Name ______________________ Class ____________ Date ________

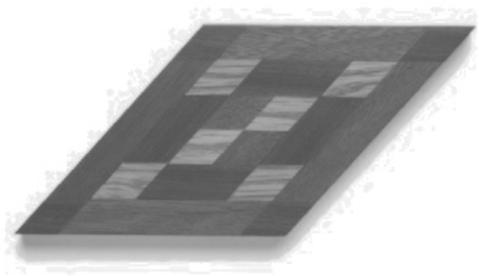
9.1 Properties of Parallelograms

Essential Question: What can you conclude about the sides, angles, and diagonals of a parallelogram?

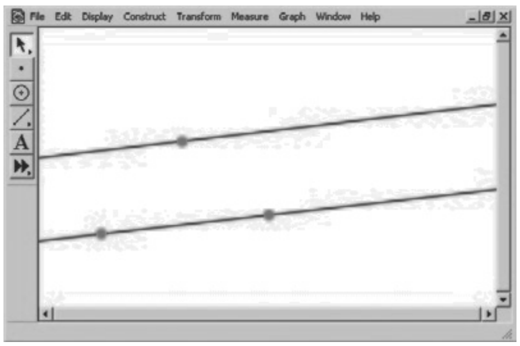
Resource Locker

Explore Investigating Parallelograms

A **quadrilateral** is a polygon with four sides. A **parallelogram** is a quadrilateral that has two pairs of parallel sides. You can use geometry software to investigate properties of parallelograms.



(A) Draw a straight line. Then plot a point that is not on the line. Construct a line through the point that is parallel to the line. This gives you a pair of parallel lines.



(B) Repeat Step A to construct a second pair of parallel lines that intersect those from Step A.

(C) The intersections of the parallel lines create a parallelogram. Plot points at these intersections. Label the points *A*, *B*, *C*, and *D*.

Identify the *opposite sides* and *opposite angles* of the parallelogram.

Opposite sides: ____

Opposite angles: ____

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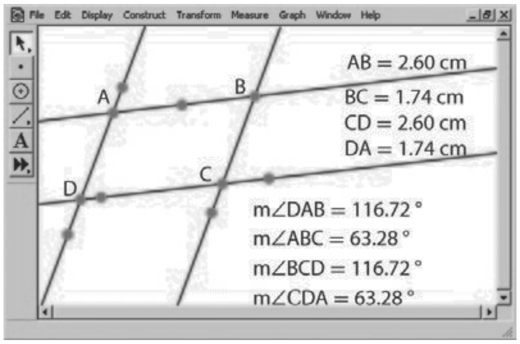
(D) Measure each angle of the parallelogram.

Measure the length of each side of the parallelogram. You can do this by measuring the distance between consecutive vertices.



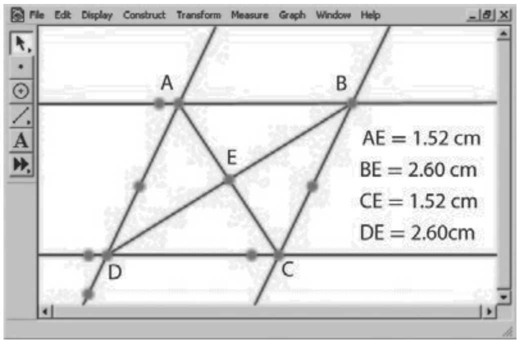
(E) Then drag the points and lines in your construction to change the shape of the parallelogram. As you do so, look for relationships in the measurements. Make a conjecture about the sides and angles of a parallelogram.

Conjecture: ______________________________

____

(F) A segment that connects two nonconsecutive vertices of a polygon is a **diagonal**. Construct diagonals $\overline{AC}$ and $\overline{BD}$. Plot a point at the intersection of the diagonals and label it *E*.

(G) Measure the length of $\overline{AE}$, $\overline{BE}$, $\overline{CE}$, and $\overline{DE}$.



