Name _____ Class _____ Date _____

6.3 HL Triangle Congruence

Essential Question: **What does the HL Triangle Congruence Theorem tell you about two triangles?**

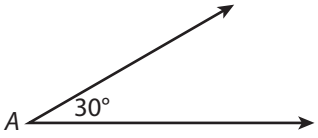
Resource Locker

Explore Is There a Side-Side-Angle Congruence Theorem?

You have already seen several theorems for proving that triangles are congruent. In this Explore, you will investigate whether there is a SSA Triangle Congruence Theorem.

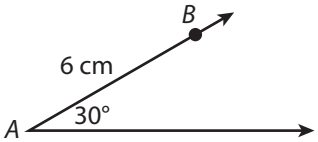
Follow these steps to draw $\triangle ABC$ such that $m\angle A = 30°$, $AB = 6$ cm, and $BC = 4$ cm. The goal is to determine whether two side lengths and the measure of a non-included angle (SSA) determine a unique triangle.

A Use a protractor to draw a large 30° angle on a separate sheet of paper. Label it $\angle A$.



B Use a ruler to locate point B on one ray of $\angle A$ so that $AB = 6$ cm.

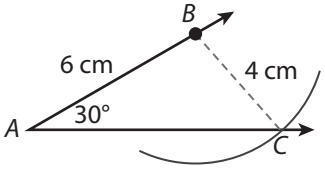
C Now draw $\overline{BC}$ so that $BC = 4$ cm. To do this, open a compass to a distance of 4 cm. Place the point of the compass on point B and draw an arc. Plot point C where the arc intersects the side of $\angle A$. Draw $\overline{BC}$ to complete $\triangle ABC$.



D What do you notice? Is it possible to draw only one $\triangle ABC$ with the given side length? Explain.

_____

_____



Reflect

1. Do you think that SSA is sufficient to prove congruence? Why or why not?

_____

_____

_____

2. **Discussion** Your friend said that there is a special case where SSA can be used to prove congruence. Namely, when the non-included angle was a right angle. Is your friend right? Explain.

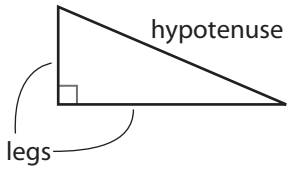
_____

_____

_____

Explain 1 Justifying the Hypotenuse-Leg Congruence Theorem

In a right triangle, the side opposite the right angle is the **hypotenuse**. The two sides that form the sides of the right angle are the **legs**.



You have learned four ways to prove that triangles are congruent.

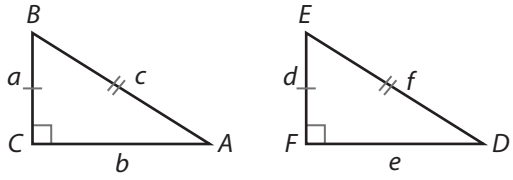
- Angle-Side-Angle (ASA) Congruence Theorem
- Side-Angle-Side (SAS) Congruence Theorem
- Side-Side-Side (SSS) Congruence Theorem
- Angle-Angle-Side (AAS) Congruence Theorem

The Hypotenuse-Leg (HL) Triangle Congruence Theorem is a special case that allows you to show that two right triangles are congruent.

Hypotenuse-Leg (HL) Triangle Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

Example 1 **Prove the HL Triangle Congruence Theorem.**



Given: $\triangle ABC$ and $\triangle DEF$ are right triangles; $\angle C$ and $\angle F$ are right angles.

$\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$

Prove: $\triangle ABC \cong \triangle DEF$

By the Pythagorean Theorem, $a^2 + b^2 = c^2$ and $\square^2 + \square^2 = f^2$. It is given that $\overline{AB} \cong \overline{DE}$, so $AB = DE$ and $c = f$. Therefore, $c^2 = f^2$ and $a^2 + b^2 = \square^2 + \square^2$. It is given that $\overline{BC} \cong \overline{EF}$, so $BC = EF$ and $a = d$. Substituting a for d in the above equation, $a^2 + b^2 = \square^2 + \square^2$.

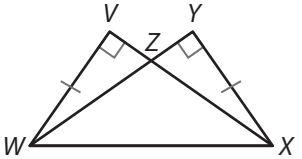
Subtracting a^2 from each side shows that $b^2 = \square^2$, and taking the square root of each side, $b = \square$.

This shows that $\overline{AC} \cong \square$.

Therefore, $\triangle ABC \cong \triangle DEF$ by _____.

