

8.2 Relative Frequency



Resource Locker

Essential Question: How can you recognize possible associations and trends between two categories of categorical data?

Explore Relative Frequencies

To show what portion of a data set each category in a frequency table makes up, you can convert the data to *relative frequencies*. The **relative frequency** of a category is the frequency of the category divided by the total of all frequencies.



The frequency table below shows the results of a survey Kenesha conducted at school. She asked 80 randomly selected students whether they preferred basketball, football, or soccer.

Favorite Sport	Basketball	Football	Soccer	Total
Frequency	20	32	28	80

A Use the frequencies to make a relative frequency table expressed with decimals.

Favorite Sport	Basketball	Football	Soccer	Total
Relative Frequency	$\frac{20}{80} = 0.25$			$\frac{80}{80} = \square$

B Rewrite the relative frequency table using percents instead of decimals.

Favorite Sport	Basketball	Football	Soccer	Total
Relative Frequency	25%			

Reflect

1. Explain what the numerator and denominator of the ratio $\frac{20}{80}$ refer to in part A.

2. What types of numbers can you use to write relative frequencies?

Explain 1 Two-Way Relative Frequency Tables

Two types of relative frequencies are found in a relative frequency table:

1. A **joint relative frequency** is found by dividing a frequency that is not in the Total row or the Total column by the grand total. It tells what portion of the total has both of the two specified characteristics.
2. A **marginal relative frequency** is found by dividing a row total or a column total by the grand total. It tells what portion of the total has a specified characteristic.

Example 1 Complete a two-way relative frequency table from the data in a two-way frequency table. Identify the joint relative frequencies and the marginal relative frequencies.

- A** For her survey about sports preferences, Kenesha also recorded the gender of each student. The results are shown in the two-way frequency table for Kenesha's data.

	Preferred Sport			
Gender	Basketball	Football	Soccer	Total
Girl	6	12	18	36
Boy	14	20	10	44
Total	20	32	28	80

To find the relative frequencies, divide each frequency by the grand total.

	Preferred Sport			
Gender	Basketball	Football	Soccer	Total
Girl	$\frac{6}{80} = 0.075$	$\frac{12}{80} = 0.15$	$\frac{18}{80} = 0.225$	$\frac{36}{80} = 0.45$
Boy	$\frac{14}{80} = 0.175$	$\frac{20}{80} = 0.25$	$\frac{10}{80} = 0.125$	$\frac{44}{80} = 0.55$
Total	$\frac{20}{80} = 0.25$	$\frac{32}{80} = 0.4$	$\frac{28}{80} = 0.35$	$\frac{80}{80} = 1$

The joint relative frequencies tell what percent of all those surveyed are in each category:

- 7.5% are girls who prefer basketball.
- 15% are girls who prefer football.
- 22.5% are girls who prefer soccer.
- 17.5% are boys who prefer basketball.
- 25% are boys who prefer football.
- 12.5% are boys who prefer soccer.

The marginal relative frequencies tell what percent of totals has a given single characteristic:

- 25% prefer basketball.
- 40% prefer football.
- 35% prefer soccer.
- 45% are girls.
- 55% are boys.

- B Millie performed a survey of students in the lunch line and recorded which type of fruit each student selected along with the gender of each student. The two-variable frequency data she collected is shown in the table.

	Fruit			
	Apple	Banana	Orange	Total
Girl	16	10	14	40
Boy	25	13	14	52
Total	41	23	28	92

	Fruit			
	Apple	Banana	Orange	Total
Girl	17.4%			
Boy	27.2%			
Total	44.6%			

The joint relative frequencies:

- are girls who selected an apple.
- are girls who selected a banana.
- are girls who selected an orange.
- are boys who selected an apple.
- are boys who selected a banana.
- are boys who selected an orange.

The marginal relative frequencies:

- selected an apple.
- selected a banana.
- selected an orange.
- are girls.
- are boys.

Reflect

3. **Discussion** Explain how you can use joint and marginal relative frequencies to check your relative frequency table.

Your Turn

Use the two-way table of data from another student survey to answer the following questions.

		Like Aerobic Exercise		
Like Weight Lifting		Yes	No	Total
Yes		7	14	21
No		12	7	19
Total		19	21	40

- Find the joint relative frequency of students surveyed who like aerobics exercise but dislike weight lifting.
- What is the marginal relative frequency of students surveyed who like weight lifting?

Explain 2 Conditional Relative Frequencies

A **conditional relative frequency** describes what portion of a group with a given characteristic also has another characteristic. A conditional relative frequency is found by dividing a frequency that is not in the Total row or the Total column by the total for that row or column.

Example 2 Use the joint relative frequencies to calculate the associated conditional relative frequencies and describe what each one means.

- A** Use the data from Example 1A. Find the conditional relative frequency that a person in Kenesha's survey prefers soccer, given that the person is a girl.

