Name ______________________ Class ____________ Date ________

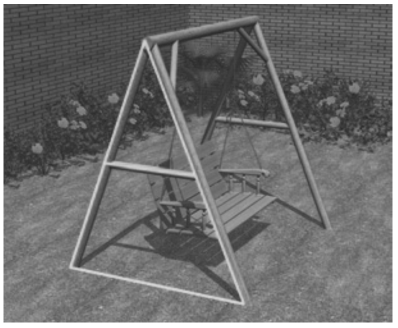
8.4 Midsegments of Triangles

Essential Question: How are the segments that join the midpoints of a triangle's sides related to the triangle's sides?

Resource Locker

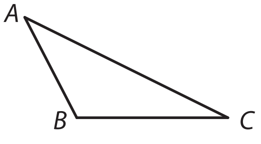
Explore Investigating Midsegments of a Triangle

The **midsegment** of a triangle is a line segment that connects the midpoints of two sides of the triangle. Every triangle has three midsegments. Midsegments are often used to add rigidity to structures. In the support for the garden swing shown, the crossbar $\overline{DE}$ is a midsegment of $\triangle ABC$

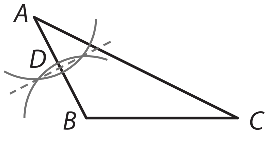


You can use a compass and straightedge to construct the midsegments of a triangle.

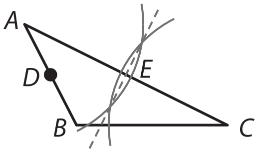
(A) Sketch a scalene triangle and label the vertices A, B, and C.



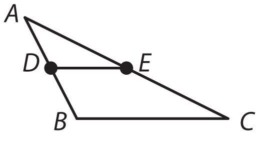
(B) Use a compass to find the midpoint of $\overline{AB}$. Label the midpoint D.



(C) Use a compass to find the midpoint of $\overline{AC}$. Label the midpoint E.



(D) Use a straightedge to draw $\overline{DE}$. $\overline{DE}$ is one of the midsegments of the triangle.



(E) Repeat the process to find the other two midsegments of $\triangle ABC$. You may want to label the midpoint of $\overline{BC}$ as F.

Reflect

1. Use a ruler to compare the length of $\overline{DE}$ to the length of $\overline{BC}$. What does this tell you about $\overline{DE}$ and $\overline{BC}$?

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2. Use a protractor to compare $m\angle ADE$ and $m\angle ABC$. What does this tell you about $\overline{DE}$ and $\overline{BC}$? Explain.

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3. Compare your results with your class. Then state a conjecture about a midsegment of a triangle.

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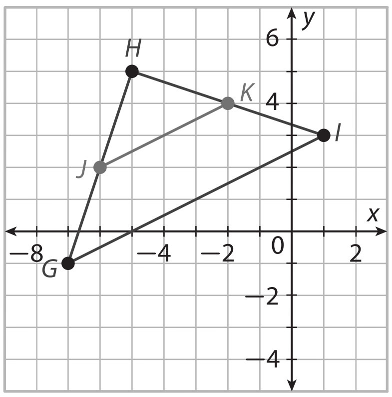
Explain 1 Describing Midsegments on a Coordinate Grid

You can confirm your conjecture about midsegments using the formulas for the midpoint, slope, and distance.

Example 1 **Show that the given midsegment of the triangle is parallel to the third side of the triangle and is half as long as the third side.**

(A) The vertices of $\triangle GHI$ are $G(-7, -1)$, $H(-5, 5)$, and $I(1, 3)$. J is the midpoint of $\overline{GH}$, and K is the midpoint of $\overline{IH}$. Show that $\overline{JK} \parallel \overline{GI}$ and $JK = \frac{1}{2}GI$. Sketch $\overline{JK}$.



Step 1 Use the midpoint formula, $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, to find the coordinates of J and K.

The midpoint of $\overline{GH}$ is $\left(\frac{-7-5}{2}, \frac{-1+5}{2}\right) = (-6, 2)$. Graph and label this point J.

The midpoint of $\overline{IH}$ is $\left(\frac{-5+1}{2}, \frac{5+3}{2}\right) = (-2, 4)$. Graph and label this point K. Use a straightedge to draw $\overline{JK}$.

Step 2 Use $\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$ to compare the slopes of $\overline{JK}$ and $\overline{GI}$.

Slope of $\overline{JK} = \frac{4-2}{-2-(-6)} = \frac{1}{2}$ Slope of $\overline{GI} = \frac{3-(-1)}{1-(-7)} = \frac{1}{2}$

Since the slopes are the same, $\overline{JK} \parallel \overline{GI}$.