Your Turn

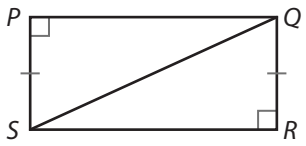
3. Determine whether there is enough information to prove that triangles $\triangle VWX$ and $\triangle YXW$ are congruent. Explain.



Explain 2 Applying the HL Triangle Congruence Theorem

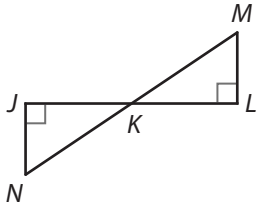
Example 2 **Use the HL Congruence Theorem to prove that the triangles are congruent.**

(A) Given: $\angle P$ and $\angle R$ are right angles. $\overline{PS} \cong \overline{RQ}$
Prove: $\triangle PQS \cong \triangle RSQ$



Statements	Reasons
1. $\angle P$ and $\angle R$ are right angles.	**1.** Given
2. $\overline{PS} \cong \overline{RQ}$	**2.** Given
3. $\overline{SQ} \cong \overline{SQ}$	**3.** Reflexive Property of Congruence
4. $\triangle PQS \cong \triangle RSQ$	**4.** HL Triangle Congruence Theorem

(B) Given: $\angle J$ and $\angle L$ are right angles. K is the midpoint of $\overline{JL}$ and $\overline{MN}$.
Prove: $\triangle JKN \cong \triangle LKM$



Statements	Reasons
1. $\angle J$ and $\angle L$ are right angles.	**1.**
2. K is the midpoint of $\overline{JL}$ and $\overline{MN}$.	**2.**
3. $\overline{JK} \cong \overline{LK}$ and $\overline{MK} \cong \overline{NK}$	**3.**
4. $\triangle JKN \cong \triangle LKM$	**4.**

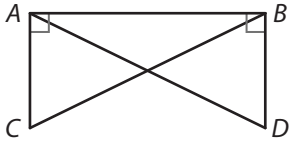
Reflect

4. Is it possible to write the proof in Part B without using the HL Triangle Congruence Theorem? Explain.

Your Turn

Use the HL Congruence Theorem to prove that the triangles are congruent.

5. Given: $\angle CAB$ and $\angle DBA$ are right angles. $\overline{AD} \cong \overline{BC}$
Prove: $\triangle ABC \cong \triangle BAD$



Elaborate

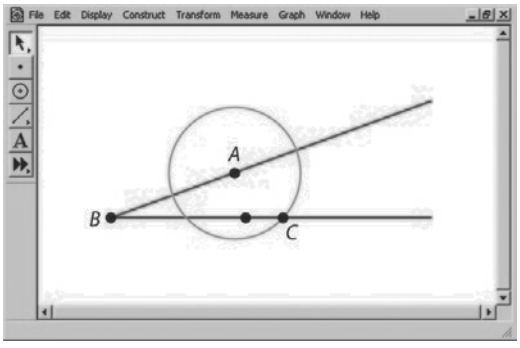
6. You draw a right triangle with a hypotenuse that is 5 inches long. A friend also draws a right triangle with a hypotenuse that is 5 inches long. Can you conclude that the triangles are congruent using the HL Congruence Theorem? If not, what else would you need to know in order to conclude that the triangles are congruent?

7. **Essential Question Check-In** How is the HL Triangle Congruence Theorem similar to and different from the ASA, SAS, SSS, and AAS Triangle Congruence Theorems?

Evaluate: Homework and Practice

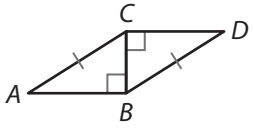
- Online Homework
- Hints and Help
- Extra Practice

1. Tyrell used geometry software to construct $\angle ABC$ so that $m\angle ABC = 20°$. Then he dragged point A so that $AB = 6$ cm. He used the software's compass tool to construct a circle centered at point A with radius 3 cm. Based on this construction, is there a unique $\triangle ABC$ with $m\angle ABC = 20°$, $AB = 6$ cm, and $AC = 3$ cm? Explain.

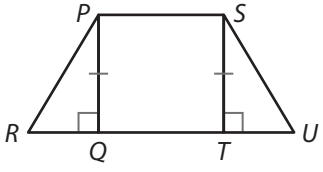


Determine whether enough information is given to prove that the triangles are congruent. Explain your answer.

