Advanced Mathematical Decision Making

In Texas, also known as

Advanced Quantitative Reasoning

Student materials

Semester 2

Unit IV: Using Recursion in Models and Decision Making
Unit V: Using Functions in Models and Decision Making
Unit VI: Decision Making in Finance
Unit VII: Networks and Graphs

This course is a project of
The Texas Association of Supervisors of Mathematics and
The Charles A. Dana Center at The University of Texas at Austin
With support from the Greater Texas Foundation

2010

Charles A. Dana Center
The University of Texas at Austin
1616 Guadalupe Street, Suite 3.206
Austin, TX 78701-1222

Fax: 512-232-1855
dana-txshop@utlists.utexas.edu
www.utdanacenter.org

About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center works to raise student achievement in K–16 mathematics and science, especially for historically underserved populations. We do so by providing direct service to school districts and institutions of higher education; to local, state, and national education leaders; and to agencies, nonprofits, and professional organizations concerned with strengthening American mathematics and science education.

The Center was founded in 1991 in the College of Natural Sciences at The University of Texas at Austin. Our original purpose—which continues in our work today—was to increase the diversity of students who successfully pursue careers in science, technology, engineering, and mathematics (STEM) fields.

We carry out our work by supporting high standards and building system capacity; developing collaborations with key state and national organizations to address emerging issues; creating and delivering professional supports for educators and education leaders; and writing and publishing education resources, including student supports.

Our staff of more than 80 researchers and education professionals has worked intensively with dozens of school systems in nearly 20 states and with 90 percent of Texas's more than 1,000 school districts. As one of the College's largest research units, the Dana Center works to further the university's mission of achieving excellence in education, research, and public service. We are committed to ensuring that the accident of where a child attends school does not limit the academic opportunities he or she can pursue.

For more information about the Dana Center and our programs and resources, see our homepage at **www.utdanacenter.org.** To access our resources (many of them free), please see our products index at **www.utdanacenter.org/products.** And for updates and background on the Advanced Mathematical Decision Making project, see **www.utdanacenter.org/amdm**.

About the Texas Association of Supervisors of Mathematics

The mission of the Texas Association of Supervisors of Mathematics is to assist in promoting effectiveness in the supervision, coordination, and teaching of mathematics, especially in the elementary and secondary fields.

TASM accomplishes this by holding meetings for the presentation and discussion of papers; by conducting public discussion groups, forums, panels, lectures, or other similar programs; by conducting or promoting investigations for the purpose of improving the teaching of mathematics; and by the publication of papers, journals, books, and reports, thus vitalizing and coordinating the work of mathematics supervisors across Texas and bringing the interests of mathematics to the attention and consideration of the larger education community in Texas.

For more information about TASM, visit its website at **www.tasmonline.net.**

About the contents of this course

The materials for the AMDM/AQR course consist of teacher and student materials for Units I through VII. *This two-volume resource contains only the student materials.* The teacher materials are composed of the student expectations, unit overviews, and unit section planners. Of course, the full 2010 AMDM/AQR instructional materials are available free to the people of Texas, as described in the copyright language above. (The materials are available to educators outside Texas by arrangement; contact dana-txshop@utlists.utexas.edu

About the development of the AMDM course

The development and production of the original AMDM student expectations, as well as the AMDM instructional materials that constitute Units I through VII of this resource, were supported by the Greater Texas Foundation, based in Bryan, Texas. Any opinions, findings, conclusions, or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the Greater Texas Foundation or The University of Texas at Austin.

AMDM student expectations provided the basis for the Texas Essential Knowledge and Skills for Advanced Quantitative Reasoning, which were adopted by the Texas State Board of Education in January 2011. The AQR TEKS have only minor editorial differences from the original AMDM student expectations. Thus, we have updated the AMDM student expectations to match the wording in the AQR TEKS.

- *AMDM student expectations* were developed by a group of 20 people, consisting primarily of Texas mathematics educators and mathematicians at the secondary and postsecondary level, with additional assistance from mathematicians outside Texas. These AMDM student expectations reflect many months of development and feedback from across the state—and from expert reviewers outside the state—as described in updates presented to the Texas State Board of Education in November 2007, June 2008, and March 2009.
- The TEKS for Advanced Quantitative Reasoning have been adopted by the Texas State Board of Education. As such, they are part of state law and are thus available to all Texans at no charge.
- *The AMDM instructional materials* (Units I through VII), now available in Texas as materials for Advanced Quantitative Reasoning, were developed by mathematics teachers and faculty from Texas and beyond, with support from the Texas Association of Supervisors of Mathematics and others in Texas who donated their time or served as consultants for the project. See the acknowledgments section in the teacher materials (available via www.utdanacenter.org/amdm, as described above) for a complete listing of the authors, reviewers, advisory team members, and staff supporting this work.

  As noted in the copyright language above, the people of Texas are granted by the University a nonexclusive license in perpetuity to use the 2010 AMDM/AQR instructional materials (reflecting improvements based on the 2009–2010 pilot and a minor update based on the adoption of Advanced Quantitative Reasoning TEKS by the Texas State Board of Education) in Texas classrooms and homes for the education of our children. The full 2010 AMDM/AQR instructional materials are available free to the people of Texas via the AMDM website (www.utdanacenter.org/amdm). The materials are available to educators outside Texas by arrangement; to inquire, contact dana-txshop@utlists.utexas.edu.

This resource was produced in Microsoft Word 2004 for Mac.

Released April 2011.

Advanced Mathematical Decision Making

In Texas, also known as

Advanced Quantitative Reasoning

Unit IV: Using Recursion in Models and Decision Making

This course is a project of
The Texas Association of Supervisors of Mathematics and
The Charles A. Dana Center at The University of Texas at Austin
With support from the Greater Texas Foundation

2010

Advanced Mathematical Decision Making

In Texas, also known as

Advanced Quantitative Reasoning

Student Materials

These student materials are excerpted from one of seven units that make up the 2010 AMDM/AQR instructional materials (developed under the name Advanced Mathematical Decision Making).

Unit I: Analyzing Numerical Data
Unit II: Probability
Unit III: Statistical Studies
Unit IV: Using Recursion in Models and Decision Making
Unit V: Using Functions in Models and Decision Making
Unit VI: Decision Making in Finance
Unit VII: Networks and Graphs

Table of Contents

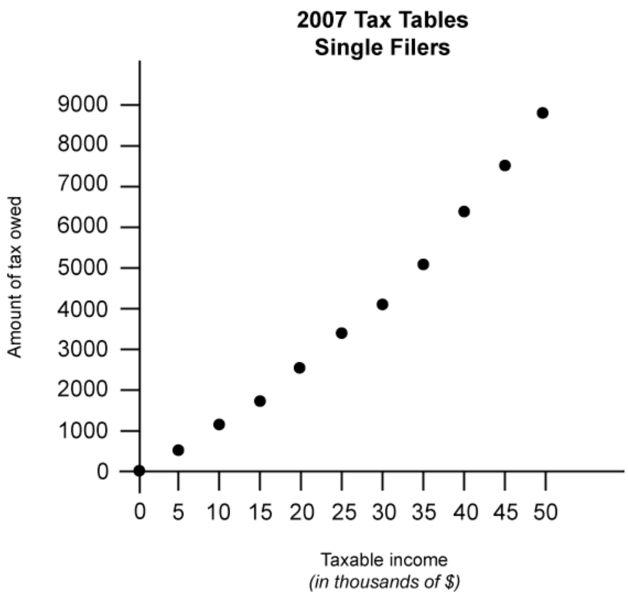
Student: ______________________________ Class: ____________________ Date:______________

Using Recursion in Models and Decision Making: Relationships in Data

IV.A Student Activity Sheet 1: Using Scatterplots in Reports

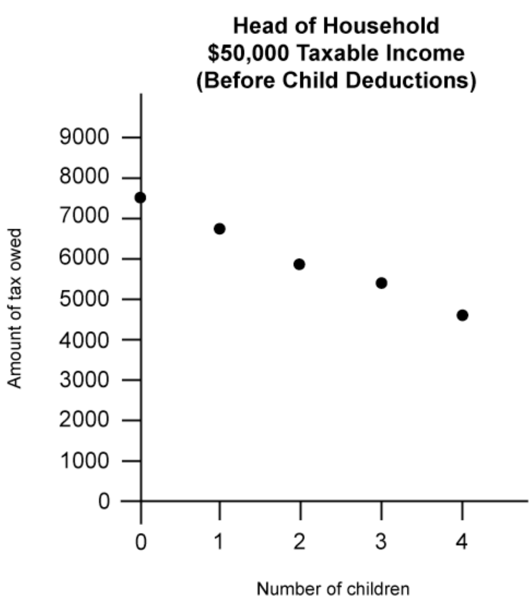
1. Consider the following graph. Who are the subjects in the study? What are the variables of interest?

Thoroughly describe the information illustrated by the graph, choosing at least two data points to help with your explanation.



(Data compiled from 2007 tax tables for Form 1040 on www.irs.gov.)

2. Look at this new graph and discuss with your partner the information illustrated. Then compare and contrast this display with the graph in Question 1.



(Data compiled from 2007 tax tables for Form 1040 on www.irs.gov.)

3. REFLECTION: Use the previous graphs to complete the following sentences.

A person with higher taxable income pays ______________________________.

A person with lower taxable income pays ______________________________.

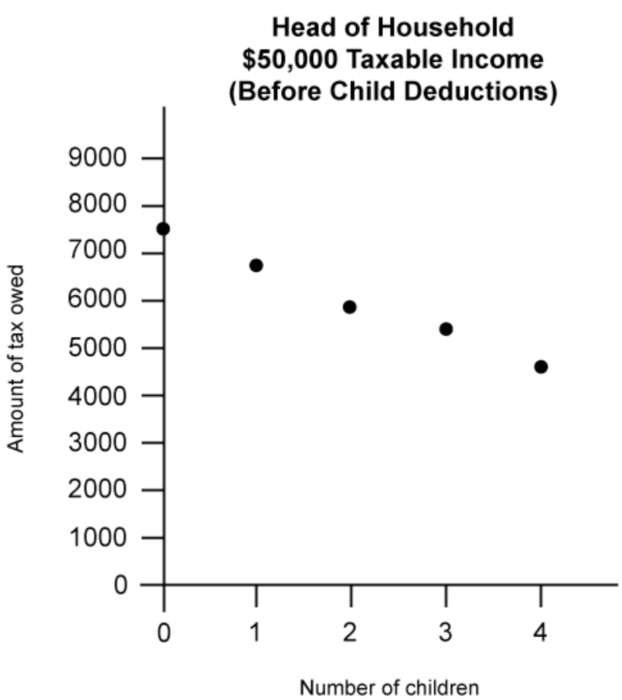
This is an example of a ______________________________ association.

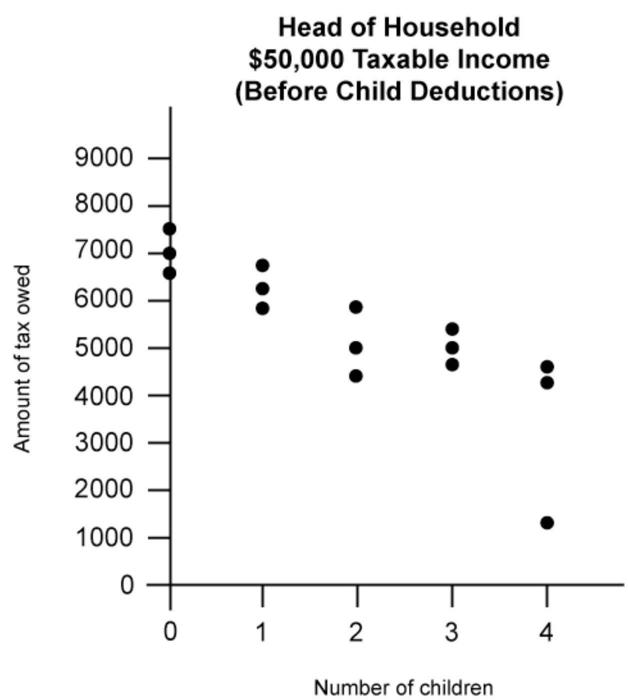
A person with fewer children pays ______________________________.

A person with more children pays ______________________________.

This is an example of a ______________________________ association.

4. In actuality, head-of-household filers with $50,000 in taxable income and the same number of children could pay different amounts of income tax, as shown by the graph on the right. These differences result from tax credits for expenses such as child care that can reduce the amount of tax owed. Compare and contrast this new graph with the original on the left.



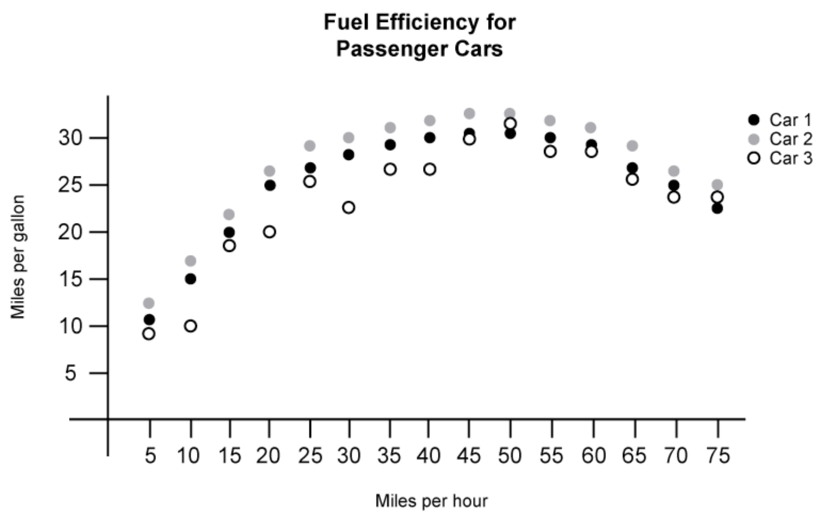


(Data compiled from 2007 tax tables for Form 1040 on www.irs.gov.)

Using Recursion in Models and Decision Making: Relationships in Data

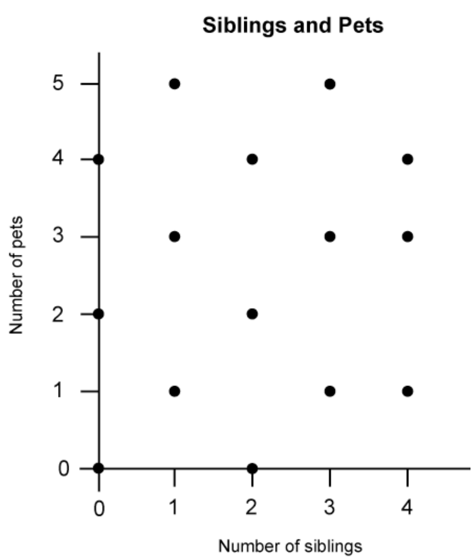
IV.A Student Activity Sheet 1: Using Scatterplots in Reports

5. Now consider the following graph. What information is displayed? Compare and contrast this graph with the others you have analyzed.



(Data compiled from www.fueleconomy.gov.)

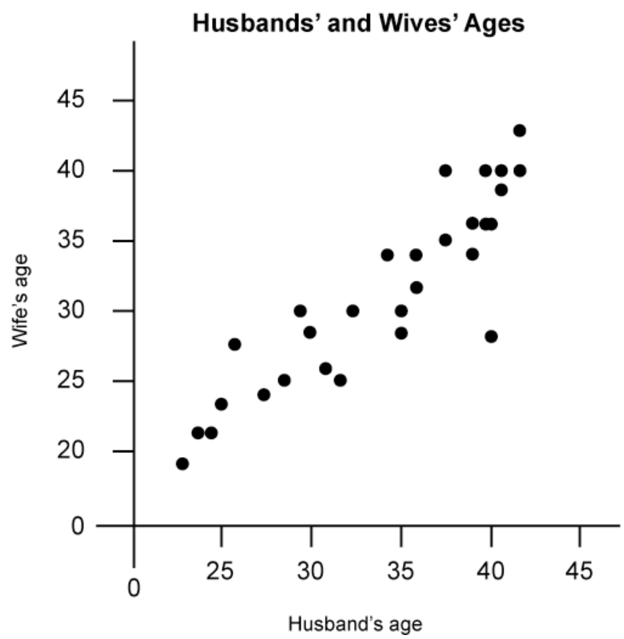
6. A survey of students asked, “How many siblings live in your house with you?” and “How many pets does your family have?” The results are displayed below. Comment on the graph, comparing and contrasting it with the previous graphs.