Step 3 Use $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to compare the lengths of $\overline{JK}$ and $\overline{GI}$.

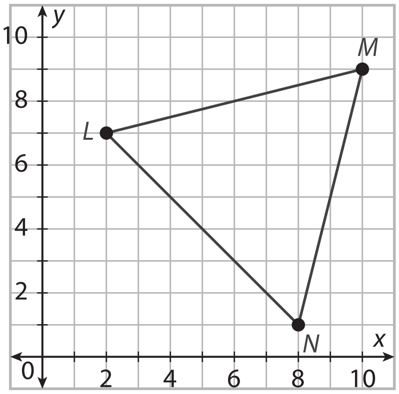
$$JK = \sqrt{(-2-(-6))^2 + (4-2)^2} = \sqrt{20} = 2\sqrt{5}$$

$$GI = \sqrt{\left(1-(-7)\right)^2 + \left(3-(-1)\right)^2} = \sqrt{80} = 4\sqrt{5}$$

Since $2\sqrt{5} = \frac{1}{2}\left(4\sqrt{5}\right)$, $JK = \frac{1}{2}GI$.

(B) The vertices of $\triangle LMN$ are $L(2, 7)$, $M(10, 9)$, and $N(8, 1)$. P is the midpoint of $\overline{LM}$, and Q is the midpoint of $\overline{MN}$.

Show that $\overline{PQ} \parallel \overline{LN}$ and $PQ = \frac{1}{2}LN$. Sketch $\overline{PQ}$.



Step 1 The midpoint of $\overline{LM} = \left(\frac{2 + \square}{2}, \frac{7 + \square}{2}\right) = (\square, \square)$. Graph and label this point P.

The midpoint of $\overline{NM} = \left(\frac{\square + \square}{2}, \frac{\square + \square}{2}\right)$

$= (\square, \square)$. Graph and label this point Q. Use a straightedge to draw $\overline{PQ}$.

Step 2 Slope of $\overline{PQ} = \frac{5 - 8}{9 - \square} = \square$ Slope of $\overline{LN} = \frac{\square - \square}{\square - \square} = \square$

Since the slopes are the same, $\overline{PQ}$ and $\overline{LN}$ are ________.

Step 3 $PQ = \sqrt{(\square - 6)^2 + (5 - 8)^2} = \sqrt{\square + 9} = \sqrt{18} = 3\sqrt{2}$

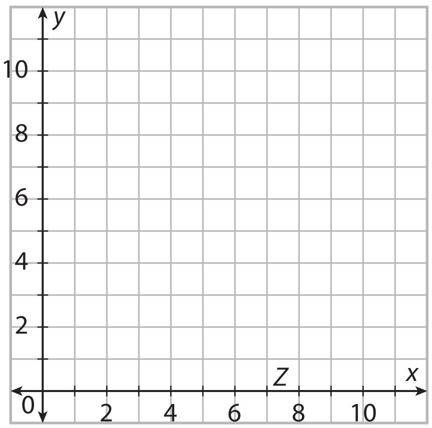
$LN = \sqrt{(\square - \square)^2 + (\square - \square)^2} = \sqrt{\square + \square} = \sqrt{\square} = 6\sqrt{\square}$

Since $\square\sqrt{\square} = \frac{1}{2}\left(\square\sqrt{\square}\right)$, $\square = \frac{1}{2}\square$.

The length of $\overline{PQ}$ is ______ the length of $\overline{LN}$.

Your Turn

4. The vertices of $\triangle XYZ$ are $X(3, 7)$, $Y(9, 11)$, and $Z(7, 1)$. U is the midpoint of $\overline{XY}$, and W is the midpoint of $\overline{XZ}$. Show that $\overline{UW} \parallel \overline{YZ}$ and $UW = \frac{1}{2}YZ$. Sketch $\triangle XYZ$ and $\overline{UW}$.



Explain 2 Using the Triangle Midsegment Theorem

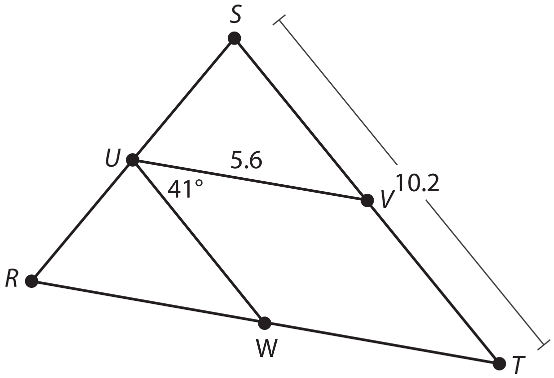
The relationship you have been exploring is true for the three midsegments of every triangle.