Divide the number of girls who prefer soccer by the total number of girls.

$$\frac{\text{Number of girls who prefer soccer}}{\text{Total number of girls}} = \frac{18}{36} = 0.5 = 50\%$$

Half of the girls in the sample prefer soccer.

- B** Use the data from Example 1B. Find the conditional relative frequency that a student in Millie's survey chose an orange, given that the student is a boy.

$$\frac{\text{Number of } \boxed{} \text{ who chose an orange}}{\text{Total number of } \boxed{}} = \frac{\boxed{}}{\boxed{}} \approx 0.269 = \boxed{}\%$$

Your Turn

Use the data from Your Turn Exercises 4 and 5 after Example 1.

- What is the conditional relative frequency that a student likes to lift weights, given that the student does not like aerobics?
- Find the conditional relative frequency that a student likes to lift weights, given that the student likes aerobics.

Explain 3 Finding Possible Associations

You can analyze two-way frequency tables to locate possible associations or patterns in the data.

Example 3 Analyze the results of the surveys to determine preferences by gender.

Kenesha is interested in the question, “Does gender influence what type of sport students prefer?” If there is no influence, then the distribution of gender within each sport preference will roughly equal the distribution of gender within the whole group. Analyze the results of Kenesha’s survey from Example 1. Determine which sport each gender is more likely to prefer.

A Analyze the data about girls that were surveyed.

Step 1: Identify the percent of all students surveyed who are girls.

$$\frac{36}{80} = 0.45 = 45\%$$

Step 2: Determine each conditional relative frequency.

Basketball	Football	Soccer
Of the 20 students who prefer basketball, 6 are girls. $\frac{6}{20} = 0.3 = 30\%$	Of the 32 students who prefer football, 12 are girls. $\frac{12}{32} = 0.375 = 37.5\%$	Of the 28 students who prefer soccer, 18 are girls. $\frac{18}{28} \approx 0.643 = 64.3\%$

Step 3: Interpret the results by comparing each conditional relative frequency to the percent of all students surveyed who are girls, 45%.

Basketball	Football	Soccer
30% < 45% Girls are less likely to prefer basketball.	37.5% < 45% Girls are less likely to prefer football.	64.3% > 45% Girls are more likely to prefer soccer.

B Analyze the data about boys that were surveyed.

Step 1: Identify the percent of all students surveyed who are boys.

$$\frac{\square}{80} = 0.\square = \square\%$$

Step 2: Determine each conditional relative frequency.

Basketball	Football	Soccer
Of the 20 students who prefer basketball, \square are boys. $\frac{\square}{20} = 0.\square = \square\%$	Of the \square students who prefer football, \square are boys. $\frac{\square}{\square} = 0.\square = \square\%$	Of the \square students who prefer soccer, \square are boys. $\frac{\square}{\square} = 0.\square = \square\%$

Step 3: Interpret the results by comparing each conditional relative frequency to the percent of all students surveyed who are boys, %.

Basketball	Football	Soccer
70% > 55%	62.5% > 55%	35.7% < 55%
Boys are more likely to prefer basketball.	Boys are more likely to prefer football.	Boys are less likely to prefer soccer.

Reflect

8. Making Connections How can the statement “6 out of the 20 students who prefer basketball are girls” be stated as a conditional relative frequency?

Your Turn

9. Analyze the data given in the Your Turn after Example 1 to determine if liking aerobic exercise influences whether a person also likes weight lifting. Explain.

 **Elaborate**

10. What does it mean to say there is an association between characteristics in a two-way frequency table?

11. Essential Question Check-In How can you use two-way frequency data to recognize possible associations between the two categories of categorical data?

Evaluate: Homework and Practice



- Online Homework
- Hints and Help
- Extra Practice

Use the table of frequency data for Exercises 1–4.

Class Survey of Favorite Colors

Favorite Color	Red	Orange	Yellow	Green	Blue	Purple	Total
Frequency	2	5	1	6	8	2	24

1. Complete the relative frequency table for this data using decimals rounded to the nearest thousandth.

Class Survey of Favorite Colors

Favorite Color	Red	Orange	Yellow	Green	Blue	Purple	Total
Relative Frequency							

2. Complete the relative frequency table for this data using percents rounded to the nearest tenth.

Class Survey of Favorite Colors

Favorite Color	Red	Orange	Yellow	Green	Blue	Purple	Total
Relative Frequency							

3. What is the relative frequency of having blue as a favorite color, expressed as a decimal? 4. Which color is a favorite color with a relative frequency of 25%?

The following frequency data shows the number of states, including the District of Columbia, that favored each party in the presidential popular vote in 1976 and in 2012.