(H) Drag the points and lines in your construction to change the shape of the parallelogram. As you do so, look for relationships in the measurements in Step G. Make a conjecture about the diagonals of a parallelogram.

Conjecture: ______________________________

Reflect

1. *Consecutive angles* are the angles at consecutive vertices, such as ∠*A* and ∠*B*, or ∠*A* and ∠*D*. Use your construction to make a conjecture about consecutive angles of a parallelogram.

Conjecture: ______________________________

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2. **Critique Reasoning** A student claims that the perimeter of $\triangle AEB$ in the construction is always equal to the perimeter of $\triangle CED$. Without doing any further measurements in your construction, explain whether or not you agree with the student's statement.

Explain 1 Proving Opposite Sides Are Congruent

The conjecture you made in the Explore about opposite sides of a parallelogram can be stated as a theorem. The proof involves drawing an *auxiliary line* in the figure.

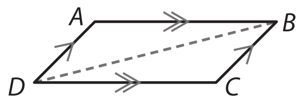
Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

Example 1 **Prove that the opposite sides of a parallelogram are congruent.**

Given: $ABCD$ is a parallelogram.

Prove: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$



Statements	Reasons
1. $ABCD$ is a parallelogram.	**1.**
2. Draw $\overline{DB}$.	**2.** Through any two points, there is exactly one line.
3. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$	**3.**
4. $\angle ADB \cong \angle CBD$ $\angle ABD \cong \angle CDB$	**4.**
5. $\overline{DB} \cong \overline{DB}$	**5.**
6.	**6.** ASA Triangle Congruence Theorem
7. $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$	**7.**

Reflect

3. Explain how you can use the rotational symmetry of a parallelogram to give an argument that supports the above theorem.

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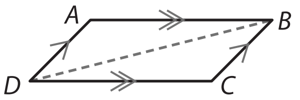
Explain 2 Proving Opposite Angles Are Congruent

The conjecture from the Explore about opposite angles of a parallelogram can also be proven and stated as a theorem.

Theorem
If a quadrilateral is a parallelogram, then its opposite angles are congruent.

Example 2 **Prove that the opposite angles of a parallelogram are congruent.**



Given: $ABCD$ is a parallelogram.

Prove: $\angle A \cong \angle C$ (A similar proof shows that $\angle B \cong \angle D$.)

Statements	Reasons
1. $ABCD$ is a parallelogram.	**1.**
2. Draw $\overline{DB}$.	**2.**
3. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$	**3.**
4.	**4.** Alternate Interior Angles Theorem
5.	**5.** Reflexive Property of Congruence
6.	**6.** ASA Triangle Congruence Theorem
7.	**7.**

Reflect

4. Explain how the proof would change in order to prove $\angle B \cong \angle D$.

5. In Reflect 1, you noticed that the consecutive angles of a parallelogram are supplementary. This can be stated as the theorem, *If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.*

Explain why this theorem is true.

Explain 3 Proving Diagonals Bisect Each Other

The conjecture from the Explore about diagonals of a parallelogram can also be proven and stated as a theorem. One proof is shown on the facing page.

Theorem
If a quadrilateral is a parallelogram, then its diagonals bisect each other.