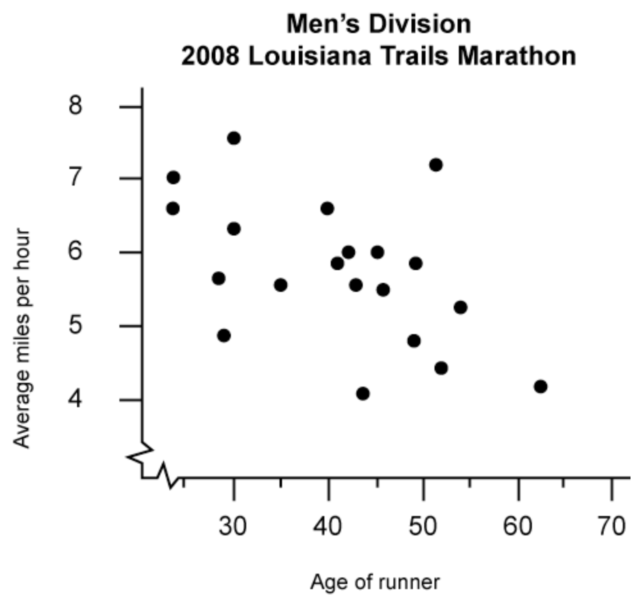


7. When analyzing a display of bivariate statistics, you need to consider the following:

- Form—Does the graph exhibit a linear or nonlinear pattern?
- Direction—Does the graph exhibit a positive relationship, a negative relationship, or neither?
- Relative strength—Are the data points tightly clustered along the line or curve (strongly associated) or are they more scattered (weakly associated)?

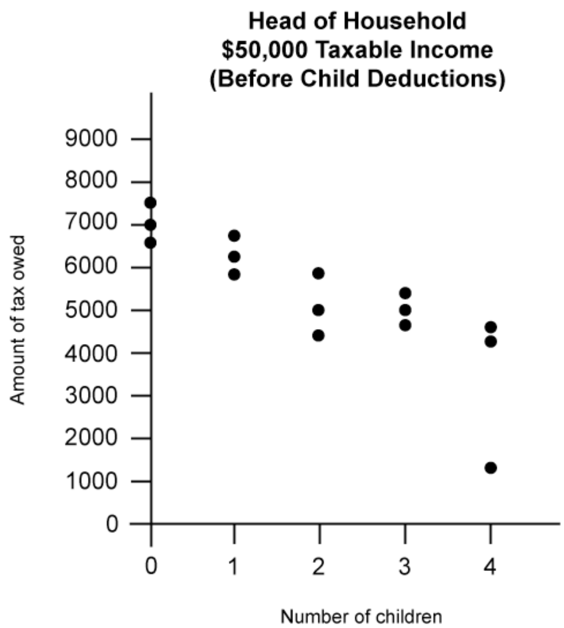
Using these guidelines, analyze the following graphical displays. Conduct your analysis in the context of the situation.





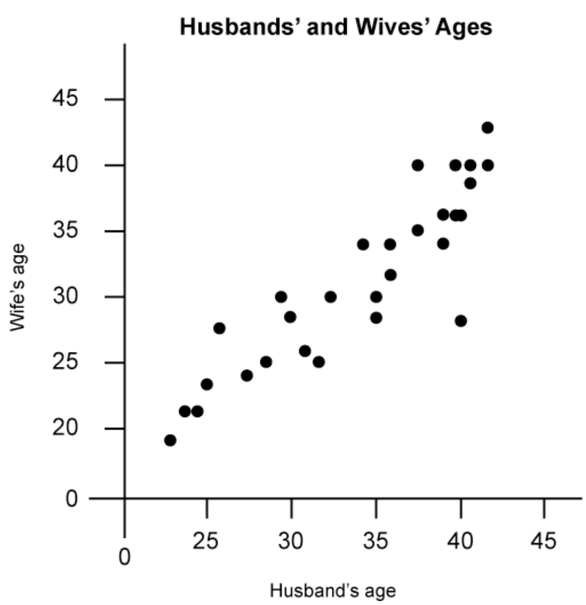
(Data adapted from marathonguide.com.)

8. The following graph illustrates the fact that for a designated filing status and taxable income level, the amount of tax owed depends on the number of children. Does this sound like a cause-and-effect relationship or simply a matter of an association between the variables?



(Data compiled from 2007 tax tables for Form 1040 on www.irs.gov.)

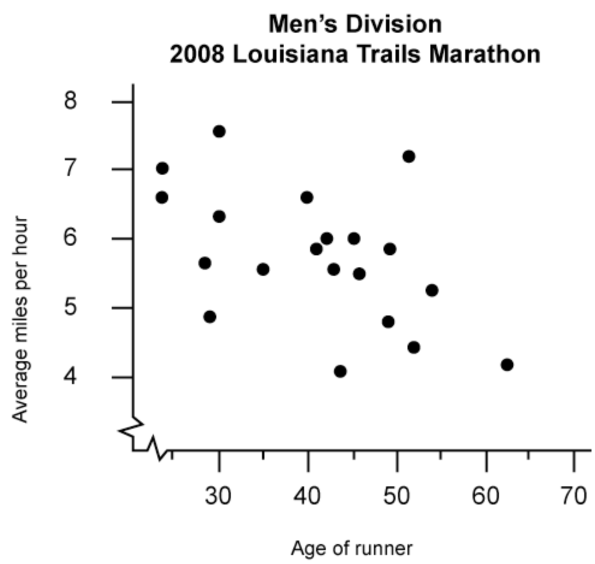
9. The graph shown below illustrates that in general older men have wives about their age and younger men have wives about their age. Does this sound like a cause-and-effect relationship or simply a matter of an association between the variables?



Using Recursion in Models and Decision Making: Relationships in Data

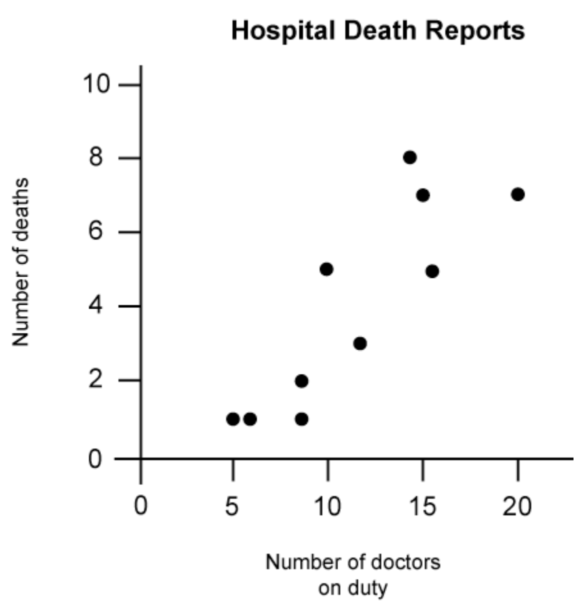
IV.A Student Activity Sheet 1: Using Scatterplots in Reports

10. EXTENSION: A news report noted, "As men age, they begin to run slower." Does this report imply cause and effect or association? What is your opinion of this implication?



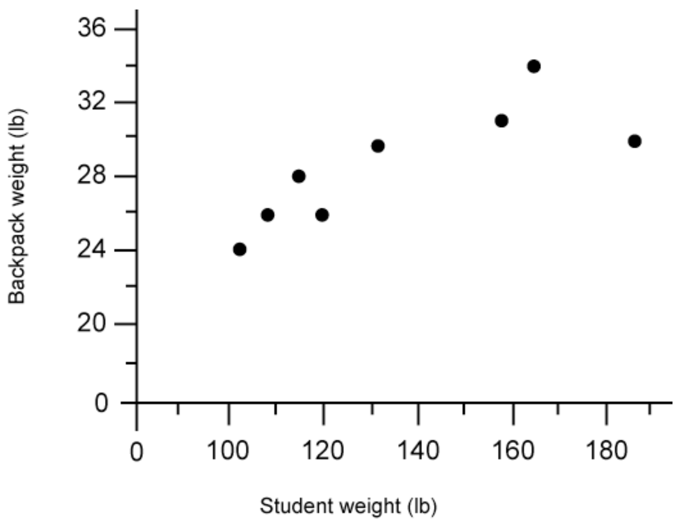
(Data adapted from marathonguide.com)

11. EXTENSION: A special report on the evening news exposed a startling fact: When more doctors are on duty at a hospital, more deaths occur. Does this mean that doctors are killing patients? What are some other explanations?



12. **EXTENSION:** In the last 40 years, spending on education has increased, while SAT scores have gone down. Sketch a scatterplot that represents these trends. Does increased spending cause a drop in SAT scores? Explain your reasoning.

13. **EXTENSION:** Doctors have become concerned about the effect of backpack weights on students' backs. Studies showed that, in general, students weighing more carry heavier backpacks. Write an analysis of the situation for the school newspaper. Clearly indicate in the analysis whether this is a situation of cause and effect or association.



14. **EXTENSION:** In the computer lab, conduct searches for examples of causation and correlation. If you find a misleading report, write a new report that clarifies the issue. If you cannot find a misleading report, describe how such a report might be written and how you would improve on it.

Using Recursion in Models and Decision Making: Relationships in Data

IV.A Student Activity Sheet 2: Recursion and Linear Functions

1. Coen decides to take a job with a company that sells magazine subscriptions. He is paid \$20 to start selling and then earns \$1.50 for each subscription he sells. Fill in the following table, showing the amount of money (***M***) Coen earns for selling ***n*** subscriptions. Use the process column to note what is happening in each line.

n	Process	M_n
0		$M_0 =$
		$M_1 =$
		$M_2 =$
		$M_3 =$
		$M_4 =$

2. Write a recursive rule for the amount of money Coen can earn selling magazine subscriptions.

3. **REFLECTION**: The rule in Question 2 defines a term (M_{n+1}) with respect to the term that precedes it (M_n). Write a rule that defines a term (M_n) with respect to the term that precedes it (M_{n-1})? How is this rule similar to and different from the rule you wrote in Question 2?

4. Write an explicit function rule for the ***n***th term in the sequence describing the amount of money Coen can earn. Describe any domain restrictions in your rule. How is this rule related to the rules you wrote in Question 2?

Using Recursion in Models and Decision Making: Relationships in Data

IV.A Student Activity Sheet 2: Recursion and Linear Functions

5. Use sequence notation to enter the data from your table in Question 1 in a graphing calculator, if your calculator has this capability. Limit your lists to 50 entries each. How do you expect the scatterplot of your data to look? Justify your reasoning.

6. How much does Coen earn if he sells 100 magazine subscriptions? Which rule did you use to answer this question? Why did you choose that rule?

7. Coen is trying to earn enough money to buy a new MP3 player. He needs $225 to cover the cost and tax on the MP3 player. How many magazine subscriptions does Coen need to sell to buy his new MP3 player? Justify your answer. Which rule did you use to answer this question? Why did you choose that rule?

8. Your phone service allows you to add international long distance to your phone. The cost is a $5 flat fee each month and 3¢ a minute for calls made. Write a recursive rule describing your monthly cost for international calls. Then write a function rule for the ***n*** minutes of calls made in a month.

9. REFLECTION

- How are recursive rules different from explicit function rules for modeling linear data?
- How are they the same?
- When are recursive rules more useful than function rules?
- When are function rules more useful?

10. EXTENSION: Think of a situation that can be described by a linear function. Model the situation using a recursive rule and a function rule. Write a question that is better answered using the recursive rule and give the solution.

Student: ______________________________ Class: ____________________ Date:______________

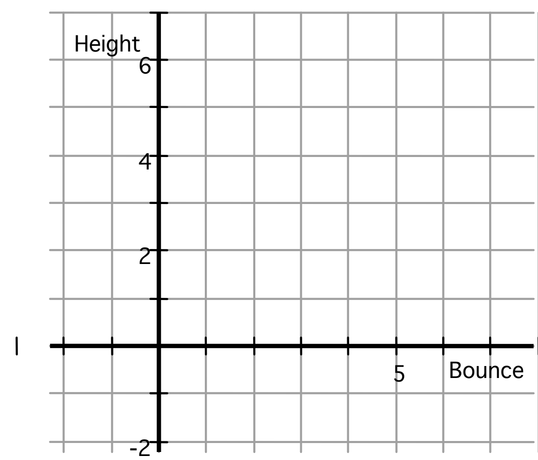
Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

IV.B Student Activity Sheet 3: Recursion and Exponential Functions

Different balls bounce at various heights depending on things like the type of ball, the pressure of air in the ball, and the surface on which it is bounced. The rebound percentage of a ball is found by determining the quotient of the rebound height (that is the height of each bounce) to the height of the ball before that bounce, converted to a percentage.

1. Collect data on a bouncing ball that show the maximum height of at least five bounces of the ball. Then make a scatterplot of the maximum height as a function of the bounce number. (Let Bounce 0 be the initial drop height of the ball.)

Bounce No.	Height
0	
1	
2	
3	
4	
5	



2. Find the average rebound percentage for your ball. Show your work.

Bounce No.	Height	Process	Rebound Percentage
0			
1			
2			
3			
4			
5			

Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

IV.B Student Activity Sheet 3: Recursion and Exponential Functions

3. Tennis balls are sealed in a pressurized container to maintain the rebound percentage of the balls. A tennis ball has a rebound percentage of 55% when it is taken out of the pressurized can. Suppose a tennis ball is dropped from a height of 2 meters onto a tennis court. Use the rebound rate given to predict the height of the ball's first seven bounces.

Bounce No.	**Process**	**Height (m)**
0	(initial drop height given)	2
1		
2		
3		
4		
5		
6		
7		

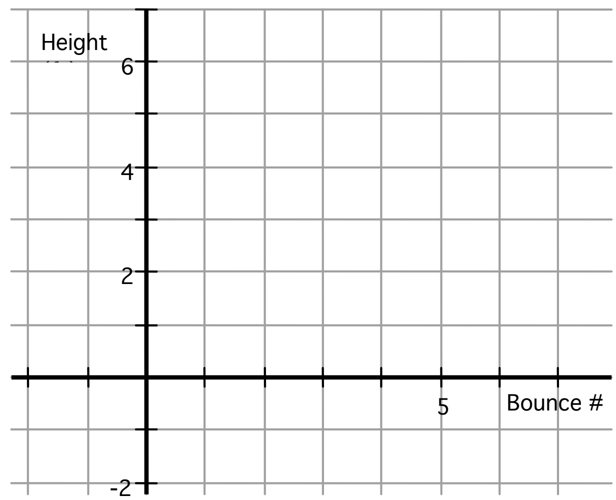
4. Write a recursive rule for the height of the ball for each successive bounce.

5. Describe, in words, how the height of each bounce is calculated from the height of the previous bounce.

Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

IV.B Student Activity Sheet 3: Recursion and Exponential Functions

6. Enter the bounce height data into a graphing calculator. Make a scatterplot and then sketch the graph below.



7. What kind of function might model the tennis ball bounce situation? Explain your reasoning with a table of values or other representation.

8. Look back at the table you generated in Question 3. Write a function rule for bounce height in terms of bounce number. Graph the function rule with the scatterplot on your graphing calculator to see if the function rule models the data.

9. What is the height of the fifth bounce of a new tennis ball if the initial drop height is 10 meters above the ground? Use a function rule to find your answer.

10. Suppose a new tennis ball is dropped from a height of 20 feet. How many times does it bounce before it has a bounce height of less than 4 inches (the diameter of the ball)? Explain your solution.

11. What is the total vertical distance that the ball from Question 10 has traveled after six bounces? Explain your answer.