Triangle Midsegment Theorem

The segment joining the midpoints of two sides of a triangle is parallel to the third side, and its length is half the length of that side.

You explored this theorem in Example 1 and will be proving it later in this course.

Example 2 **Use triangle *RST*.**



(A) Find UW.

By the Triangle Midsegment Theorem, the length of midsegment $\overline{UW}$ is half the length of $\overline{ST}$.

$UW = \frac{1}{2}ST$

$UW = \frac{1}{2}(10.2)$

$UW = 5.1$

(B) Complete the reasoning to find m $\angle SVU$.

____________	$\overline{UW} \parallel \overline{ST}$
____________	$m\angle SVU = m\angle VUW$
Substitute ____ for ______	$m\angle SVU = \boxed{}$

Reflect

5. How do you know to which side of a triangle a midsegment is parallel?

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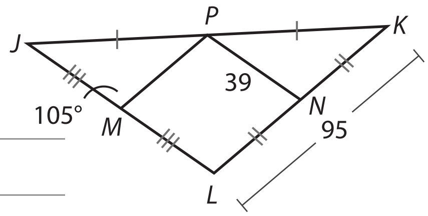
Your Turn

6. Find JL, PM, and $m\angle MLK$.



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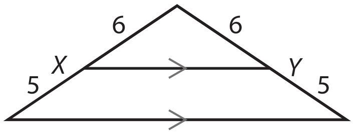
Elaborate

7. Discussion Explain why $\overline{XY}$ is NOT a midsegment of the triangle.



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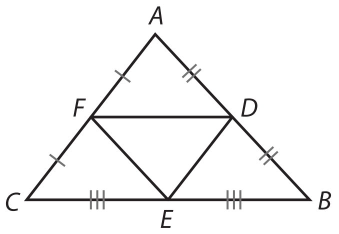
8. **Essential Question Check–In** Explain how the perimeter of $\triangle DEF$ compares to that of $\triangle ABC$.



Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. Use a compass and a ruler or geometry software to construct an obtuse triangle. Label the vertices. Choose two sides and construct the midpoint of each side; then label and draw the midsegment. Describe the relationship between the length of the midsegment and the length of the third side.

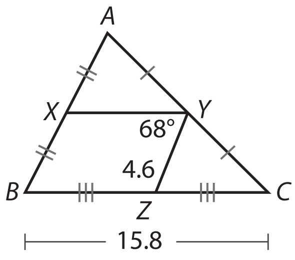
2. The vertices of $\triangle WXY$ are $W(-4, 1)$, $X(0, -5)$, and $Y(4, -1)$. A is the midpoint of $\overline{WY}$, and B is the midpoint of $\overline{XY}$. Show that $\overline{AB} \parallel \overline{WX}$ and $AB = \frac{1}{2}\ WX$.

3. The vertices of $\triangle FGH$ are $F(-1, 1)$, $G(-5, 4)$, and $H(-5, -2)$. X is the midpoint of $\overline{FG}$, and Y is the midpoint of $\overline{FH}$. Show that $\overline{XY} \parallel \overline{GH}$ and $XY = \frac{1}{2}GH$.

4. One of the vertices of $\triangle PQR$ is $P(3, -2)$. The midpoint of $\overline{PQ}$ is $M(4, 0)$. The midpoint of $\overline{QR}$ is $N(7, 1)$. Show that $\overline{MN} \parallel \overline{PR}$ and $MN = \frac{1}{2}PR$.

5. One of the vertices of $\triangle ABC$ is $A(0, 0)$. The midpoint of $\overline{AC}$ is $J\left(\frac{3}{2}, 2\right)$. The midpoint of $\overline{BC}$ is $K\,(4, 2)$. Show that $\overline{JK} \parallel \overline{BA}$ and $JK = \frac{1}{2}\,BA$.

Find each measure.



6. XY

7. BZ

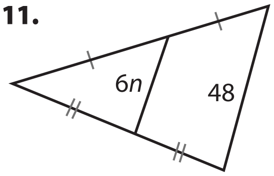
8. AX

9. $m\angle YZC$

10. $m\angle BXY$

Algebra **Find the value of n in each triangle.**

11.