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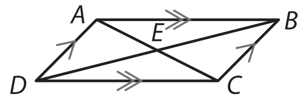
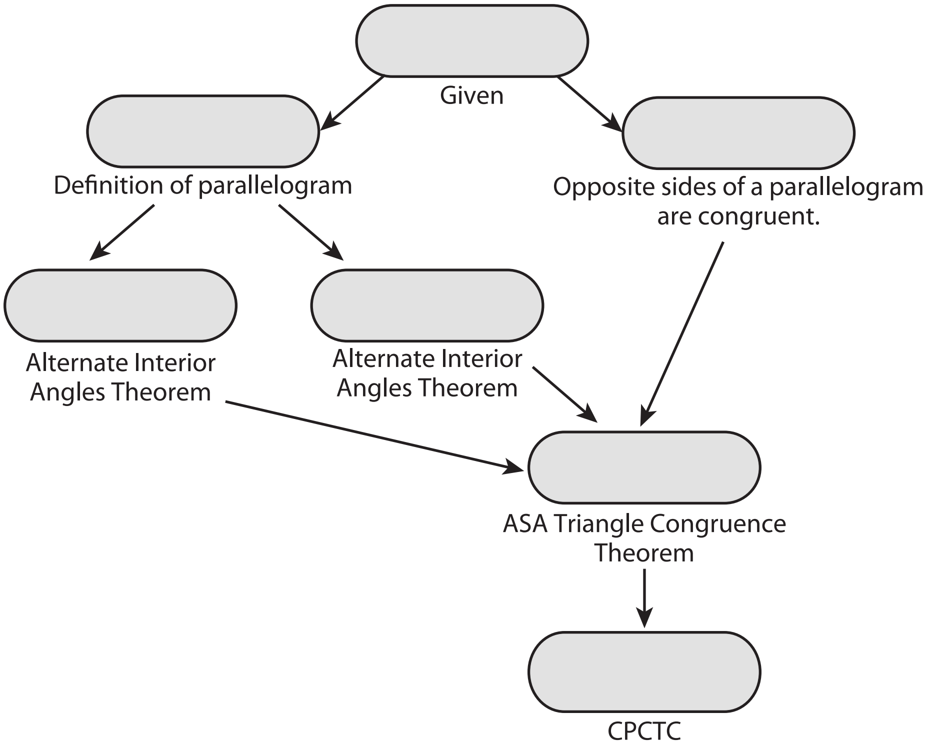
Example 3 **Complete the flow proof that the diagonals of a parallelogram bisect each other.**

Given: *ABCD* is a parallelogram.

Prove: $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$





Reflect

6. **Discussion** Is it possible to prove the theorem using a different triangle congruence theorem? Explain.

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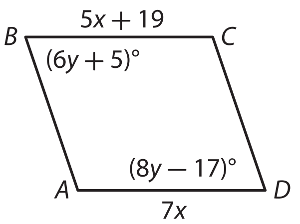
Explain 4 Using Properties of Parallelograms

You can use the properties of parallelograms to find unknown lengths or angle measures in a figure.

Example 4 ***ABCD* is a parallelogram. Find each measure.**



Ⓐ AD

Use the fact that opposite sides of a parallelogram are congruent, so $\overline{AD} \cong \overline{CB}$ and therefore $AD = CB$.

Write an equation. $7x = 5x + 19$

Solve for x. $x = 9.5$

$AD = 7x = 7(9.5) = 66.5$

Ⓑ $m\angle B$

Use the fact that opposite angles of a parallelogram are congruent,

so $\angle B \cong \angle \boxed{}$ and therefore $m\angle B = m\angle \boxed{}$.

Write an equation. $6y + 5 =$ ______

Solve for y. ______ $= y$

$m\angle B = (6y + 5)° = \left(6\left(\boxed{}\right) + 5\right)° = \boxed{}°$

Reflect

7. Suppose you wanted to find the measures of the other angles of parallelogram *ABCD*. Explain your steps.

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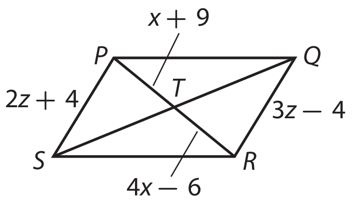
Your Turn

$PQRS$ is a parallelogram. Find each measure.