12. **REFLECTION:** How can you decide if a data set can be modeled by an exponential function? How are recursive rules different from function rules for modeling exponential data? How are they the same?

13. EXTENSION: As our population uses more antibiotics for minor infections, bacteria adapt and become resistant to the medications that are available. Methicillin-resistant *Staphylococcus aureus* (MRSA) is strain of *Staphylococcus* bacteria that is resistant to antibiotics. MRSA causes skin and respiratory infections and can be fatal. The Center for Disease Control (CDC) reported that in the United States 94,360 MRSA infections occurred in 2005, with 18,650 of these cases resulting in deaths. To put this number in perspective, CDC reported 16,316 deaths in 2005 related to AIDS.

A research lab is observing the growth of a new strain of MRSA in an agar dish. The initial area occupied by the bacteria is 2 square millimeters. In previous experiments with MRSA, scientists observed the bacteria to increase by 20% each week.

- Fill in the table, showing the increase in area of the bacteria over eight weeks.
- Write a recursive rule for the area (***a***) of the bacteria after ***n*** weeks.
- Then make a scatterplot of the data.

Week	Process	Area (mm^2)
0		2
1		
2		
3		
4		
5		
6		
7		
8		
n		

a. How are the data in this situation the same and different from the data in the rebound rate situation? Consider the data, recursive rule, and graph in your response.

b. Write a function rule that models the area of MRSA bacteria in terms of the number of weeks they grow. Describe where the values in the function rule appear in the data. Graph the function rule with a scatterplot to check the rule.

c. Use the function rule to predict the area of the MRSA bacteria after 20 weeks.

14. EXTENSION

- Research a set of population data. Use data from a country's census report or other state/national population data or animal population data.
- Cite the source of the data.
- Decide if the data, or a portion of the data, can be modeled by an exponential function.
- Support your decision with a mathematical argument.
- Find an appropriate model for the data.
- Based on your model, make a prediction about the population.

Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

IV.B Student Activity Sheet 4: Comparing Models

Derrick is trying to save money for the down payment on a used car. His parents have said that, in an effort to help him put aside money, they will pay him 10% interest on the money Derrick accumulates each month. At the moment, he has saved $200.

1. Suppose Derrick does not add any money to the savings. Write a recursive rule and an explicit function rule that model the money Derrick will accumulate with only the addition of the interest his parents pay.

2. How long will it take Derrick to save at least $2,000 for the down payment if the only additions to his savings account are his parents' interest payments?

3. In an effort to speed up the time needed to save $2,000, Derrick decides to take on some jobs in his community. Suppose he commits to adding $50 per month to his savings, starting with the initial deposit from his parents. Fill in the table, showing the amount of money Derrick will have over several months.

Months	Process	Dollars
0		200
1		
2		
3		
4		
5		
6		
7		
8		

4. Make a scatterplot of the data you generated in the table and compare the scatterplot to the function rule you found for Question 1. How does adding $50 per month to Derrick's savings change the way in which his money grows?

Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

IV.B Student Activity Sheet 4: Comparing Models

5. How long will it take Derrick to save $2,000 for the down payment if he continues to add $50 every month? Explain how you arrived at your answer.

6. **REFLECTION:** How would you write a recursive routine to model this situation? A function rule? Explain your reasoning for each type of rule and compare your responses.

7. **EXTENSION:** Suppose Derrick changes the amount of money he adds to his savings each month to $100. How does this affect the time it takes to save $2,000? How much does he have to add to the savings each month to have enough money for the down payment on his car in six months? Explain your responses.

Student: ______________________________ Class: ____________________ Date:______________

Using Recursion in Models and Decision Making: Recursion Using Rate of Change

IV.C Student Activity Sheet 5: Newton's Law of Cooling

Have you ever noticed that a container of cold liquid, such as a glass of iced tea, creates condensation on the outside of the container? Or that a cup of hot coffee does not always stay hot?

What happened to the temperatures of those cups of liquid? In this activity, you will investigate changes in the temperature of a liquid over time.

1. Suppose your teacher poured a cup of hot coffee at the beginning of class, set it on her desk, and then forgot about it. What would happen to the temperature of the coffee over time? Why do you think this is so?

2. Sketch a graph of the coffee's temperature over time.

Using Recursion in Models and Decision Making: Recursion Using Rate of Change

IV.C Student Activity Sheet 5: Newton's Law of Cooling

3. Collect a set of data showing the temperature (degrees Fahrenheit) of a cup of hot liquid, such as coffee, changing over time (minutes). Then complete the column that asks for the difference between the temperature of the liquid and the room temperature. The remaining columns will be used in the next question.

Time (t)	Temperature of Liquid (L)	Temperature of Liquid (L) – Room Temperature (T)	First Differences (ΔT)	Successive Ratios

Using Recursion in Models and Decision Making: Recursion Using Rate of Change

IV.C Student Activity Sheet 5: Newton's Law of Cooling

4. Complete the table by computing the first differences and successive ratios. Use your calculator or spreadsheet to help with the computation.

5. Is the relationship between time and temperature linear or exponential? How do you know?

6. Use the information in the table to build a recursive rule for the difference in temperature from room temperature for each successive temperature reading.

7. What does the constant in the recursive rule represent?

8. Use your graphing calculator to make a scatterplot of temperature versus time. Sketch your results.

9. How does your scatterplot compare to the graph you sketched at the beginning? Explain any differences.

10. How does your scatterplot support the type of relationship you chose in Question 5?

11. The general form for an exponential function is $y = a(b)^x$, where a represents the initial condition and b represents the successive ratio, or base of the exponential function. Using data from your table, write a function rule to describe the temperature of the coffee (y) as a function of time (x).

12. What do the constants in the function rule represent?

13. Graph the function rule over the data in your scatterplot. Sketch your results.

14. **REFLECTION:** Compare the recursive rule and explicit function rule that you wrote in the previous questions. What do you notice?

If you repeated this experiment in a room that was much cooler, what changes in your data would you expect? Why do you think so?

15. What would a scatterplot of the change in temperature (ΔT) versus the difference between the liquid's temperature and the room temperature (T) look like? Sketch your prediction, if needed.

Using Recursion in Models and Decision Making: Recursion Using Rate of Change

IV.C Student Activity Sheet 5: Newton's Law of Cooling

16. Use your graphing calculator to make a scatterplot of the change in temperature ($\Delta \boldsymbol{T}$) versus the difference between the liquid's temperature and the room temperature ($\boldsymbol{T}$). Sketch your graph.

How does your graph compare to your prediction?

What kind of function appears to model your graph?

17. EXTENSION: Use an appropriate regression routine to find a function rule to model your scatterplot of $\Delta \boldsymbol{T}$ versus $\boldsymbol{T}$. Record your function rule, rounding to an appropriate number of places.

Graph the function rule over your scatterplot. How well does it fit the data?

Recall that a proportional relationship between the independent and dependent variables satisfies three criteria:

- The graph is linear and passes through the origin.
- The function rule is of the form $\boldsymbol{y} = \boldsymbol{kx}$.
- The ratio $\frac{y}{x}$ is constant for all corresponding values of $\boldsymbol{x}$ and $\boldsymbol{y}$.

18. EXTENSION: Is your function rule from Question 17 a proportional relationship? Defend your answer using all three criteria.

Graph:

Function Rule:

Table:

Using Recursion in Models and Decision Making: Recursion Using Rate of Change

IV.C Student Activity Sheet 5: Newton's Law of Cooling

19. REFLECTION: Suppose the relationship between the change in temperature and the difference between the liquid's temperature and the ambient temperature is proportional. Write a proportionality statement to show the relationship between the two variables.

	is proportional to	

20. EXTENSION: Choose a simple exponential function to validate the proportionality according to all criteria.

21. EXTENSION: Use the Internet to research other situations that could be modeled using a function from the same family as the one for Newton's Law of Cooling. Obtain a data set and generate a function model. Cite your sources. Support your choice of function model with mathematical reasoning. Make a prediction from your data set using your model. How reasonable is your prediction?

Student: ___________________________ Class: ____________________ Date:______________

Using Recursion in Models and Decision Making: Recursion Using Rate of Change

IV.C Student Activity Sheet 6: Rates of Change in Exponential Models

1. Consider the exponential function $y = 2^x$. Fill in the table of values for the function, and find the rate of change between consecutive values in the function (Δy). What pattern do you see for Δy change?

x	y	Δy
0		
1		
2		
3		
4		
5		

2. In Student Activity Sheet 3, you learned about the sometimes fatal antibiotic-resistant staph bacteria methicillin-resistant *Staphylococcus aureus* (MRSA) growing in an agar dish. The initial area occupied by the bacteria in the agar dish is 2 square millimeters, and they increase in area by 20% each week. The table below gives the area of the bacteria over several weeks. Use the table to describe the rate of growth of the area of the bacteria (Δa).

x (no. of weeks)	a (area in mm^2)	Δa
0	2	
1	2.4	
2	2.88	
3	3.456	
4	4.147	
5	4.977	

Using Recursion in Models and Decision Making: Recursion Using Rate of Change

IV.C Student Activity Sheet 6: Rates of Change in Exponential Models

3. Suppose a quantity increases at a rate proportional to the quantity, and the constant of proportionality is 0.2. The initial quantity is 2. Write a difference equation that describes the statement above, and find several values of this quantity.

x	Δy	y
0		2
1		
2		
3		
4		

4. Use spreadsheet software to generate about 75 values of the table you started in Question 3. The spreadsheet allows you to use a recursive rule to generate the data.

5. The agar dish that the MRSA bacteria are growing in has an area of 1,000 square millimeters. The growth of the bacteria is limited in the lab by the size of the agar dish. The growth of the bacteria can still be modeled by a proportional difference equation, but now the rate of increase of the bacteria's area is directly proportional to the bacteria's area and the difference between the agar dish's area and the fungus's area. This constant of proportionality is the ratio of the original constant of proportionality (0.2) and the maximum area the bacteria can reach (1,000 square millimeters). The difference equation can be written as follows:

$$\frac{\Delta A}{\Delta t} = \frac{0.2}{1{,}000} A(1{,}000 - A)$$

Let $\Delta t = 1$ to simplify your work, and then use the difference equation to find the new values of A. Use a spreadsheet to calculate about 75 values.

6. Compare the data generated in the unrestricted and restricted growth models. Record your observations.

7. Use spreadsheet software or your graphing calculator to make a graph of the restricted growth model. Sketch the graph below. What observations can you make about the graph?

8. **REFLECTION**: The unrestricted bacteria growth models exponential growth and has a common ratio of 1.2. Use the spreadsheet to find the ratio between successive values in the restricted growth model. What do you notice? How does this support the graph in Question 7?

9. A rancher has decided to dedicate a 400-square-mile portion of his ranch as a black bear habitat. Working with his state, he plans to bring 10 young black bears to the habitat in an effort to grow the population. His research shows that the annual growth rate of black bears is about 0.8. Black bears thrive when the population density is no more than about 1.5 black bears per square mile.

   a. What is the maximum sustainable number of black bears for the habitat?

   b. Write a recursive rule showing the restricted growth in population for the black bears. (***Hint:*** The constant of proportionality is the ratio of the unrestricted growth rate and the maximum sustainable population.)

   c. Make a table and graph showing the yearly population of the black bears in the habitat. (Include enough years to show the population reaching the maximum sustainable population.)

   d. When will the population of bears in the habitat reach 500?

   e. The rancher wants to repopulate the state with black bears. The rancher's original plan was to release the bears from his ranch when the population reaches 500. Do you think this is a good decision based on the growth rate within the habitat over time? If you agree with the rancher, support the decision with your data and graph. If you disagree, propose a different target population value to the rancher; again, support your proposal with the data and graph.

10. **EXTENSION**: Research population data, either of humans in various parts of the world or animal species. You need to find data over a significant time, not just a few years. Cite your source. Make a scatterplot of the data. Do the data show exponential growth or do they show signs that the population's growth is slowing? What limitations does the population you are analyzing have? Could you predict a maximum population? Support your prediction.

Student: ___________________________ Class: ____________________ Date:______________

Using Recursion in Models and Decision Making: Recursion in Cyclical Models

IV.D Student Activity Sheet 7: Modeling the Singapore Flyer

On February 11, 2008, Singapore opened a new observation wheel called the Singapore Flyer. At the time of its opening, this giant Ferris wheel was the tallest in the world. The Singapore Flyer consists of an observation wheel with a diameter of 150 meters atop a boarding terminal, giving the structure an overall height of 165 meters. Twenty-eight air-conditioned capsules rotate on the outside of the wheel to provide unobstructed views of the city. The wheel rotates at a constant rate of 26 centimeters per second. This is slow enough that the wheel does not need to stop for loading and unloading unless there are special passenger needs.

1. Using graph paper, draw an accurate diagram of the wheel showing the dimensions given above. Use a compass or other tool to accurate draw the circle.

2. On the next page, fill in the table showing the height of a single capsule changing as it rotates counterclockwise from the boarding terminal around the wheel. To do this, calculate the circumference of the wheel.

  a. How many minutes does it take a capsule to make one complete revolution around the wheel? (Round to the nearest minute.) Explain your process.

  b. Before completing the table, explain how the angle values provided in the table are correct.

  c. The first inscribed angle that models the situation after one minute is shown. Use additional diagrams and inscribed right triangles to determine more values of the total height as a given capsule continues to rotate through one complete revolution. Use trigonometry to calculate the corresponding values of ***a*** and complete the table, finding the height of the capsule at the various intervals of time.

Using Recursion in Models and Decision Making: Recursion in Cyclical Models

IV.D Student Activity Sheet 7: Modeling the Singapore Flyer

Time (min)	Angle of Rotation from Boarding Station	Vertical Leg of Right Triangle (*a*)	Process (If you decide to change your process, explain your decision.)	Total Height of Capsule (m)
1	12°			
30	360°			

3. Create a graph showing the height changing as a given capsule rotates through one complete revolution of the wheel. Show at least 10 well-spaced data points on your graph.

4. Use spreadsheet software or a graphing calculator to model the height of a capsule as it continues to rotate around the wheel. Show at least 90 minutes of rotation. Create a graph of the data.

5. On your graph, label the period and amplitude of the curve. How do these values on your mathematical model relate to the physical context of the Singapore Flyer?

6. **REFLECTION:** In this problem, your classmates used different methods to solve this problem. Now that you have seen the different processes, which were most useful? If you had to do this problem using a different process than the one you used, which method would you choose? What information would you need to know? What calculations would you have to do differently? How were trigonometric ratios used in the different processes??

7. **EXTENSION**

   - Research another gigantic Ferris wheel. Find the dimensions of the wheel and how long it takes for the wheel to complete one revolution. Using that information, make a rough sketch of the height of a capsule over time.

   OR

   - What other situations can you think of that repeat themselves and could possibly be modeled with a periodic function? (Some examples include volume of air in lungs over time during rest, a pendulum swinging, the bounce of a sp***rin***g, sea levels affected by tides, and sound waves.)

Advanced Mathematical Decision Making

In Texas, also known as

Advanced Quantitative Reasoning

Unit V: Using Functions in Models and Decision Making

This course is a project of
The Texas Association of Supervisors of Mathematics and
The Charles A. Dana Center at The University of Texas at Austin
With support from the Greater Texas Foundation

2010

Advanced Mathematical Decision Making

In Texas, also known as

Advanced Quantitative Reasoning

Student Materials

These student materials are excerpted from one of seven units that make up the 2010 AMDM/AQR instructional materials (developed under the name Advanced Mathematical Decision Making).

Unit I: Analyzing Numerical Data
Unit II: Probability
Unit III: Statistical Studies
Unit IV: Using Recursion in Models and Decision Making
Unit V: Using Functions in Models and Decision Making
Unit VI: Decision Making in Finance
Unit VII: Networks and Graphs

Table of Contents

Student: ______________________________ Class: ______________________ Date:________________

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 1: Analyzing Linear Regression Equations

One factor that talent scouts look for in potentially competitive swimmers is the ratio of their height to their arm span. For most people, arm span is generally equal to height. Consider U.S. Olympic swimmer Michael Phelps, who is 6 feet, 4 inches (193 centimeters) tall with an arm span of 6 feet, 7 inches (200 centimeters). In fact, the U.S. swim team found that its male swimmers have an average height of 187.1 centimeters and an average arm span of 192.9 centimeters. Of course, other factors influence the success of a swimmer, but coaches often look at a swimmer's physical attributes, including arm span, to determine which strokes he or she should focus on.