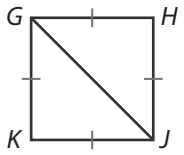
2. $\triangle ABC$ and $\triangle DCB$



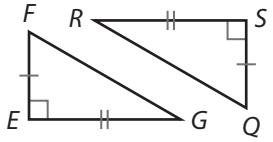
3. $\triangle PQR$ and $\triangle STU$



4. $\triangle GKJ$ and $\triangle JHG$

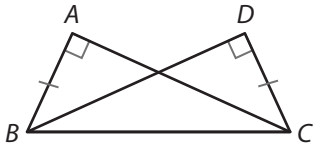


5. $\triangle EFG$ and $\triangle SQR$

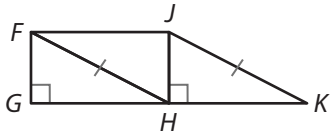


Write a two-column proof, using the HL Congruence Theorem, to prove that the triangles are congruent.

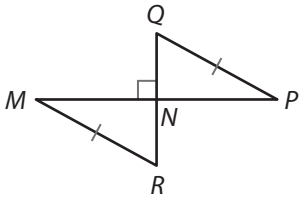
6. Given: $\angle A$ and $\angle B$ are right angles. $\overline{AB} \cong \overline{DC}$
Prove: $\triangle ABC \cong \triangle DCB$



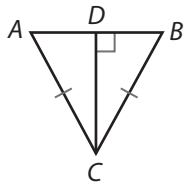
7. Given: $\angle FGH$ and $\angle JHK$ are right angles.
H is the midpoint of $\overline{GK}$. $\overline{FH} \cong \overline{JK}$
Prove: $\triangle FGH \cong \triangle JHK$



8. Given: $\overline{MP}$ is perpendicular to $\overline{QR}$.
N is the midpoint of $\overline{MP}$. $\overline{QP} \cong \overline{RM}$
Prove: $\triangle MNR \cong \triangle PNQ$

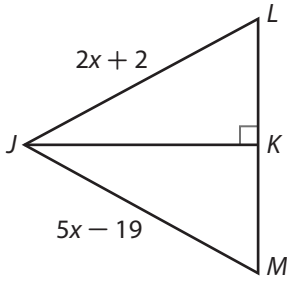


9. Given: $\angle ADC$ and $\angle BDC$ are right angles. $\overline{AC} \cong \overline{BC}$
Prove: $\overline{AD} \cong \overline{BD}$

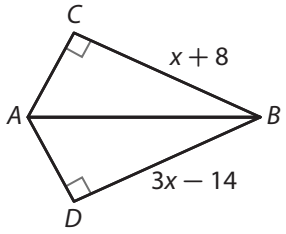


Algebra **What value of x will make the given triangles congruent? Explain.**

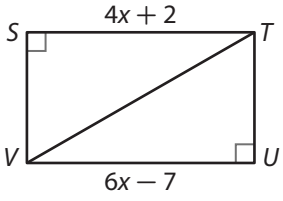
10. $\triangle JKL$ and $\triangle JKM$



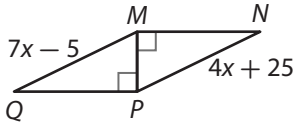
11. $\triangle ABC$ and $\triangle ABD$



12. $\triangle STV$ and $\triangle UVT$

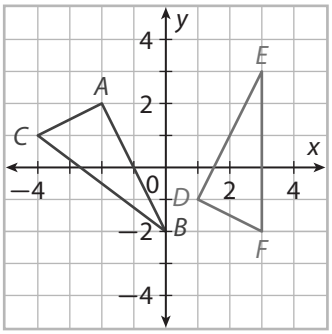


13. $\triangle MPQ$ and $\triangle PMN$

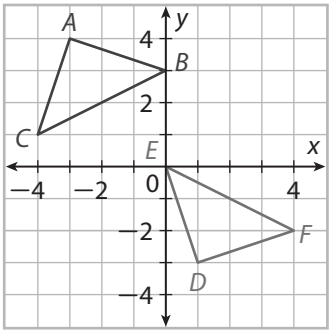


Algebra **Use the HL Triangle Congruence Theorem to show that $\triangle ABC \cong \triangle DEF$. (*Hint*: Use the Distance Formula to show that appropriate sides are congruent. Use the slope formula to show that appropriate angles are right angles.)**

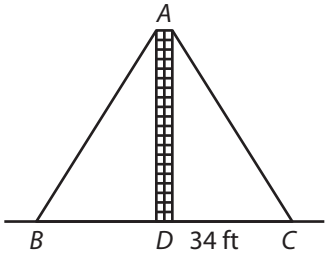
14.