8. QR

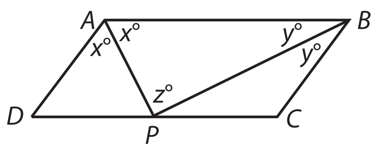


9. PR

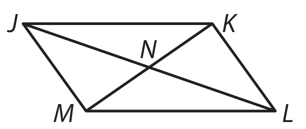
Elaborate

10. What do you need to know first in order to apply any of the theorems of this lesson?

11. In parallelogram $ABCD$, point P lies on $\overline{DC}$, as shown in the figure. Explain why it must be the case that $DC = 2AD$. Use what you know about base angles of an isosceles triangle.



12. **Essential Question Check-In** $JKLM$ is a parallelogram. Name all of the congruent segments and angles in the figure.



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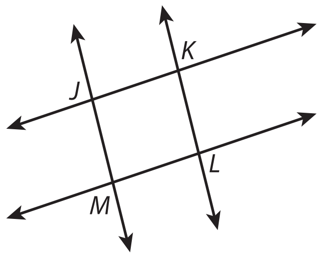
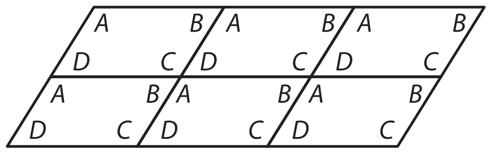
Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

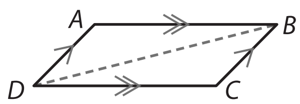
1. Pablo traced along both edges of a ruler to draw two pairs of parallel lines, as shown. Explain the next steps he could take in order to make a conjecture about the diagonals of a parallelogram.

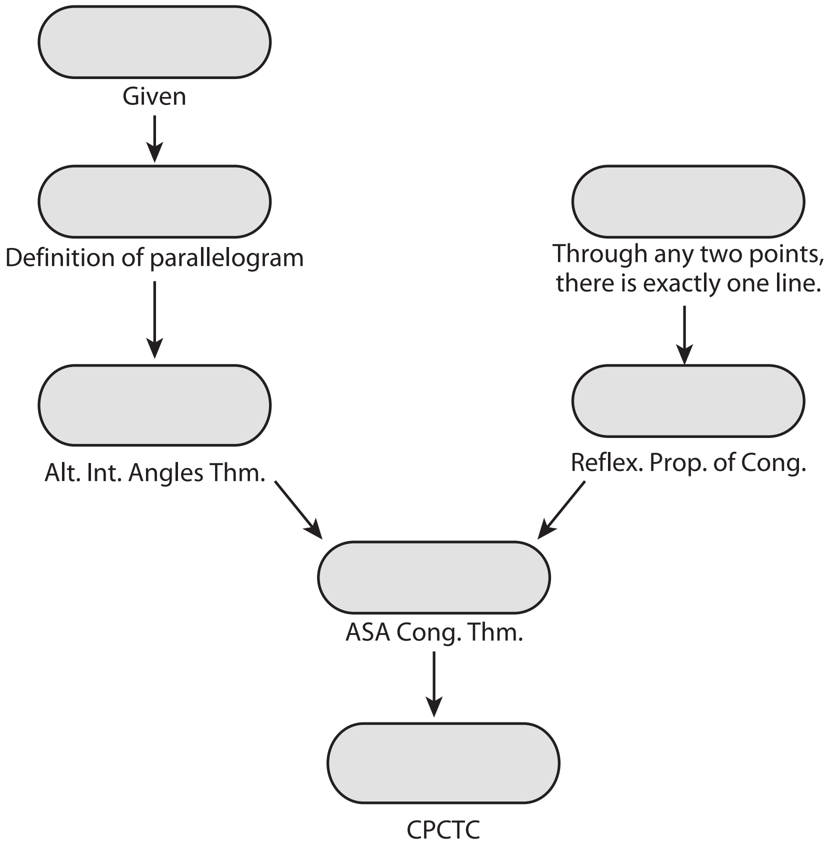


2. Sabina has tiles in the shape of a parallelogram. She labels the angles of each tile as $\angle A$, $\angle B$, $\angle C$, and $\angle D$. Then she arranges the tiles to make the pattern shown here and uses the pattern to make a conjecture about opposite angles of a parallelogram. What conjecture does she make? How does the pattern help her make the conjecture?