At a local competitive swim club, the coach measured the height and arm span of his top 10 swimmers. The data are shown in the table below.

Height (cm)	Arm Span (cm)
172	173
173	175
179	182
180	185
183	187
186	189
187	186
190	195
191	191
192	196

1. Enter the data given in the table into your graphing calculator and make a scatterplot. Sketch the graph below and describe it in words.

2. Use what you know about the situation and the data to find a function model for this data set. Explain your reasoning.

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 1: Analyzing Linear Regression Equations

3. Use your graphing calculator to compute a regression analysis of the swimmers' arm spans in relation to their height. What does the information from the calculator tell you? How does the equation given by the calculator compare to the function you found in Question 2?

4. Work in a group of four students. Each group member enters one of the data sets below into a graphing calculator, makes a scatterplot, and performs a linear regression analysis. Compare the graphs and the values of the correlation coefficients (***r***). Record an observation about how the value of ***r*** describes the strength and direction of the relationship between the variables.

x	*y*
-2	-4
-1	-2
0	0
1	2
2	4

x	*y*
-2	2
-1	-3
0	0
1	-2
2	5

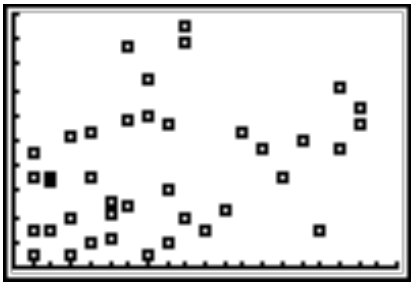
x	*y*
-2	9
0	0
1	7
5	-2
7	4

x	*y*
0	8
1	5
2	4
3	1
4	0

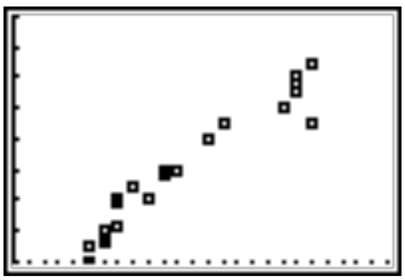
Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 1: Analyzing Linear Regression Equations

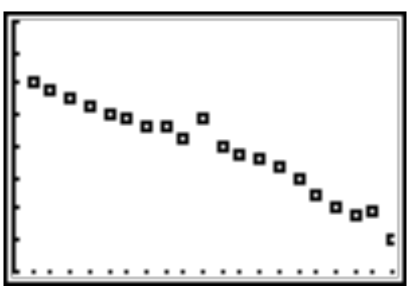
5. Consider each scatterplot below. Draw a line to match each ***r***-value to a scatterplot.



r = 0.972



r = 0.333



r = -0.976

6. **REFLECTION**: Does a strong correlation indicate a cause-and-effect relationship between variables? Give examples to justify your response.

7. **EXTENSION:** Think of a situation that might have a linear relationship. Research the situation to find data relating the variables and perform a linear regression analysis on the data. Make sure your data set is of ample size. Use the regression analysis to determine a model and describe the strength of the model.

Student: ______________________________ Class: ____________________ Date: ______________

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 2: Comparing Linear and Exponential Functions

Coen sells magazine subscriptions. He is paid \$20 to start selling and then earns \$1.50 for each subscription he sells. The table shows the amount of money (***M***) Coen earns for selling ***n*** subscriptions.

n	*M*
0	\$20.00
1	\$21.50
2	\$23.00
3	\$24.50
4	\$26.00

1. In previous work, you wrote a linear function rule describing the amount of money Coen earns as a function of the number of subscriptions he sells. What do the domain and range of this situation represent?

2. Fill in the blanks below to find the differences between the given entries in the table. For each table, make a statement summarizing the relationship between changes in the domain and changes in the range.

n	*M*
0	\$20.00
1	\$21.50
2	\$23.00
3	\$24.50
4	\$26.00

n	*M*
0	\$20.00
1	\$21.50
2	\$23.00
3	\$24.50
4	\$26.00

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 2: Comparing Linear and Exponential Functions

3. Suppose Coen's earning structure changed so that for every magazine subscription he sold, he made 1.5 times his previous earnings. Again, assume that he starts with $20 for 0 subscriptions sold. Make a table showing Coen's earnings.

n	M
0	
1	
2	
3	
4	

4. In Question 2, you analyzed changes in the domain values and their impact on the values in the range. Now analyze the new data set you found in Question 3. Do these data show the same kind of "add-add" relationship as in the linear relationship in Question 2? Describe the effect on values in the range for this new set of data when values in the domain are changed incrementally by adding 1. Is this relationship the same when adding 2 to each domain value? Adding 5? Explain your answers.

5. **REFLECTION:** Describe a fundamental difference between linear and exponential functions based on a look at tables of values. How is the rate of change of a **linear function** different than the rate of change of an **exponential function**?

6. **EXTENSION:** Describe two additional "add-add" relationships that exist in real-world applications, and provide at least two representations of the relationships. Describe two additional "add-multiply" relationships that exist in real-world applications, and provide at least two representations of the relationships. Be prepared to share your examples with the class.

Student: ___________________________________ Class: ________________________ Date:________________

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

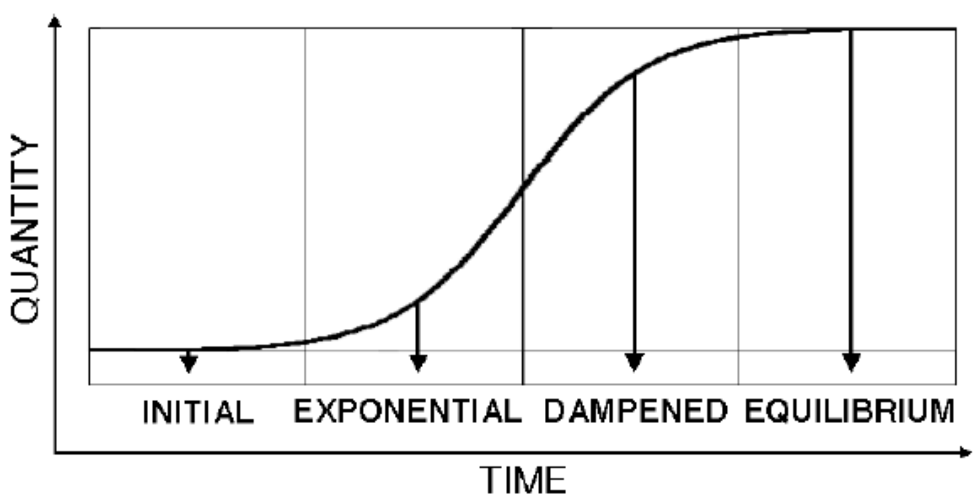
V.A Student Activity Sheet 3: Growth Model

H1N1—two letters and two numbers—are memorable as the most recent and perhaps greatest public health concern of this decade. The outbreak of this strain of influenza as most similar outbreaks can be simulated using mathematical techniques and models you are familiar with.

The simulation in this activity may create duplications or repetitions. For example, two people may both infect the same person. What are other possibilities of duplications or repetitions in a random number generating based simulation?

These duplications and repetitions are a desired aspect of the simulation because they signal the change from one stage of the simulation to the next stage.

The four stages of are labeled in the following graph. Remember the scenario you are considering here—the spread of the flu virus.



Intervals showing four stages of growth

1. What is happening with the spread of the flu virus in the graph?

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions
V.A Student Activity Sheet 3: Growth Model

2. Use the following simulation procedure to complete the table on the next page. This simulates the introduction of the flu virus to a closed environment or population by means of a single infected individual.

Imagine a total population of 100 individuals. Each number from 0-99 in the Hundreds Chart represents an individual, with the number 0 used to portray the original host. Use the Hundreds Chart to keep track of the infected individuals by crossing off their number on the list as they become infected.

Hundreds Chart

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Day 1: The original host infects a person represented by a randomly generated number. Generate a random integer between and including 0 and 99 using your graphing calculator or some other random number generating tool. Mark that person in the chart.

Day 2: The two infected people from Day 1 now infect two people, so generate two random integers.

Continue to simulate the rest of the days, completing the table of data up to Day 6.

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 3: Growth Model

Day	Number of initially infected people	Number of newly infected people	Total number of infected people
1	1	1	1
2	1		
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 3: Growth Model

3. How is the number of infected people growing? What function would you use to model these data?

4. Make a scatterplot of the data from Days 1–6. Determine and record the model that best fits the data set. How do you know this model is best?

5. What are the independent and dependent variables in this model?

6. Graph your function rule over your scatterplot of Days 1-6 data. How well does the function rule fit your data?

7. Use your regression equation to predict the number of infected persons by Day 10. What conclusions can you draw from the data and predictions to this point?

8. Add Days 7-9 to the table of simulated data.

9. REFLECTION: What do you expect to occur as additional days are simulated? Why do expect this?

10. Complete the table, recording your simulations through Day 15.

11. Make a scatterplot of the day related to the total number of people infected with the flu virus.

12. You should recognize this graph from your work in the previous unit as the *logistic* graph. Use the regression capabilities of your graphing calculator to determine the function rule that best fits this data. Then graph this function rule over the scatterplot.

13. How well does the function rule fit the data?

14. EXTENSION: The graph of the *logistic function* displays *asymptotic behavior*. Investigate the meaning of an *asymptote* and describe why this graph in fact demonstrates this behavior. Describe another scenario where the data and resulting graph are similar to this type of graph and behavior.

Student: ___________________________ Class: ____________________ Date:______________

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

You may have noticed that during the winter the days are shorter and during the summer the days are longer. How much longer are days during the summer? Does the length of summer days change depending on the latitude of a place?

You will investigate these questions using data from four different cities at four different latitudes:

- Houston, Texas—30˚N latitude
- Philadelphia, Pennsylvania—40˚N latitude
- Winnipeg, Manitoba, Canada—50˚N latitude
- Porto Alegre, Brazil—30˚S latitude (addressed in Student Activity Sheet 5)

The data in the tables for this activity describe the length of daylight for the year 2009. The data table is based on two assumptions:

- The length of daylight is defined as the amount of elapsed time between sunrise and sunset.
- Because 2009 is not a leap year, there are 365 days in the year.

Which city would you expect to have more daylight during the summer, Houston or Philadelphia? Why do you think so?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

Part A: Houston

1. Make a scatterplot of the length of daylight by day number for Houston on the blank grid provided at the end of this activity sheet (Length of Daylight for Cities). To make the graph easier, make January 1 = Day 1 and December 31 = Day 365. In addition, graph the length of daylight in terms of minutes.

2. Enter the data into the stat lists of your graphing calculator. Use the calculator to make a scatterplot of the length of daylight by day number for Houston. Sketch your graph and describe your axes and scaling.

3. Use your calculator to generate a sinusoidal regression model. Record the equation (round values to the nearest hundredth) in the Summary Table at the end of this activity sheet. Factor the value of ***b*** from the quantity (***bx - c***) and include that form of the equation as well.

4. Graph your model over your scatterplot. How well does the model fit your data?

Date	Day Number	Houston	
		HH:MM	Min.
Jan. 1	1	10:17	617
Feb. 1	32	10:48	648
March 1	60	11:34	694
Apr. 1	91	12:29	749
May 1	121	13:20	800
June 1	152	13:57	837
July 1	182	14:01	841
Aug. 1	213	13:33	813
Sept. 1	244	12:45	765
Oct. 1	274	11:52	712
Nov. 1	305	11:00	660
Dec. 1	335	10:23	623

Source: U.S. Naval Observatory, www.usno.navy.mil

5. Connect the points on your paper scatterplot with a smooth curve to represent the regression model.

6. Use your calculator to determine the maximum and minimum values for the length of daylight by day in Houston. Record these ordered pairs in your Summary Table and label them on your scatterplot. To which dates do these values correspond?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

Part B: Philadelphia

1. Make a scatterplot of the length of daylight by day number for Philadelphia. Plot the points on the same grid that you used for the Houston scatterplot.

2. Enter the data for Philadelphia into a third list and graph the scatterplots for Houston and Philadelphia on the same screen. Sketch your graph and describe your axes and scaling.

3. Use your calculator to generate a sinusoidal regression model for the Philadelphia data. Record the equation (round values to the nearest hundredth) on the Summary Table. Factor the value of ***b*** from the quantity (***bx* - *c***) and include that form of the equation as well. Graph your model over your scatterplot. How well does the model fit your data?

4. Connect the points on your paper scatterplot with a smooth curve to represent the regression model for Philadelphia.

Date	Day Number	Philadelphia	
		HH:MM	Min.
Jan. 1	1	9:23	563
Feb. 1	32	10:11	611
March 1	60	11:19	679
Apr. 1	91	12:41	761
May 1	121	13:56	836
June 1	152	14:46	886
July 1	182	14:57	897
Aug. 1	213	14:15	855
Sept. 1	244	13:03	783
Oct. 1	274	11:46	706
Nov. 1	305	10:28	628
Dec. 1	335	9:33	573

Source: U.S. Naval Observatory, www.usno.navy.mil

5. How do the regression models compare for Houston and Philadelphia?

   Similarities:

   Differences:

6. Use your calculator or graph to determine the maximum and minimum values for the length of daylight in Philadelphia. Record these ordered pairs in the Summary Table and label them on your paper scatterplot. To which dates do these values correspond?

7. How does the maximum length of daylight for Philadelphia compare to the maximum length of daylight for Houston?

8. **REFLECTION:** How does your answer to Question 7 compare to the prediction you made at the beginning of this activity?

9. Determine the intersection points of the regression models for Houston and Philadelphia. Record these ordered pairs in the Summary Table and label them on your scatterplot.

10. What do the intersection points mean in the context of this situation? ***Hint:*** Recall that your scatterplot shows the ordered pairs (Day Number, Length of Daylight) for Houston and Philadelphia.

11. **REFLECTION:** When is there more daylight in Houston than in Philadelphia? Is this what you expected? Why or why not?

    When is there less daylight in Houston than in Philadelphia? Is this what you expected? Why or why not?

12. What is the difference in latitude between Houston and Philadelphia?

13. What is the difference in latitude between Philadelphia and Winnipeg?

14. What would you expect a scatterplot of length of daylight by day number for Winnipeg to look like? Why?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

Part C: Winnipeg

1. Make a scatterplot of the length of daylight by day for Winnipeg. Plot the points on the same grid that you used for the other two scatterplots.

2. Enter the data for Winnipeg into a fourth list and graph all three scatterplots on the same screen. Sketch your graph and describe your axes and scaling.

3. Use your calculator to generate a sinusoidal regression model for the Winnipeg data. Record the equation in your Summary Table (round values to the nearest hundredth). Factor the value of ***b*** from the quantity (***bx* - *c***) and include that form of the equation as well. Graph your model over your scatterplot. How well does the model fit your data?

4. Connect the points on your paper scatterplot with a smooth curve to represent the regression model for Winnipeg.

Date	**Day Number**	**Winnipeg**	
		HH:MM	**Min.**
Jan. 1	1	8:12	492
Feb. 1	32	9:23	563
March 1	60	11:01	661
Apr. 1	91	12:56	776
May 1	121	14:43	883
June 1	152	16:04	964
July 1	182	16:15	975
Aug. 1	213	15:11	911
Sept. 1	244	13:28	808
Oct. 1	274	11:37	697
Nov. 1	305	9:46	586
Dec. 1	335	8:25	505

Source: U.S. Naval Observatory, www.usno.navy.mil

5. How do the regression models compare for all three cities?

   Similarities:

   Differences:

6. Use your calculator to determine the maximum and minimum values for the length of daylight in Winnipeg. Record these ordered pairs in your Summary Table and label them on the paper scatterplot. To which dates do these values correspond?