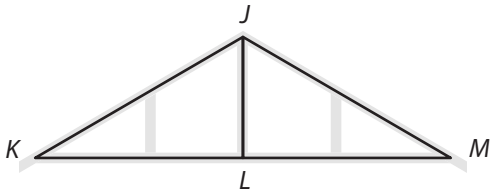
15.



16. **Communicate Mathematical Ideas** A vertical tower is supported by two guy wires, as shown. The guy wires are both 58 feet long. Is it possible to determine the distance from the bottom of guy wire $\overline{AB}$ to the bottom of the tower? If so, find the distance. If not, explain why not.



17. A carpenter built a truss, as shown, to support the roof of a doghouse.

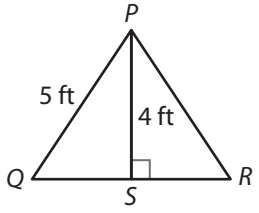


a. The carpenter knows that $\overline{KJ} \cong \overline{MJ}$. Can the carpenter conclude that $\triangle KJL \cong \triangle MJL$? Why or why not?

b. **What If?** Suppose the carpenter also knows that $\angle JLK$ is a right angle. Can the carpenter now conclude that $\triangle KJL \cong \triangle MJL$? Explain.

18. **Counterexamples** Denise said that if two right triangles share a common hypotenuse, then the triangles must be congruent. Sketch a figure that serves as a counterexample to show that Denise's statement is not true.

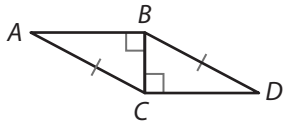
19. **Multi-Step** The front of a tent is covered by a triangular flap of material. The figure represents the front of the tent, with $\overline{PS} \perp \overline{QR}$ and $\overline{PQ} \cong \overline{PR}$. Jonah needs to determine the perimeter of $\triangle PQR$ so that he can replace the zipper on the tent. Find the perimeter. Explain your steps.



20. A student is asked to write a two-column proof for the following.

Given: $\angle ABC$ and $\angle DCB$ are right angles. $\overline{AC} \cong \overline{BD}$

Prove: $\overline{AB} \cong \overline{DC}$



Assuming the student writes the proof correctly, which of the following will appear as a statement or reason in the proof? Select all that apply.

A. ASA Triangle Congruence Theorem

B. $\overline{BC} \cong \overline{BC}$

C. $\angle A \cong \angle D$

D. Reflexive Property of Congruence

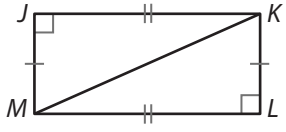
E. CPCTC

F. HL Triangle Congruence Theorem

H.O.T. Focus on Higher Order Thinking

21. **Analyze Relationships** Is it possible for a right triangle with a leg that is 10 inches long and a hypotenuse that is 26 inches long to be congruent to a right triangle with a leg that is 24 inches long and a hypotenuse that is 26 inches long? Explain.

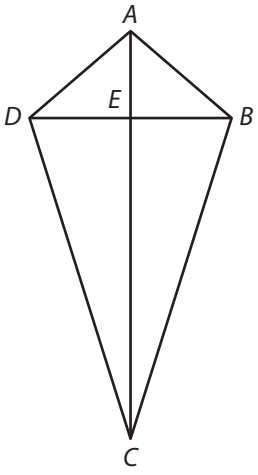
22. **Communicate Mathematical Ideas** In the figure, $\overline{JK} \cong \overline{LM}$, $\overline{JM} \cong \overline{LK}$, and $\angle J$ and $\angle L$ are right angles. Describe how you could use three different congruence theorems to prove that $\triangle JKM \cong \triangle LMK$.



23. **Justify Reasoning** Do you think there is an LL Triangle Congruence Theorem? That is, if the legs of one right triangle are congruent to the legs of another right triangle, are the triangles necessarily congruent? If so, write a proof of the theorem. If not, provide a counterexample.

Lesson Performance Task

The figure shows kite *ABCD*.



a. What would you need to know about the relationship between $\overline{AC}$ and $\overline{DB}$ in order to prove that $\triangle ADE \cong \triangle ABE$ and $\triangle CDE \cong \triangle CBE$ by the HL Triangle Congruence Theorem?

b. Can you prove that $\triangle ADC$ and $\triangle ABC$ are congruent using the HL Triangle Congruence Theorem? Explain why or why not.

c. How can you prove that the two triangles named in Part b are in fact congruent, even without the additional piece of information?