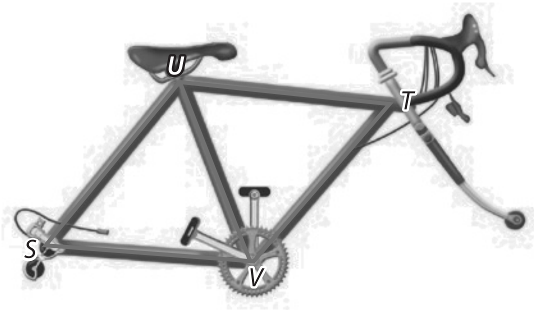
26. **Justify Reasoning** Triangles *ABC* and *DEF* are constructed with the following angles: $m\angle A = 65°$, $m\angle B = 60°$, $m\angle D = 65°$, $m\angle F = 55°$. Also, $AB = DE = 7$ units. Are the two triangles congruent? Explain.

27. **Algebra** A bicycle frame includes $\triangle VSU$ and $\triangle VTU$, which lie in intersecting planes. From the given angle measures, can you conclude that $\triangle VSU \cong \triangle VTU$? Explain.

$m\angle VUS = (7y - 2)°$ $\qquad$ $m\angle VUT = \left(5\frac{1}{2}x - \frac{1}{2}\right)°$

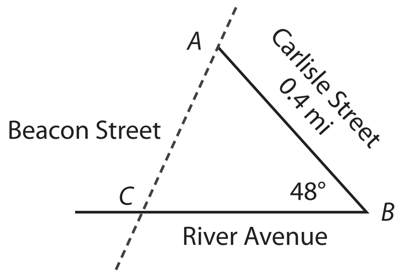
$m\angle USV = 5\frac{2}{3}y°$ $\qquad$ $m\angle UTV = (4x + 8)°$

$m\angle SVU = (3y - 6)°$ $\qquad$ $m\angle TVU = 2x°$



Lesson Performance Task

A mapmaker has successfully mapped Carlisle Street and River Avenue, as shown in the diagram. The last step is to map Beacon Street correctly. To save time, the mapmaker intends to measure just one more angle or side of the triangle.



a. Which angle(s) or side(s) could the mapmaker measure to be sure that only one triangle is possible? For each angle or side that you name, justify your answer.

b. Suppose that instead of measuring the length of Carlisle Street, the mapmaker measured $\angle A$ and $\angle C$ along with $\angle B$. Would the measures of the three angles alone assure a unique triangle? Explain.