7. Use your scatterplot to compare the points of intersection for all three graphs. What do they mean in the context of this situation?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

8. The town of Seward, Alaska, is at 60°N latitude, just south of Anchorage, Alaska. What would you expect the length of daylight during the summer months to be in Seward compared to Winnipeg? The winter months?

9. What relationship do you think there is between a city's latitude and the amount of daylight it receives throughout the year?

10. **REFLECTION:** Describe how this application of sinusoidal regression and latitude as related to length of daylight is similar to the model of the Singapore Flyer. Compare and contrast the two situations with regard to similarities and differences of the model, scatterplot(s), and the functional relationship.

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

Part D: Connections to Sinusoidal Functions

The parent function $y = \sin(x)$ can be transformed using four parameters. Each parameter describes a certain characteristic of the graph.

$y = A\sin[B(x - C)] + D$

- ***A*** represents the *amplitude* of the graph. The amplitude is the vertical distance from the horizontal axis of the graph to the maximum value or the minimum value of the graph. The amplitude is also equal to half of the difference between the maximum and minimum values.
- ***B*** represents the *angular frequency* of the graph. The angular frequency describes how many crests or troughs of the graph are present within a 360°or 2π portion of the domain of the graph. The angular frequency is also found by dividing 2π by the *period*, which is the horizontal distance between two consecutive maximum or minimum values.
- ***C*** represents the *phase shift*, or horizontal translation of a sine function.
- ***D*** represents a *vertical translation* of the graph. The line $y = D$ is the equation of the sinusoidal axis, which is the horizontal line representing the distance that is midway between the crests and troughs of the graph.

1. Look at the Houston row on the Summary Table. Subtract the maximum value of daylight from the minimum value of daylight, and then divide the difference by 2. How does this value compare to the amplitude (***A***) in the regression model?

2. Repeat the process of subtraction and division from Question 1 for Philadelphia and Winnipeg. What does this value suggest about the relationship between the maximum/minimum values and the amplitude for all three cities?

3. Divide 2π by the number of days in a year. How does the result compare to the angular frequency (***B***) for all three cities?

4. If the period of a sine function is the number of units before the cycle begins to repeat, why would the period of the regression model include a quotient with the number of days in a year?

5. How does the value for ***C*** (phase shift) in the factored form of your regression equations compare to the ***x***-coordinate of the first intersection points?

6. How many minutes are there in 12 hours? Why would the vertical translation (***D***) be a number that is close to this value?

7. Why did the values of ***B***, ***C***, and ***D*** remain close to the same for the regression models for all three cities? Why did the value of ***A*** change for the models?

8. **EXTENSION:** Sun path diagrams show the path of the sun as it travels across the sky from sunrise to sunset at a given point on the surface of Earth. Because the sun's path varies each day, a sun path diagram reveals the part of the sky where the sun would be located for an observer on the ground at that point.

Investigate cities at other latitudes, including those closer to the poles and the equator. Prepare a short presentation for the class.

Some cities whose data can be obtained via the Internet (www.gaisma.com) include the following:

- 80°N: Longyearbyen, Norway (78°N)
- 70°N: Barrow, Alaska (71°N)
- 60°N: Seward, Alaska; St. Petersburg, Russia; Anchorage, Alaska (61°N)
- 20°N: Guadalajara, Mexico; Mexico City (19°N); Honolulu, Hawaii (21°N)
- 10°N: Caracas, Venezuela; San Jose, Costa Rica
- 0°: Quito, Ecuador; Kampala, Uganda; Pontianak, Indonesia
- 10°S: Rio Branco, Brazil; Lima, Perú
- 20°S: Belo Horizonte, Brazil; Port Hedland, Australia
- 30°S: Durban, South Africa; Perth, Australia
- 40°S: Valdivia, Chile; San Carlos de Bariloche, Argentina
- 50°S: Stanley, Falkland Islands
- 60°S: Villa Las Estrellas, Chilean Antarctic Territory

Student: ______________________________ Class: ____________________ Date:______________

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

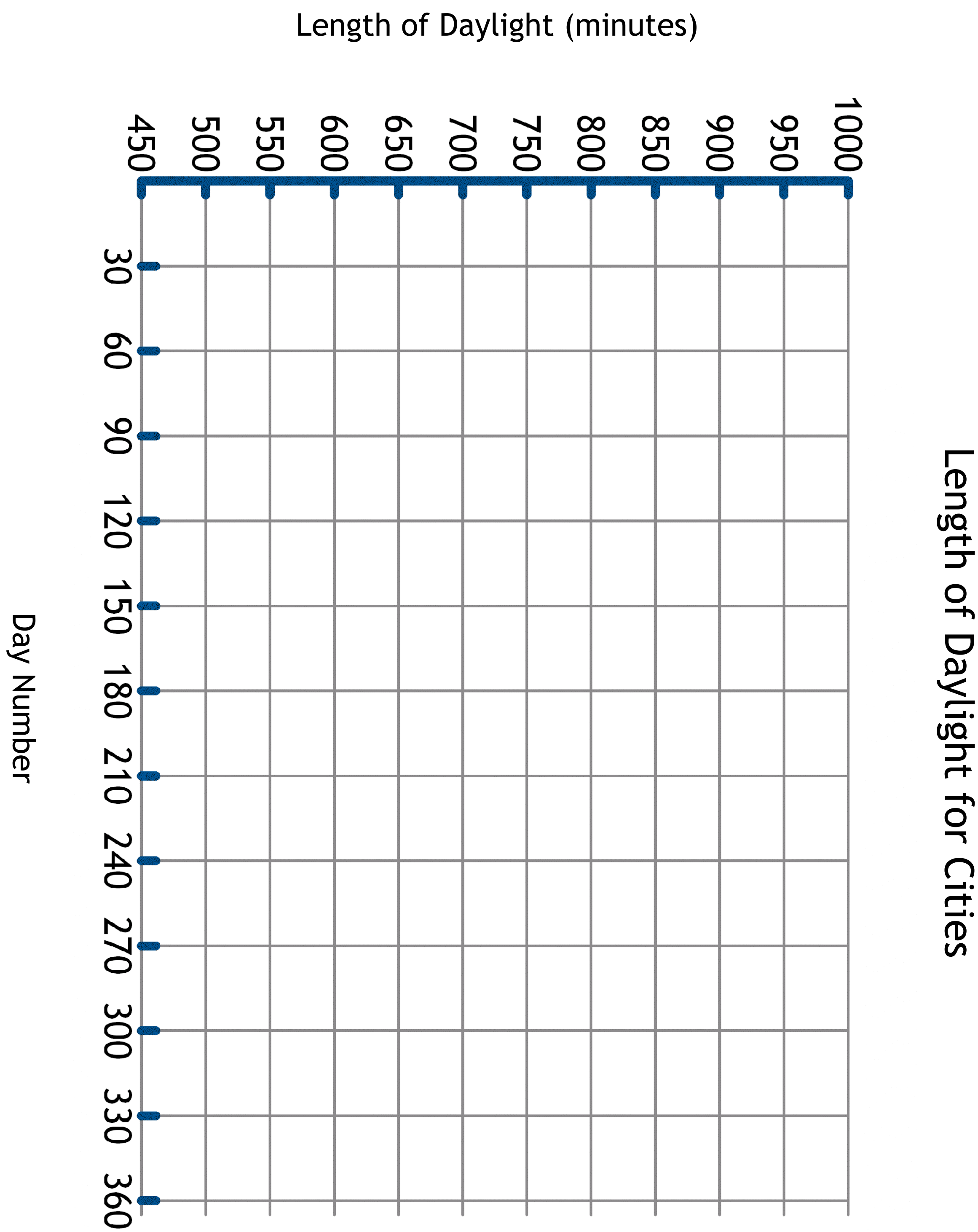
Summary Table for Length of Daylight

City	Regression Model	Maximum	Minimum	First Intersection	Second Intersection
Houston	Calculator form: Factored ***B***:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:		
Philadelphia	Calculator form: Factored ***B***:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:
Winnipeg	Calculator form: Factored ***B***:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:
Porto Alegre	Calculator form: Factored ***B***:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:

Student: ______________________ Class: ______________ Date: ___________

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight



Student: ______________________________ Class: ____________________ Date:______________

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 5: Crossing the Equator

You investigated the relationship between a city's latitude and the length of daylight it experiences throughout the year. You did so by making scatterplots and finding regression models for the functional relationship between the day of the year and the length of daylight for three different cities at three different latitudes in the Northern Hemisphere:

- Houston, Texas—30°N latitude
- Philadelphia, Pennsylvania—40°N latitude
- Winnipeg, Manitoba, Canada—50°N latitude

In this activity, you will investigate the relationship between two cities that are the same distance from the equator, but on opposite sides of it: Houston, Texas, and Porto Alegre, Brazil.

Remember that the data in the tables for this activity describe the length of daylight for the year 2009 for each day. The data table is based on two assumptions:

- The length of daylight is defined as the amount of elapsed time between sunrise and sunset.
- Because 2009 is not a leap year, there are 365 days in the year.

You will need your Summary Table and scatterplots from Student Activity Sheet 4.

1. Porto Alegre, Brazil, is located in the Southern Hemisphere at 30°S latitude. Houston, Texas, is located in the Northern Hemisphere at 30°N latitude. How do you think the graphs of the length of daylight by day would compare for the two cities? Sketch your prediction, if needed, and explain why it might be true.

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 5: Crossing the Equator

2. Make a scatterplot of the length of daylight by day in Porto Alegre, Brazil. Plot the points on the same grid that you used for the scatterplots from the previous activity.

Date	Day Number	Houston		Porto Alegre	
		HH:MM	Min.	HH:MM	Min.
Jan. 1	1	10:17	617	14:03	843
Feb. 1	32	10:48	648	13:29	809
March 1	60	11:34	694	12:42	762
Apr. 1	91	12:29	749	11:45	705
May 1	121	13:20	800	10:55	655
June 1	152	13:57	837	10:19	619
July 1	182	14:01	841	10:15	615
Aug. 1	213	13:33	813	10:42	642
Sept. 1	244	12:45	765	11:30	690
Oct. 1	274	11:52	712	12:23	743
Nov. 1	305	11:00	660	13:17	797
Dec. 1	335	10:23	623	13:56	836

Source: U.S. Naval Observatory, www.usno.navy.mil

3. How does the scatterplot for Porto Alegre compare to the scatterplot for Houston? Does this match your prediction? Why do you think this is so?

4. Use your calculator to generate a scatterplot of length of daylight by day for Houston. You may need to re-enter the data into your data lists. In addition, graph the regression equation that you found for Houston.

5. Enter the data for Porto Alegre into a third list and graph both scatterplots on the same screen. Sketch your graph and describe the axes and scaling.

6. Use your calculator to generate a sinusoidal regression model for the Porto Alegre data. Record the equation (round values to the nearest hundredth) in the Summary Table. Factor the value of ***b*** from the quantity (***bx* - *c***) and include that form of the equation as well.

7. Graph your model over your scatterplot. How well does the model fit your data?

8. Connect the points on your paper scatterplot with a smooth curve to represent the regression model.

9. How do the regression models for Houston and Porto Alegre compare?

Similarities:

Differences:

10. Use your calculator to determine the maximum and minimum values for the length of daylight in Porto Alegre. Record these ordered pairs in the Summary Table and label them on your scatterplot. To which dates do these values correspond?

11. How does the maximum length of daylight for Porto Alegre compare to the maximum length of daylight for Houston?

12. How does the minimum length of daylight for Porto Alegre compare to the minimum length of daylight for Houston?

13. **REFLECTION:** Based on your observations of Porto Alegre and Houston, what would you conclude about the longest and shortest days for two cities on opposite sides of the equator?

14. Determine the intersection points of the regression models for Houston and Porto Alegre. Mark these points on your scatterplot and record them in your Summary Table.

15. What do the intersection points mean in the context of this situation? ***Hint:*** Recall that your scatterplot shows the ordered pairs (Day Number, Length of Daylight) for Houston and Porto Alegre.

16. How do the intersection points for the graphs of Houston, Philadelphia, Winnipeg, and Porto Alegre compare? What do these points mean in terms of the context of this situation?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 5: Crossing the Equator

17. Suppose you made a scatterplot of the length of daylight by day for Philadelphia (40°N latitude) and San Carlos de Bariloche, Argentina (40°S latitude). Based on what you noticed about the graphs for Houston and Porto Alegre, what would you expect the two scatterplots to look like?

18. **REFLECTION:** What generalization could you make about the relationship between the length of daylight over time for two cities that are the same distance from the equator but on opposite sides of it (like Houston and Porto Alegre)?

19. **EXTENSION:** What would you expect a scatterplot of the length of daylight by day to look like for a city like Quito, Ecuador, which lies on the equator? Why do you think this is so? Use the Internet to find data for Quito and test your conjecture.

Student: ___________________________ Class: ____________________ Date: _______________

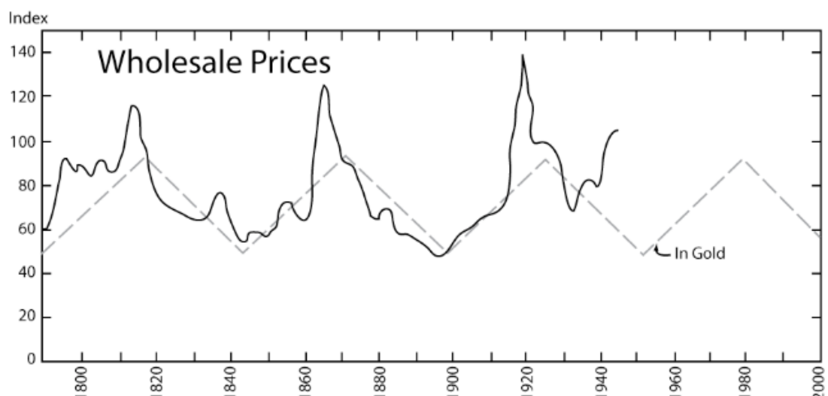
Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 6: Making Decisions from Cyclical Functions in Finance

Economists look for cycles to make predictions about the economy. Market traders also look for patterns in the prices of financial items such as stock, commodities, and currency value to make trades that yield the most money. In a cycle, prices rise and fall with a predictable regularity. If market traders can identify where in a cycle prices are, they can make decisions to increase their profit.

In 1947, economists Edward R. Dewey and Edwin R. Dakin published *Cycles—The Science of Predictions*, in which they identified a 54-year cycle in the wholesale price of goods. *Wholesale prices* are the prices that store owners pay the people who produce the goods (such as milk, gasoline, or chocolate chip cookies) to purchase the items to sell in their stores.

Dewey and Dakin presented a graph like the one shown below. The graph shows wholesale prices of goods in the United States in terms of a wholesale price index (WPI). The dashed line traces out the 54-year cycle that Dewey and Dakin describe.



1. According to the graph, in what years do there appear to be *peaks*, or relative maximum values in the wholesale prices?

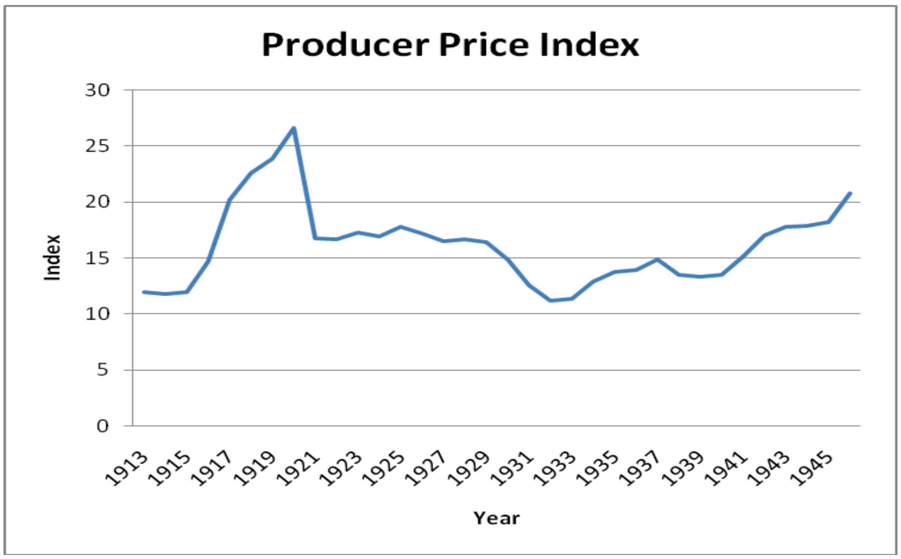
2. In what years do there appear to be *valleys*, or relative minimum values?