3. Complete the flow proof that the opposite sides of a parallelogram are congruent. Given: $ABCD$ is a parallelogram.
Prove: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$





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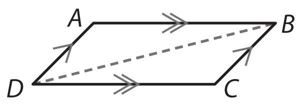
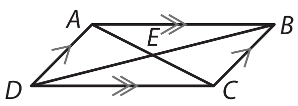
4. Write the proof that the opposite angles of a parallelogram are congruent as a paragraph proof.



Given: $ABCD$ is a parallelogram.

Prove: $\angle A \cong \angle C$ (A similar proof shows that $\angle B \cong \angle D$.)

5. Write the proof that the diagonals of a parallelogram bisect each other as a two-column proof.

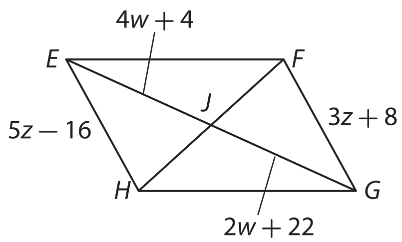


Given: $ABCD$ is a parallelogram.

Prove: $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$

Statements	Reasons
1.	1.

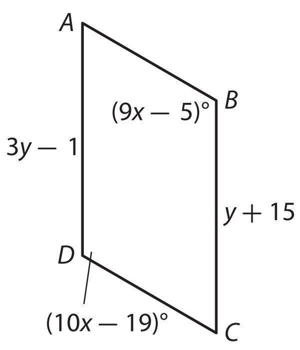
***EFGH* is a parallelogram. Find each measure.**



6. FG

7. EG

***ABCD* is a parallelogram. Find each measure.**



8. $m\angle B$

9. AD

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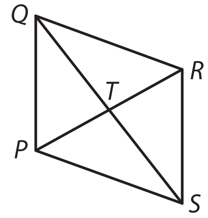
A staircase handrail is made from congruent parallelograms. In ▱$PQRS$, $PQ = 17.5$, $ST = 18$, and $m\angle QRS = 110°$. Find each measure. Explain.



10. RS

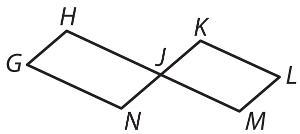
11. QT

12. $m\angle PQR$

13. $m\angle SPQ$

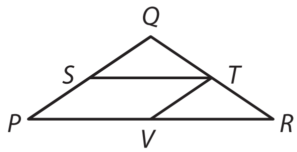
Write each proof as a two-column proof.

14. Given: $GHJN$ and $JKLM$ are parallelograms.
Prove: $\angle G \cong \angle L$



Statements	Reasons
1.	1.

15. Given: $PSTV$ is a parallelogram. $\overline{PQ} \cong \overline{RQ}$
Prove: $\angle STV \cong \angle R$



Statements	Reasons
1.	1.

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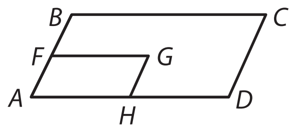
16. Given: *ABCD* and *AFGH* are parallelograms.
Prove: $\angle C \cong \angle G$



Statements	Reasons
1.	**1.**

Justify Reasoning **Determine whether each statement is always, sometimes, or never true. Explain your reasoning.**

17. If quadrilateral *RSTU* is a parallelogram, then $\overline{RS} \cong \overline{ST}$.

18. If a parallelogram has a 30° angle, then it also has a 150° angle.

19. If quadrilateral *GHJK* is a parallelogram, then $\overline{GH}$ is congruent to $\overline{JK}$.

20. In parallelogram *ABCD*, $\angle A$ is acute and $\angle C$ is obtuse.

21. In parallelogram *MNPQ*, the diagonals $\overline{MP}$ and $\overline{NQ}$ meet at *R* with $MR = 7$ cm and $RP = 5$ cm.