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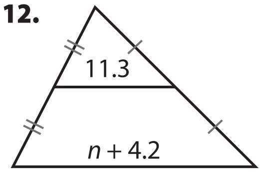

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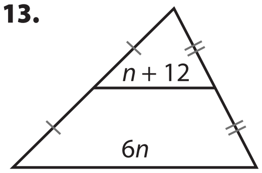

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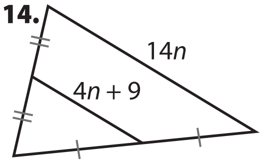
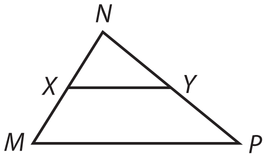


15. Line segment XY is a midsegment of $\triangle MNP$. Determine whether each of the following statements is true or false. Select the correct answer for each lettered part.



a. $MP = 2XY$ ◯ True ◯ False

b. $MP = \frac{1}{2}XY$ ◯ True ◯ False

c. $MX = XN$ ◯ True ◯ False

d. $MX = \frac{1}{2}NX$ ◯ True ◯ False

e. $NX = YN$ ◯ True ◯ False

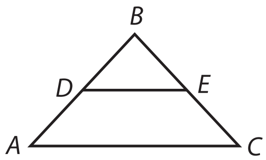
f. $XY = \frac{1}{2}MP$ ◯ True ◯ False

16. What do you know about two of the midsegments in an isosceles triangle? Explain.

17. Suppose you know that the midsegments of a triangle are all 2 units long. What kind of triangle is it?

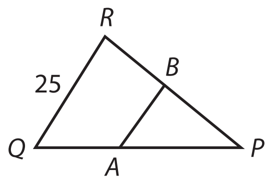
18. In $\triangle ABC$, $m\angle A = 80°$, $m\angle B = 60°$, $m\angle C = 40°$. The midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are D, E, and F, respectively. Which midsegment will be the longest? Explain how you know.

19. **Draw Conclusions** Carl's Construction is building a pavilion with an A-frame roof at the local park. Carl has constructed two triangular frames for the front and back of the roof, similar to $\triangle ABC$ in the diagram. The base of each frame, represented by $\overline{AC}$, is 36 feet long. He needs to insert a crossbar connecting the midpoints of $\overline{AB}$ and $\overline{BC}$, for each frame. He has 32 feet of timber left after constructing the front and back triangles. Is this enough to construct the crossbar for both the front and back frame? Explain.



20. **Critique Reasoning** Line segment AB is a midsegment in $\triangle PQR$. Kayla calculated the length of $\overline{AB}$. Her work is shown below. Is her answer correct? If not, explain her error.

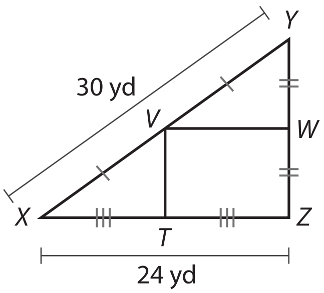
$$2(QR) = AB$$
$$2(25) = AB$$
$$50 = AB$$



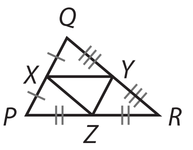
21. Using words or diagrams, tell how to construct a midsegment using only a straightedge and a compass.

H.O.T. Focus on Higher Order Thinking

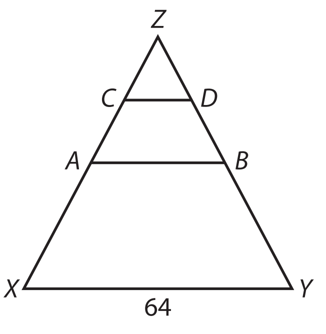
22. Multi–Step A city park will be shaped like a right triangle, and there will be two pathways for pedestrians, shown by $\overline{VT}$ and $\overline{VW}$ in the diagram. The park planner only wrote two lengths on his sketch as shown. Based on the diagram, what will be the lengths of the two pathways?



23. Communicate Mathematical Ideas $\triangle XYZ$ is the midsegment of $\triangle PQR$. Write a congruence statement involving all four of the smaller triangles. What is the relationship between the area of $\triangle XYZ$ and $\triangle PQR$?



24. Copy the diagram shown. $\overline{AB}$ is a midsegment of $\triangle XYZ$. $\overline{CD}$ is a midsegment of $\triangle ABZ$.



a. What is the length of $\overline{AB}$? What is the ratio of AB to XY?

b. What is the length of $\overline{CD}$? What is the ratio of CD to XY?

c. Draw $\overline{EF}$ such that points E and F are $\frac{3}{4}$ the distance from point Z to points X and Y. What is the ratio of EF to XY? What is the length of $\overline{EF}$?

d. Make a conjecture about the length of non-midsegments when compared to the length of the third side.

Lesson Performance Task

The figure shows part of a common roof design using very strong and stable triangular *trusses*. Points B, C, D, F, G, and I are midpoints of $\overline{AC}$, $\overline{AE}$, $\overline{CE}$, $\overline{GE}$, $\overline{HE}$ and $\overline{AH}$ respectively. What is the total length of all the stabilizing bars inside $\triangle AEH$? Explain how you found the answer.