	2012 Election		
1976 Election	Democrat	Republican	Total
Democrat	12 = <input type="text"/>	12 = <input type="text"/>	24 = <input type="text"/>
Republican	15 = <input type="text"/>	12 = <input type="text"/>	27 = <input type="text"/>
Total	27 = <input type="text"/>	24 = <input type="text"/>	51 = <input type="text"/>

5. Complete the table above with relative frequencies using percents.
6. What percent switched from Democrat in 1976 to Republican in 2012? What type of frequency is this?
7. What percent voted Republican in 1976? What type of frequency is this?

The results of a survey of 45 students and the foreign language they are studying are shown in the two-way frequency table.

Gender	Language			Total
	Chinese	French	Spanish	
Girl	2	8	15	25
Boy	4	4	12	20
Total	6	12	27	45

8. Fill in the table of two-way relative frequencies using decimals, rounded to the nearest thousandth.

Gender	Language			Total
	Chinese	French	Spanish	
Girl				
Boy				
Total				

9. What fraction of the surveyed students are boys taking Spanish? 10. What fraction of the surveyed students are taking Chinese?

In some states, a driver of a vehicle may not use a handheld cell phone while driving. In one state with this law, 250 randomly selected drivers were surveyed to determine the association between drivers who know the law and drivers who obey the law.

The results are shown in the table below.

11. Complete the table of two-way relative frequencies using percents.

Obeys the Law	Knows the Law		Total
	Yes	No	
Yes	160 =	45 =	
No	25 =	20 =	
Total			



12. What is the relative frequency of drivers who know and obey the law? 13. What is the relative frequency of drivers who know the law?

© Houghton Mifflin Harcourt Publishing Company. Image credit: © Leena Robinson/Shutterstock

Refer to the election data from Exercises 5–7. Answer using percents rounded to the nearest tenth.

- 14.** What is the conditional relative frequency of a state's popular vote being won by the Democrat in 2012, given that it was won by the Democrat in 1976?
- 15.** What is the conditional relative frequency of a state's popular vote being won by the Democrat in 1976, given that it was won by the Democrat in 2012?

Refer to the language data from Exercises 8–10. Answer using decimals rounded to the nearest thousandth.

- 16.** What fraction of girls are studying French? **17.** What fraction of Spanish students are boys?

Refer to the cell phone law data from Exercises 11–13. Answer using percents rounded to the nearest tenth.

- 18.** What percent of drivers obey the law despite not knowing the law?
- 19.** What is the conditional relative frequency of drivers who obey the law, given that they know the law?

Use the previously described data to determine whether there are associations between the categories surveyed.

- 20.** Refer to the election data from Exercises 5–7. Is there an association between the party that won the popular vote in a state in 1976 and in 2012?
- 21.** Refer to the language data from Exercises 8–10. Can you use gender to predict a preference for taking Spanish?

22. Refer to the language data from Exercises 8–10. Is there an association between gender and a preference for French?

23. Refer to the cell phone law data from Exercises 11–13. Most drivers who don't know that it is illegal to operate a cell phone while driving obey the law anyway, presumably out of a general concern for safe driving. Does this mean there is no association between knowledge of the cell phone law and obeying the cell phone law?

24. Multipart Classification Classify each statement as describing a *joint*, *marginal*, or *conditional* relative frequency.

- a.** In a study on age and driving safety, 33% of drivers were considered younger and a high accident risk.
- b.** In a study on age and driving safety, 45% of older drivers were considered a high accident risk.
- c.** In a study on age and driving safety, 67% of drivers were classified as younger.
- d.** In a pre-election poll, 67% of the respondents who preferred the incumbent were men.
- e.** In a pre-election poll, 33% of women preferred the challenger.
- f.** In a pre-election poll, 16% of respondents were men who preferred the challenger.

H.O.T. Focus on Higher Order Thinking

- 25. Explain the Error** In the survey on gender and fruit selection (Example 1B), Millicent notices that given a preference for oranges, the conditional relative frequencies of a student being a boy or a girl are the same. She concludes that there is no association between gender and orange preference. Explain her error.
- 26. Communicate Mathematical Ideas** Can a joint relative frequency be greater than either of the conditional relative frequencies associated with it? Explain your reasoning.
- 27. Explain the Error** Refer to the cell phone data from Exercises 11–13. Cole found the conditional relative frequency that a driver surveyed does not know the law, given that the driver obeys the law, by dividing 45 by 250. Explain Cole's error.

Lesson Performance Task

Eighty students were surveyed about playing an instrument. The results are shown in the two-way frequency table.

Gender	Play an Instrument		Total
	Yes	No	
Female	28	17	45
Male	20	15	35
Total	48	32	80

- a. Complete the two-way relative frequency table for the data.

Gender	Play an Instrument		Total
	Yes	No	
Female			
Male			
Total			

- b. What percent of the students surveyed play an instrument?
What percent of the males surveyed do not play an instrument? Identify what type of frequency each percent is.
- c. Is there an association between the sex of a student and whether the student plays an instrument? Explain.