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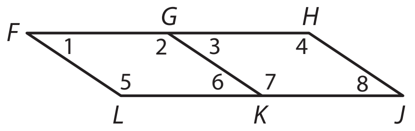
22. **Communicate Mathematical Ideas** Explain how you can use the rotational symmetry of a parallelogram to give an argument that supports the fact that opposite angles of a parallelogram are congruent.

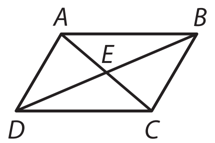
23. To repair a large truck or bus, a mechanic might use a parallelogram lift. The figure shows a side view of the lift. *FGKL*, *GHJK*, and *FHJL* are parallelograms.



a. Which angles are congruent to $\angle 1$? Explain.

b. What is the relationship between $\angle 1$ and each of the remaining labeled angles? Explain.

24. **Justify Reasoning** *ABCD* is a parallelogram. Determine whether each statement must be true. Select the correct answer for each lettered part. Explain your reasoning.



A. The perimeter of *ABCD* is $2AB + 2BC$. ○ Yes ○ No

B. $DE = \frac{1}{2} DB$ ○ Yes ○ No

C. $\overline{BC} \cong \overline{DC}$ ○ Yes ○ No

D. $\angle DAC \cong \angle BCA$ ○ Yes ○ No

E. $\triangle AED \cong \triangle CEB$ ○ Yes ○ No

F. $\angle DAC \cong \angle BAC$ ○ Yes ○ No

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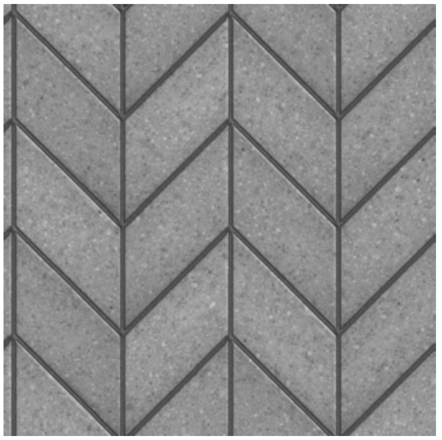
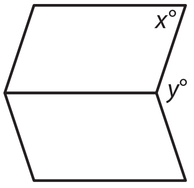
H.O.T. Focus on Higher Order Thinking

25. Represent Real-World Problems A store sells tiles in the shape of a parallelogram. The perimeter of each tile is 29 inches. One side of each tile is 2.5 inches longer than another side. What are the side lengths of the tile? Explain your steps.



26. Critique Reasoning A student claims that there is an SSSS congruence criterion for parallelograms. That is, if all four sides of one parallelogram are congruent to the four sides of another parallelogram, then the parallelograms are congruent. Do you agree? If so, explain why. If not, give a counterexample. Hint: Draw a picture.

27. Analyze Relationships The figure shows two congruent parallelograms. How are x and y related? Write an equation that expresses the relationship. Explain your reasoning.



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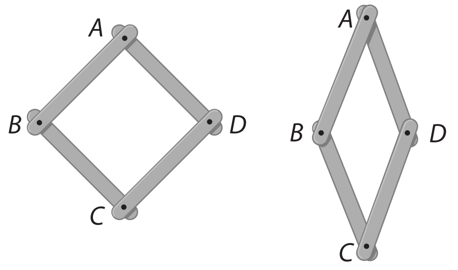
Lesson Performance Task

The principle that allows a scissor lift to raise the platform on top of it to a considerable height can be illustrated with four freezer pop sticks attached at the corners.



Answer these questions about what happens to parallelogram *ABCD* when you change its shape as in the illustration.

a. Is it still a parallelogram? Explain.

b. Is its area the same? Explain.

c. Compare the lengths of the diagonals in the two figures as you change them.

d. Describe a process that might be used to raise the platform on a scissor lift.

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