3. If there is a 54-year cycle between peaks and valleys, in what years should the next few maximum and minimum points occur?

Using Functions in Models and Decision Making: Cyclical Functions
V.B Student Activity Sheet 6: Making Decisions from Cyclical Functions in Finance

In 1978, the U.S. Bureau of Labor Statistics (BLS) reclassified the WPI that Dewey and Dakin used into the Producer Price Index (PPI). In 1982, the BLS reset the benchmark for the PPI to 100.0 for the annual value of the PPI. As a result, historical data had to be recalibrated to be used for comparisons over time.

4. The graph shows the PPI as it was recalibrated using an index of 100.0 to represent the value for 1982. How does this graph compare to the one used by Dewey and Dakin for their 1947 book?



5. The table at the right contains data from the BLS describing the commodity prices as measured by the PPI for certain years since 1940 (1982 = 100). Make a line graph of the PPI by year.

Year	PPI
1940	13.5
1944	17.9
1948	27.7
1952	29.6
1956	30.3
1960	31.7
1964	31.6
1968	34.2
1972	39.8

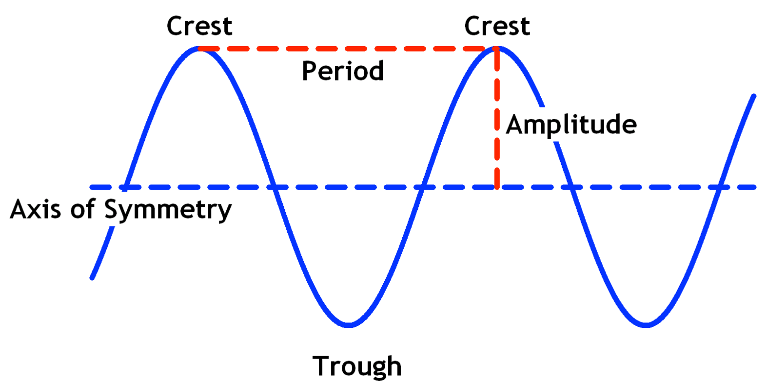
Year	PPI
1976	61.1
1980	89.8
1984	103.7
1988	106.9
1992	117.2
1996	127.7
2000	132.7
2004	146.7
2008	189.7

6. Compare your scatterplot to the 54-year cycle described by Dewey and Dakin. Is there a maximum value where the Dewey and Dakin model predicts there to be one? Why or why not?

Is there a minimum value where the Dewey and Dakin model predicts there to be one? Why or why not?

7. Does the trend in your scatterplot reveal the cyclical pattern Dewey and Dakin described in 1947?

Businesses use other cyclical models to describe seasonal phenomena. They refer to key attributes in cyclical models as shown below.



The *crest* is the maximum height of a wave, and the *trough* is the minimum height of a wave. The *period* is the distance between two consecutive crests or two consecutive troughs. The *axis of symmetry* is a horizontal line that runs exactly halfway between the crests and troughs. The *amplitude* is the distance between a crest or trough and the axis of symmetry.

8. Suppose that a particular business owner has determined that the function

$$\boldsymbol{y} = 200\sin(0.524(\boldsymbol{x} + 3.139)) + 400$$

can be used to determine the number of employees ($\boldsymbol{y}$) that he requires for month $\boldsymbol{x}$, where $\boldsymbol{x} = 1$ corresponds to January 1.

Use your calculator to graph this function. Sketch your graph using the horizontal values from 1 to 12 and vertical values from 0 to 700.

9. **EXTENSION:** Recall that sine functions can be represented using the general form $y = A\sin(B(x - C)) + D$, where

- ***A*** represents the amplitude,
- ***B*** represents the angular frequency,
- ***C*** represents a factor of a horizontal translation, and
- ***D*** represents the vertical translation.

For this function, determine the values of ***A*****,** ***B*****,** ***C*****,** and ***D***.

A =

B =

C =

D =

10. Find the length of one cycle by dividing 2π by the frequency (***B***).

11. What is the vertical translation? Graph the line ***y*** = ***D*** on your graphing calculator.

12. Determine the maximum and minimum values for number of employees. In what months do they occur?

13. How does the amplitude, combined with the vertical translation, describe the variation in number of employees needed for any given month?

14. When would you expect the next maximum value in the cycle to occur?

15. Change your viewing window so that you can see two full cycles of the graph, and determine the next maximum value. How does this compare with your prediction?

16. Suppose the economic conditions change, and the business owner needs between 300 and 900 employees during the seasonal cycle. Which parameters should change? What should the new numbers be?

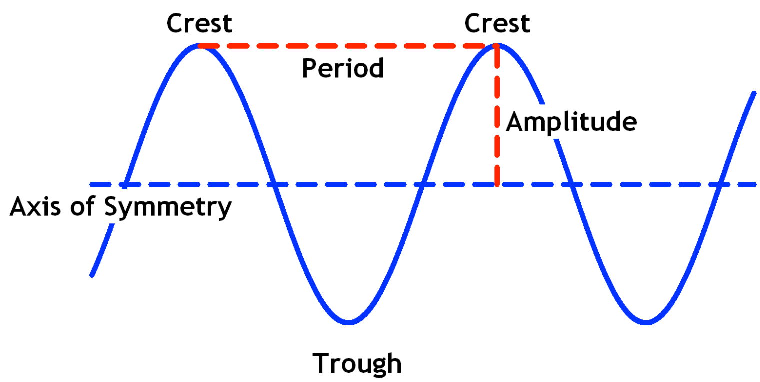
17. **REFLECTION:** What other types of employment might be cyclical in nature?

Student: ___________________________________ Class: ________________________ Date:_________________

Using Functions in Models and Decision Making: Cyclical Functions

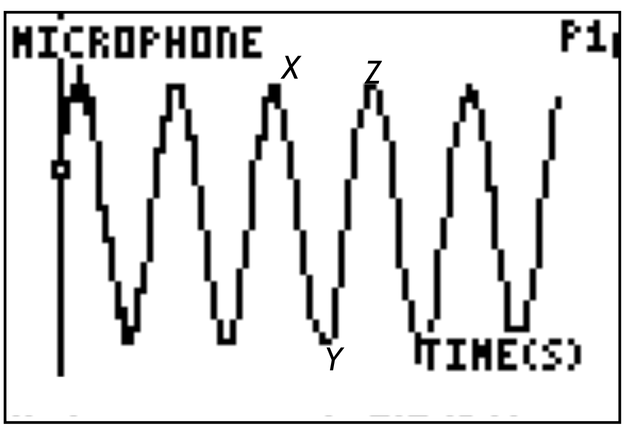
V.B Student Activity Sheet 7: Making Decisions from Cyclical Functions in Science and Economics

Recall from your science class that sound travels in waves. A wave has several important parts:



The *crest* is the maximum height of a wave, and the *trough* is the minimum height of a wave. The *period* is the distance between two consecutive crests or two consecutive troughs. The *axis of symmetry* is a horizontal line that runs exactly halfway between the crests and troughs. The *amplitude* is the distance between a crest or trough and the axis of symmetry.

Mr. Licefi's math class used a calculator-based laboratory (CBL) and a microphone to collect the following sound data. Notice that Points *X*, *Y*, and *Z* are labeled in the graph.



X (0.0054, 6.5)
Y (0.0065, 2.5)
Z (0.0076, 6.5)

1. If *X* and *Z* each represent a crest, what is the period of the sound wave? (Do not forget your units!)

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 7: Making Decisions from Cyclical Functions in Science and Economics

2. The frequency of a sound wave can be found by taking the reciprocal of the period. What is the frequency of this sound wave? The unit for frequency is hertz.

3. If *B* represents a trough, what is the amplitude of the sound wave?

4. In a sound wave, the frequency represents the pitch of the sound, and the amplitude represents the volume. For the sound wave that Mr. Licefi's class measured, what is the pitch and volume?

5. What amplitude is required to produce a sound wave that is twice as loud?

6. What are the domain and range of the function that models the sound wave?

7. If the sound that Mr. Licefi's class measured lasted for 8 seconds and stayed the same pitch (from Question 4), what are the domain and range of the sound wave?

8. Compare the domain and range for the function that models the sound wave and the domain and range for the sound wave itself. Explain any similarities or differences.

Mrs. Kline's economics class was studying a data set that gives the price per pound of ground beef for the month of January from 1980 to 1996.

Year	Year Number	Cost (dollars)
1980	0	1.821
1981	1	1.856
1982	2	1.794
1983	3	1.756
1984	4	1.721
1985	5	1.711
1986	6	1.662
1987	7	1.694
1988	8	1.736

Year	Year Number	Cost (dollars)
1989	9	1.806
1990	10	1.907
1991	11	1.996
1992	12	1.926
1993	13	1.970
1994	14	1.892
1995	15	1.847
1996	16	1.799

Source: U.S. Bureau of Labor Statistics

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 7: Making Decisions from Cyclical Functions in Science and Economics

9. Use your graphing calculator to make a scatterplot of cost by year number.

10. Does the data set appear to be cyclical? Explain your reasoning.

11. An economics textbook suggests that the function $y = 0.169\sin[0.52(x + 2.78)] + 1.82$ can be used to model the data approximately. Graph this function over your scatterplot to verify that suggestion. Describe the axes and scaling, and sketch your graph.

12. **EXTENSION:** Recall that sine functions can be represented using the general form $y = A\sin(B(x - C)) + D$, where

- A represents the amplitude,
- B represents the angular frequency,
- C represents a factor of a horizontal translation, and
- D represents the vertical translation.

For this function, determine the values of ***A***, ***B***, ***C***, and ***D***.

A =

B =

C =

D =

13. Find the length of one cycle by dividing 2π by the frequency (B).

14. How well does the suggested function model the data?

15. Use the regression equation to predict the cost per pound of ground beef in January 2009.

16. Use the Internet to determine the actual cost per pound of ground beef in January 2009.

17. How well did your model predict the cost of ground beef in January 2009? Why do you think the model performed this way?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 7: Making Decisions from Cyclical Functions in Science and Economics

18. **REFLECTION:** What can you say about using a cyclical model to predict values beyond a given data set?

OR

How well could ocean waves be modeled using a sinusoidal function?

19. **EXTENSION:** What other natural or business phenomena could be modeled using a cyclical model? How well do you think those models could predict future values?

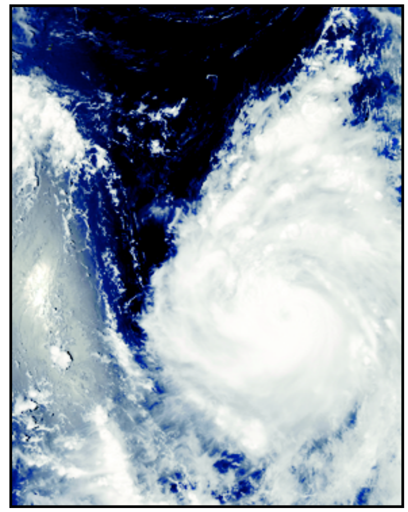
OR

Using a CBL and a microphone probe, capture your own data from sound waves that you generate. Then compare these data to the data used in the lesson.

Student: ___________________________ Class: ___________________ Date:______________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions



Texas experiences a wide variety of weather, including hurricanes. Coastal residents often feel the direct effects of hurricanes when they make landfall along the coast. Cities and towns that are directly hit by a hurricane can sometimes take years to rebuild. Galveston is one such city.

Galveston was almost completely destroyed by the storm that hit in 1900, the deadliest hurricane in U.S. history. Rebuilding after the storm took several years, partly because residents raised the elevation of the entire city and built the Galveston Seawall to protect the city. Other towns were not so resilient. In 1886, residents of Indianola completely abandoned the ruins of their town on the shores of Matagorda Bay after it was wiped away by a strong hurricane.

Meteorologists use the Saffir-Simpson scale to describe the strength of a hurricane. This scale is based on a combination of wind speed and barometric pressure. The faster the wind speed and the lower the barometric pressure, the higher the rating of the hurricane on the Saffir-Simpson scale.

Saffir-Simpson Scale

Category	Wind Speed (miles per hour)
1	74–95
2	96–110
3	111–130
4	131–155
5	156 and above

Many hurricanes have struck the Texas coast, but there have been no recorded Category 5 hurricanes, which are the strongest, most destructive storms. Although many Caribbean and Central American nations have been pounded by Category 5 hurricanes, the United States has been hit by only three: the 1935 Labor Day Hurricane, which struck the Florida keys; Hurricane Camille, which struck Pass Christian, Mississippi, in 1969; and Hurricane Andrew, which struck near Homestead, Florida, in 1992.

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions

The following table shows the year, wind speed, and Saffir-Simpson category for some hurricanes that have made landfall on the Texas coast. This table also includes the Category 5 storms that have hit the United States.

Texas Hurricanes

Hurricane	Year	Wind Speed (miles per hour)	Category
Indianola Storm	1886	155	4
Galveston Storm	1900	125	3
Brownsville Storm	1933	100	2
Labor Day Storm*	1935	161	5
Audrey	1957	100	2
Debra	1959	105	2
Carla	1961	150	4
Beulah	1967	140	4
Camille*	1969	190	5
Celia	1970	130	3
Allen	1980	115	3
Alicia	1983	115	3
Bonnie	1986	86	1
Andrew*	1992	167	5
Bret	1999	115	3
Claudette	2003	90	1
Rita	2005	115	3
Dolly	2008	86	1
Ike	2008	110	2

*Storm did not make landfall in Texas.
Source: National Hurricane Center

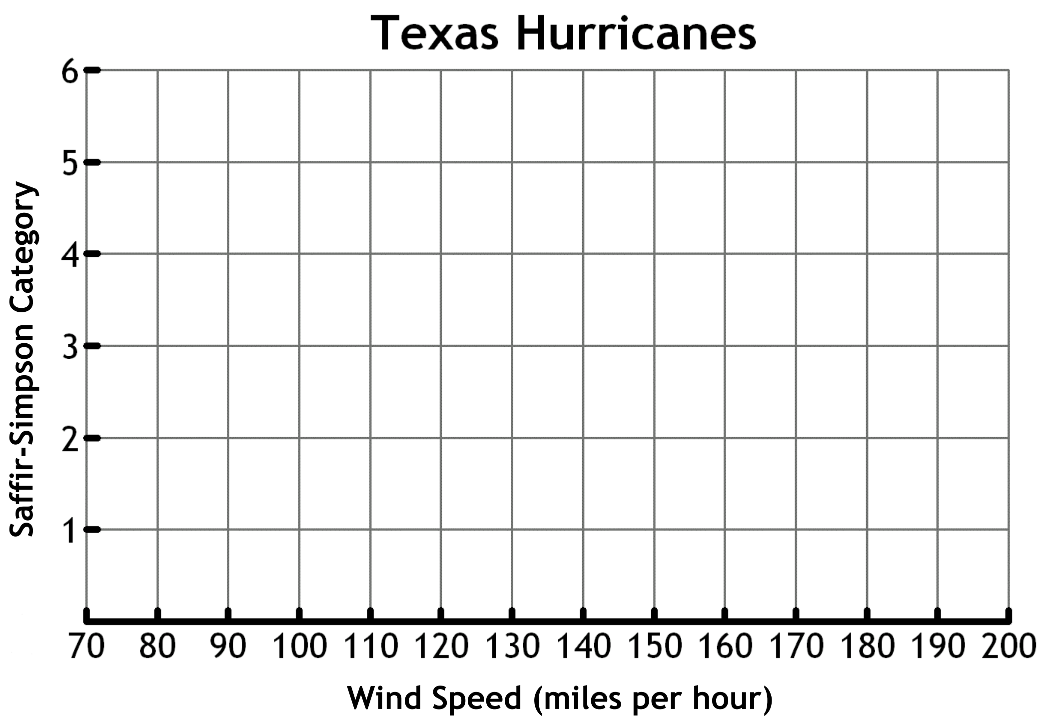
Student: ________________________________ Class: ______________________ Date:_______________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions

1. Write a dependency statement that describes the relationship between the two variables, wind speed and Saffir-Simpson category.

2. Make a scatterplot of the Saffir-Simpson category versus wind speed for the hurricanes listed in the table.



3. Now mark the wind speed endpoints for each Saffir-Simpson category on the scatterplot. Connect those endpoints with a line segment. For example, along the line for Category 1, mark the wind speeds 74 and 95 [that is, the points (74, 1) and (95, 1)] and then connect them with a line segment.

4. Is it possible for a hurricane to be rated between Category 1 and Category 2? Why or why not?

Student: ___________________________ Class: ___________________ Date:_____________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions

Hurricane wind speeds are difficult to measure precisely. Thus, most hurricane wind speeds are estimated to the nearest 5 miles per hour. Suppose a new technology were invented that allowed meteorologists to measure hurricane wind speeds very precisely.

5. If a hurricane had a wind speed of 95.1 miles per hour, what category would it be rated? How do you know?

6. Revise the Saffir-Simpson scale so that you can rate hurricanes with wind speeds that lie between the existing categories.

Revised Saffir-Simpson Scale

Category	Wind Speed (miles per hour)
1	
2	
3	
4	
5	

7. When graphing inequalities, how do you represent an endpoint that does not include *or equal to*?

8. Use a closed or open endpoint to revise your scatterplot for the new hurricane rating scale.

9. What kind of function does your new scatterplot represent?

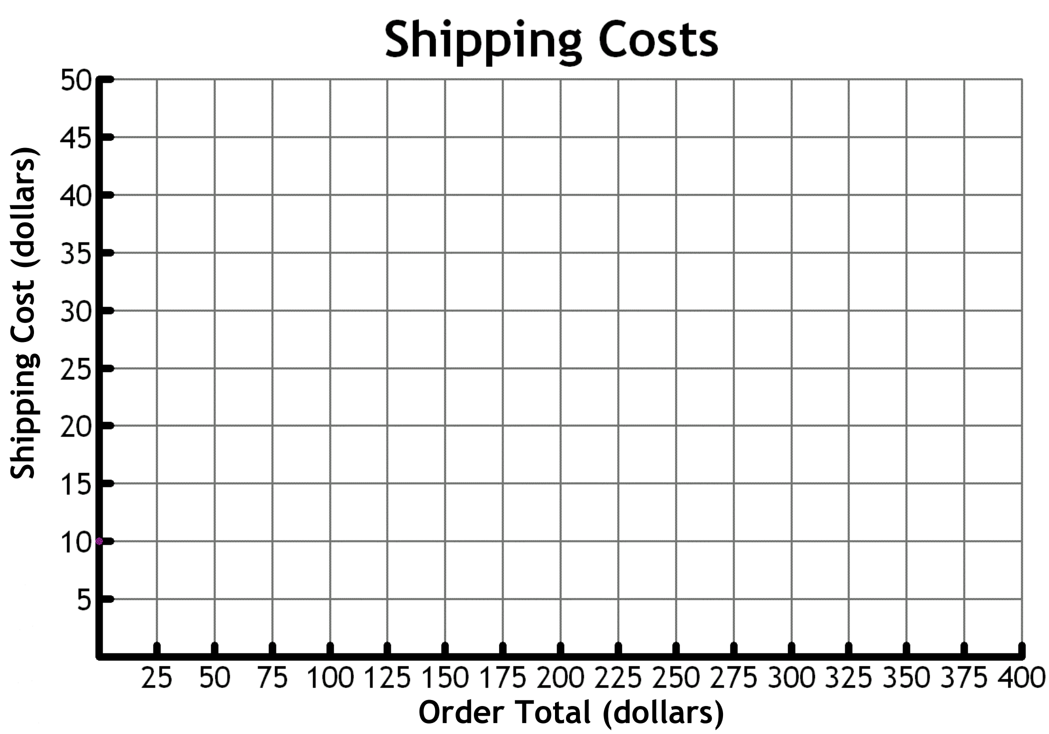
Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions

An online store uses a step function to determine shipping costs.

Order Total	Shipping Costs	
	Continental United States	Europe
Less than $25.00	$5.00	
$25.00-$74.99	$10.00	
$75.00-$124.99	$15.00	
$125.00-$349.99	$20.00	
$350.00 and greater	$25.00	

10. Use a colored pencil to make a graph of shipping costs versus the order total.



Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions

11. For orders shipped to Europe, the shipping cost for the United States is doubled. Fill in the table to show the shipping costs to Europe. Then use a different colored pencil to make a graph of the shipping costs to Europe versus the order total.

12. How do the two graphs compare?

13. **REFLECTION:** How do step functions compare to linear functions?

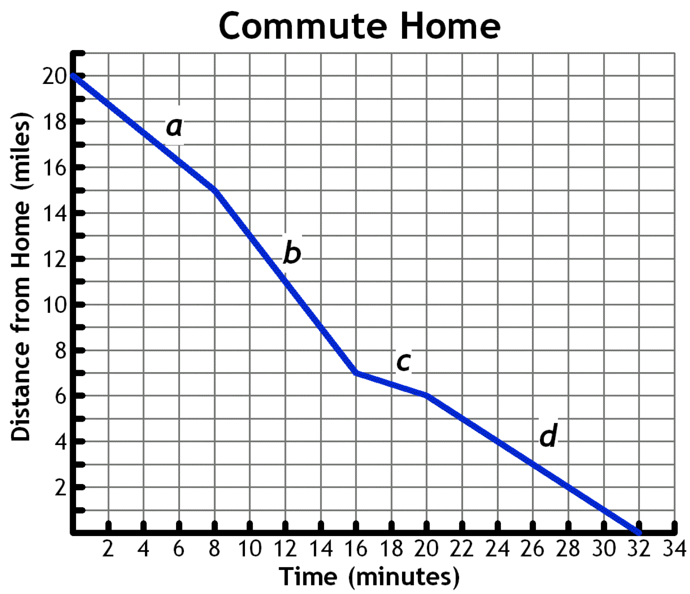
14. **REFLECTION:** How is multiplying a step function by a constant multiplier similar to multiplying the slope of a linear function by a constant multiplier?

15. **EXTENSION:** What other situations can be modeled using a step function? Use the Internet to collect data and generate a graph of a situation. How does your graph compare to those in this activity?

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 9: Another Piecewise Function

Mrs. Washington lives 20 miles from her office and drives her car to and from work every day. The graph below shows her distance from home over time as she drove home from work one day.



1. Write a dependency statement expressing the relationship between the two variables, distance and time.

The following table will be used to answer Questions 2, 6, and 8.

Segment	Slope	Equation of Line	Domain	Range
a				
b				
c				
d				

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 9: Another Piecewise Function

2. Find the slope of each line segment in the graph of Mrs. Washington's commute. Record your results in the table.

3. How did you find the slope of each segment?

4. What does the slope of a line segment represent in the context of this situation?

5. Is the slope an increasing or decreasing rate of change? What does this mean in the context of this situation?

6. Find the equations of the four line segments in the graph. Record your results in the table.

7. How did you determine the equations of the lines?

8. Identify the domain and range of the line that describes each segment of Mrs. Washington's commute. Use inequality symbols to indicate the domain and range, and record your results in the table.

9. Graph the line that represents Segment ***a*** in your graphing calculator. To do this, set your viewing window to match the graph at the beginning of the activity.

10. Now, restrict the domain of the line. If possible, use graphing technology. Sketch your graph. Explain why the graph looks like it does.

11. Graph the line that represents Segment ***b***. Restrict the domain of the line as needed. What do you expect the graph to look like? Sketch your prediction before you actually draw or display the graph.

12. How does your prediction compare with what the graph looks like? Explain any differences.

13. Repeat the procedure to graph the lines for Segment ***c*** and Segment ***d***. Sketch your final graph.

14. What piece of information did you need to enter into the calculator to tell it which parts of the four lines it should graph?

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 9: Another Piecewise Function

15. **REFLECTION:** Describe earlier types of functions that can be analyzed using the terminology used with step and piecewise functions. Give an example of an application of the function.

The height of a diver above a body of water as a function of time can be given using two different functions: a constant function for the time the diver is on the diving board and a quadratic function for the time when the diver jumps off the board and falls toward the water.

Rafael is on vacation with his family in Acapulco, Mexico. La Quebrada is a famous cliff that is about 35 meters above the ocean surface. For many years, divers have jumped off La Quebrada into the Pacific Ocean. Rafael has signed up to go cliff diving.

16. Rafael stands on the cliff, 35 meters above the ocean surface below. What function describes his height above the ocean surface (***h***) as a function of time (***t***) while he stands on the cliff?

Rafael is next. He walks to the edge of the cliff and stands still for 3 seconds. Then he dives off the cliff. As soon as he leaves the cliff, his height above the ocean surface can be found using the function $h = -4.9(t - 3)^2 + 35$, where ***h*** represents Rafael's height from the ocean surface and ***t*** represents the time since Rafael stood at the edge of the cliff.

17. Fill in the table below to describe Rafael's height above the ocean surface over time.

	Function, *h(t)*	**Domain**
Standing still		
Free-fall motion		

18. Use the domain restrictions to graph Rafael's height above the ocean surface over time on your graphing calculator, if possible. Describe the domain, range, and scaling and sketch the graph.

19. **EXTENSION**: What other situations could be modeled using piecewise functions like the ones used to describe Mrs. Washington's commute or Rafael's cliff-diving experience? Investigate one of the situations and prepare a brief report for the class regarding your

Student: ______________________________ Class: ____________________ Date: ______________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 10: Concentrations of Medicine

Part A

Have you ever taken a medication that your doctor warned you would not take effect for a few days? In this activity, you will investigate why that is the case.

Consider the allergy medicine Sneeze-B-Gone. The regular adult dose is 20 milligrams. As with all medicines, the body gradually filters Sneeze-B-Gone out of the bloodstream. The rate at which the medicine is filtered out is called the *flush rate*. For Sneeze-B-Gone, the flush rate is 30%. In other words, 24 hours after the pill is taken, 30% of Sneeze-B-Gone has flushed out of the body.

1. If 30% of Sneeze-B-Gone has flushed out of the body after 24 hours, what percent of Sneeze-B-Gone remains?

2. Use your calculator's recursion feature to fill in the table below, assuming that an adult is taking one 20-milligram dose per day.

3. At what value does the amount of Sneeze-B-Gone in the bloodstream level off? How many days does it take for that to happen?

4. What type of function could model the amount of Sneeze-B-Gone in the bloodstream as a function of time? Explain your choice.

5. What would you expect a graph of the amount of Sneeze-B-Gone in the bloodstream as a function of time to look like? Explain your prediction.

Day	Sneeze-B-Gone in Bloodstream (in mg)	Day	Sneeze-B-Gone in Bloodstream (in mg)
1	20	11	
2	34	12	
3	43.8	13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 10: Concentrations of Medicine

6. Recall that the general form for exponential decay functions is $\boldsymbol{y} = \boldsymbol{a}(\boldsymbol{b})^{\boldsymbol{x}}$, where $\boldsymbol{a}$ represents the starting amount of the substance and $\boldsymbol{b}$ represents the rate of decay. For a 20-milligram dose and a 30% flush rate, what exponential function could describe the amount of Sneeze-B-Gone in the bloodstream ($\boldsymbol{y}$) as a function of time ($\boldsymbol{x}$)? (Do not forget that $\boldsymbol{b}$ represents the percent of Sneeze-B-Gone that remains in the bloodstream.)

7. Since the patient did not begin taking the medicine until Day 1, adjust your function rule by subtracting 1 from the exponent. Graph the function on your graphing calculator. Sketch your graph and describe your viewing window.

8. If time ($\boldsymbol{x}$) is given in terms of the number of days, what happens to the amount of Sneeze-B-Gone in the patient's bloodstream at the start of Day 2 when the patient takes a second pill? How does this affect the graph?

9. Use what you learned about step and piecewise functions in previous activities to restrict the domain of the graph. Sketch your new graph.

10. For Day 2, enter the function $\boldsymbol{y} = 34 \bullet 0.7^{x-1}$ into your calculator. What do the constants 34, 0.7, and 2 represent? Sketch the new graph.

11. Based on the functions for Day 1 and Day 2, write a function from the data in your table for Day 3 and a function for Day 4.

12. Graph both of these new functions. What patterns do you notice? What do you expect the graph for Day 5 to look like?

13. Test your prediction by writing a function for Day 5.

14. **REFLECTION:** Assume the patient takes 20 milligrams of Sneeze-B-Gone every day. If you extend the graph to Day 20 or beyond, what would the minimum amount of Sneeze-B-Gone in the bloodstream be? The maximum amount?

Student: ___________________________ Class: ____________________ Date:______________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 10: Concentrations of Medicine

Part B

1. Suppose a patient requires a 30-milligram dose of Sneeze-B-Gone. Use home screen recursion on your calculator to fill in the table.

2. At what value does the amount of Sneeze-B-Gone in the bloodstream level off? How many days does it take for that to happen?

3. How does the function rule for the 20-milligram dose change for a 30-milligram dose? Write the new function rule for the portion of the graph between Day 1 and Day 2.

4. How do you think those changes would affect the graph of the new function rule?

5. Use your graphing calculator to test your prediction. Sketch your graph.

Day	**Sneeze-B-Gone in Bloodstream (in mg)**	**Day**	**Sneeze-B-Gone in Bloodstream (in mg)**
1	30	11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

6. When the amount of Sneeze-B-Gone in the bloodstream levels off for a patient taking a 30-milligram daily dose, what are the minimum and maximum amounts of Sneeze-B-Gone in the bloodstream within a given day?

7. Suppose a patient requires a 40-milligram dose of Sneeze-B-Gone. Based on what you have observed so far, what would you expect the function rule and graph to look like?

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 10: Concentrations of Medicine

8. Use recursion on your calculator to fill in the table.

9. At what value does the amount of Sneeze-B-Gone in the bloodstream level off? How many days does it take for that to happen? You may need to extend the values in the table.

10. How does the function rule for the 30-milligram dose change with a 40-milligram dose? Write the new function rule for the portion of the graph between Day 1 and Day 2.

11. How do you think those changes would affect the graph of the new function rule?

12. Use your graphing calculator to test your prediction. Sketch your graph.

Day	**Sneeze-B-Gone in Bloodstream (in mg)**
1	40
2	
3	
4	
5	
6	
7	
8	
9	
10	

Day	**Sneeze-B-Gone in Bloodstream (in mg)**
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

13. When the amount of Sneeze-B-Gone in the bloodstream levels off for a patient taking a 40-milligram dose, what are the minimum and maximum amounts of Sneeze-B-Gone in the bloodstream within a given day?

14. **REFLECTION:** How does an increase in dose affect the amount of Sneeze-B-Gone in the bloodstream when the amount levels off?

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 10: Concentrations of Medicine

15. Fill in the table below. What relationships do you notice?

Dose	**Flush Rate**	**Leveled-off Amount**	$\frac{\textbf{Dose}}{\textbf{Flush Rate}}$
20			
30			
40			

16. **REFLECTION:** If you were a doctor or nurse and you knew that a patient needed to have about 100 milligrams of Sneeze-B-Gone in his bloodstream for the medicine to be effective, what dose would you prescribe? Explain your decision.

17. **EXTENSION:** A new cholesterol-lowering medicine has a flush rate of 50%. For a 20-milligram dose of this medicine, how do the function rules and graph compare to those for the 20-milligram dose of Sneeze-B-Gone with a flush rate of 30%? Use your graphing calculator to investigate. Present your work to the class.

Student: ______________________________ Class: ____________________ Date: _______________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 11: Making Decisions from Step and Piecewise Models

When electricity became widely distributed during the early part of the 20th century, state governments regulated the electricity industry as a monopoly. One electric company had the rights to generate and distribute electricity for a city or a certain part of the state. In return, the government laid out a set of rules for what the electric company could and could not do.

During the 1990s and early 21st century, many states deregulated electricity. As a result, numerous electric companies can now provide electricity for a particular area. One such company is Lights and Power. To attract customers, Lights and Power is advertising a special:

Cheapest Electricity in Town!

To 1,000 kWh—$0.11 per kWh
More than 1,000 to 1,500 kWh—$0.18 per kWh
More than 1,500 kWh—$0.25 per kWh

No hidden fees! We promise!

1. According to the advertisement, how much does the first 1,000 kilowatt-hours (kWh) of electricity cost a customer?

2. Suppose Mrs. Brown uses 1,200 kilowatt-hours of electricity. How much does she pay for the first 1,000 kilowatt-hours?

   How much does she pay for the next 200 kilowatt-hours of electricity?

   How much does she pay altogether for 1,200 kilowatt-hours of electricity?

Student: ___________________________________ Class: ________________________ Date:_________________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 11: Making Decisions from Step and Piecewise Models

3. Use the information in Lights and Power's advertisement to determine the cost of electricity for the amounts listed in the table.

Amount of Electricity (kWh)	**Process**	**Cost ($)**
700	700(0.11)	77
800		
900		
1,000		
1,100		
1,200	1,000(0.11) + (1,200 – 1,000)(0.18)	146
1,300		
1,400		
1,500		
1,600		
1,700		
1,800		
1,900		

4. Write an equation to describe the cost (***y***) of the number of kilowatt-hours of electricity (***x***) to 1,000 kilowatt-hours.

5. For what domain does your function model the cost of the first 1,000 kilowatt-hours of electricity?

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 11: Making Decisions from Step and Piecewise Models

6. Write an equation to describe the cost (***c***) of the number of kilowatt-hours of electricity (***x***) from 1,001 to 1,500 kilowatt-hours.

7. For what domain does your function model the cost of 1,001 to 1,500 kilowatt-hours of electricity?

8. Write an equation to describe the cost (***m***) of the number of kilowatt-hours of electricity (***x***) more than 1,500 kilowatt-hours.

9. For what domain does your function model the cost of more than 1,500 kilowatt-hours of electricity?

10. Write three piecewise functions, including limitations on the domain, that describe the cost of purchasing electricity from Lights and Power.

11. Use your graphing calculator to make a scatterplot of cost versus amount of electricity. Describe the axes and scaling and sketch your graph.

12. Graph your piecewise functions over your scatterplot. Use the domain restrictions. How well do the functions model the data generated by the electricity plan?

13. The function ***y*** = 0.11***x*** has a domain of all real numbers. Why is the domain of the function as it is applied in this situation restricted?

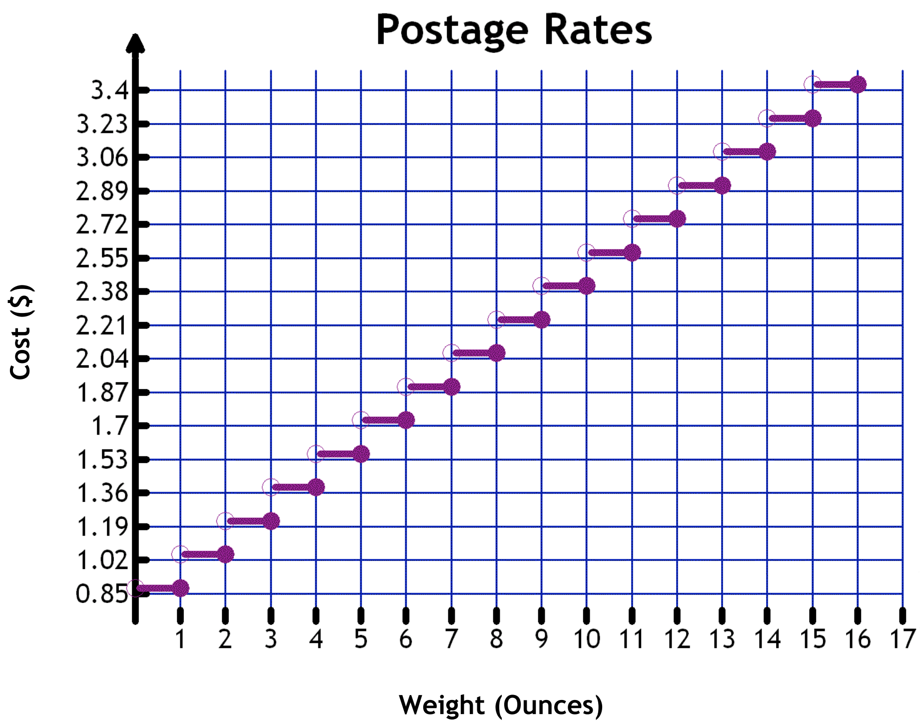
Student: ______________________________ Class: ____________________ Date:______________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 11: Making Decisions from Step and Piecewise Models

As of May 2009, the U.S. Postal Service adjusted its rates so that mailing a large envelope costs $0.88 for the first ounce and $0.17 for each additional ounce. There is a weight limit for all first-class mail—letters and parcels mailed first class cannot exceed 13 ounces.

Consider the graph below.



14. What type of function is represented by the graph? How do you know?

15. Is this type of function appropriate to represent the U.S. Postal Service rates for sending large envelopes by first-class mail? Why or why not?

16. How well does the graph represent the U.S. Postal Service rates for sending large envelopes by first-class mail? How do you know?

17. How could you modify the graph to better represent the situation?

18. **REFLECTION:** What types of situations can a step function be used to model?

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 11: Making Decisions from Step and Piecewise Models

19. REFLECTION: How are step functions similar to piecewise functions? How are they different?

20. EXTENSION: Research taxicab fares for your city or a city that you want to visit. What type of function is most appropriate to represent those fares? Generate a graph to show the fares and present your findings to the class.

21. EXTENSION: Research to determine an appropriate response to the following question. Prepare a short presentation of your findings.

Would federal income taxes be better modeled with a step function or a piecewise function?

Advanced Mathematical Decision Making

In Texas, also known as

Advanced Quantitative Reasoning

Unit VI: Decision Making in Finance

This course is a project of
The Texas Association of Supervisors of Mathematics and
The Charles A. Dana Center at The University of Texas at Austin
With support from the Greater Texas Foundation

2010

Advanced Mathematical Decision Making

In Texas, also known as

Advanced Quantitative Reasoning

Student Materials

These student materials are excerpted from one of seven units that make up the 2010 AMDM/AQR instructional materials (developed under the name Advanced Mathematical Decision Making).

Unit I: Analyzing Numerical Data
Unit II: Probability
Unit III: Statistical Studies
Unit IV: Using Recursion in Models and Decision Making
Unit V: Using Functions in Models and Decision Making
Unit VI: Decision Making in Finance
Unit VII: Networks and Graphs

Table of Contents

Student: ______________________________ Class: ____________________ Date:______________

Decision Making in Finance: Future Value of an Investment

VI.A Student Activity Sheet 1: You Have to Get Money to Make Money

1. Kafi is considering three job offers in educational publishing.

- One is a full-time position as an editor that pays a salary of $37,500 per year.
- Another is a full-time position as an e-Learning designer that pays an hourly wage of $26.50. The job assumes five 8-hour days per week.
- The final offer is for a sales representative that pays a 5% commission. Sales representatives typically sell an average of $100,000 per month in textbooks.

Record the income information for the editor, designer, and sales representative in Row 1 of Job Summary Table 1 at the end of this activity sheet.

2. Estimate the gross *annual* income for each job offer. Record your estimate in Row 3 of Job Summary Table 1. Use Row 2 for any calculations that are needed to determine the income.

3. Estimate the gross *monthly* income for each job offer. For the purposes of his comparison, Kafi assumes that each job pays monthly. Record your estimate in Row 5 of Job Summary Table 1. Use Row 4 for any calculations that are needed to determine the income.

4. Based on the gross monthly income, which job do you recommend Kafi take? Why?

5. Determine the after-tax income for each job offer. Use the following information:

- The U.S. government deducts Social Security (6.2%) and Medicare (1.45%).
- Kafi will deduct 15% of gross income to cover federal income tax.
- Kafi does not live in a state with state income tax.

Record your estimate in Row 7 of Job Summary Table 1. Use Row 6 for any calculations that are needed to determine the income.

6. Kafi determines that he needs at least $3,000 per month in after-tax income to cover his monthly expenses. Based on this budget estimate, are there any jobs that Kafi should not take? Why?

Decision Making in Finance: Future Value of an Investment

VI.A Student Activity Sheet 1: You Have to Get Money to Make Money

Another consideration in comparing jobs is the benefits each provides, such as health insurance, retirement plan, vacation time, and sick leave.

- The editor position includes two weeks of paid vacation and five paid sick days per year, paid health insurance, life insurance costing $35 per month, and a fully paid retirement plan.
- The designer position includes five paid vacation days and three paid sick days per year, paid health insurance, life insurance costing $35 per month, and a retirement plan that costs 3% of after-tax income.
- The sales position has no paid vacation or sick days, paid health insurance, paid life insurance, and a retirement plan costing $400 per month.

7. Estimate the *monthly cost* that will be deducted from Kafi's pay for benefits. Use the following information:

- Kafi plans on taking two weeks (10 days) for vacation per year.
- In the past, Kafi averaged three sick days per year.
- Kafi plans to purchase life insurance and save for his retirement.

Record your estimate in Job Summary Table 1 in Rows 8 through 12.

8. Estimate the monthly take-home income in Row 13 of Job Summary Table 1.

9. Based on the completed Job Summary Table 1, which job do you recommend that Kafi take? Explain your recommendation based on the net income.

10. Are there any factors that could affect the accuracy of the estimated net incomes? If *yes*, does this change your recommendation? Explain your reasoning.

11. Are there any other factors that Kafi should consider when deciding which job to take? If *yes*, does this change your recommendation? Explain your reasoning.

12. **EXTENSION:** You are considering two job offers: a full-time permanent position that pays $55,500 annually and a full-time contract job that pays $29 per hour. Estimate the gross annual income, gross monthly income, and the after-tax monthly income for each job offer. Record your estimates in Job Summary Table 2 at the end of this activity sheet. Use the information for calculating income, taxes, and costs that Kafi used. The contract job is self-employment, which is taxed an additional 7.65% of gross income.

    Based on the gross monthly income, which job should you take? Why?

    Based on the after-tax income, which job should you take? Why?

13. **EXTENSION:** The permanent position will cost you $95 per month in health care benefits and 4% of your after-tax income in retirement contributions. The contract job will cost you $150 per month in health care benefits and 8% of your after-tax income in retirement contributions.

    Estimate the take-home income for each job offer and record it in Job Summary Table 2. Based on this information, which job should you take? Why?

14. **REFLECTION:** Did your decision on which job to take change throughout the analysis? What does that say about the decision process for considering any job offer? When considering various job offers, what will factor into your decision?

Decision Making in Finance: Future Value of an Investment

VI.A Student Activity Sheet 1: You Have to Get Money to Make Money

Job Summary Table 1

Row No.	Job:	Editor	Designer	Sales Representative
1	Income information			
2	*Process*			
3	Gross annual income			
4	*Process*			
5	Gross monthly income			
6	*Process*			
7	After-tax monthly income			
8	*Process: Vacation*			
9	*Process: Sick leave*			
10	*Process: Health insurance*			
11	*Process: Life insurance*			
12	*Process: Retirement plan*			
13	Monthly take-home income			

Decision Making in Finance: Future Value of an Investment

VI.A Student Activity Sheet 1: You Have to Get Money to Make Money

Job Summary Table 2

Job:	Permanent Position	Contract Position
Income information		
Process		
Gross annual income		
Process		
Gross monthly income		
Process		
After-tax monthly income		
Process: Health insurance		
Process: Retirement plan		
Monthly take-home income		

Student: ___________________________ Class: ___________________ Date:______________

Decision Making in Finance: Future Value of an Investment

VI.A Student Activity Sheet 2: What Makes Money Work for You?

Amanda is analyzing how to invest $500. She is considering the two investments described below.

- **Savings accounts** are insured and vary in the way in which interest is calculated. Some accounts pay simple interest, but other accounts compound interest at varying frequencies. *Amanda is considering a savings account that pays 3.75% interest compounded annually.*
- A **certificate of deposit** (CD) is an interest-bearing instrument that is similar to a savings account—it is insured and pays interest. Unlike savings accounts, CDs have a fixed time period and usually a fixed interest rate. CDs also vary in the way in which interest is calculated. Sometimes the interest is compounded, but simple-interest CDs also exist. Simple interest is calculated only on the original deposit. The CD must be held until the date of maturity, at which time the original money deposited may be withdrawn with the accrued interest. *Amanda is considering a CD that pays 4% simple annual interest for five years.*

1. Amanda wants to evaluate each investment for the first five years. Use the spreadsheet below to record your calculations.

CD/Year	Beginning Balance	Interest Earned	Ending Balance	Savings Account/ Year	Beginning Balance	Interest Earned	Ending Balance
1	$500			1	$500		
2				2			
3				3			
4				4			
5				5			

2. If Amanda is using this investment as an emergency fund, in which should she invest? Explain your reasoning.

Decision Making in Finance: Future Value of an Investment

VI.A Student Activity Sheet 2: What Makes Money Work for You?

3. Based on the processes you used to fill in the spreadsheet in Question 1, write a function rule to model each investment. Let y represent the value of the investment at the end of any year x.

4. What types of functions did you use to model each investment option? How are the functions related to the type of interest earned in each option?

Amanda has decided to keep the investment until retirement—40 years from now. Assume that she can invest in the same CD or savings account at the same rate for the life of the investment.

5. Use your graphing calculator to graph both functions. Describe your axes and scaling and sketch your graphs.

6. Compare and contrast the graphs of the two different functions. Explain what you see in terms of the function rules and the tables.

7. Why is there a difference between the two models? Explain your answer using the information from the tables, graphs, or function rules.

8. **REFLECTION:** Which investment should Amanda use: the CD or the savings account? Explain your reasoning.

9. **EXTENSION:** One of the greatest contributors to lowering the value of money is inflation, which is a percentage representing the annual increase in the value of money. Find the current annual rate of inflation on the Internet.

   Consider the investment you recommended for Amanda. Taking inflation into account, what is her actual rate of earning on the investment? Based on your findings, would you make any recommendations to Amanda?

Student: ___________________________________ Class: ________________________ Date:_________________

Decision Making in Finance: Future Value of an Investment

VI.A Student Activity Sheet 3: Time Value of Money

The future value of an investment is the amount it will be worth after so many months or years of earning interest. The following table lists a savings account's future values in selected years.

Year	Balance
0	$2,600.00
5	$3,201.50
10	$3,942.20
15	$4,854.16
20	$5,977.16
25	$7,359.95
30	$9,062.70

1. Create a scatterplot of the given data. Label the axes and scales, and provide a title.

What type of function would best model the data? Explain your reasoning.

2. Calculate the regression equation for the given data. Graph the regression equation on the scatterplot in Question 1.

3. According to the model, what is the interest rate of the savings account?

Is the interest simple or compound? How do you know?

4. Using the model, how much will be in the account in 50 years?

5. Use the regression equation from the previous problems to write a general formula for future value of an investment compounded annually. Use the following variables:

- ***FV*** for future value
- ***t*** for time (in years)
- ***i*** for interest rate (in decimal form)
- ***PV*** for the principal or present value

Decision Making in Finance: Future Value of an Investment

VI.A Student Activity Sheet 3: Time Value of Money

6. All of the investments so far have compounded and paid interest annually. However, some investments compute interest in compounding periods that are quarterly or monthly. If the annual interest rate is divided evenly, how would the interest rate be calculated for each compounding period?

7. Write a general formula for future value that takes into account any compounding period. Use the variables from Question 5, in addition to ***n*** for number of compound periods in one year.

8. Suppose you invest $2,600 into a savings account with a 4.25% annual interest rate that compounds interest quarterly. Use the formula you wrote in Question 7 to determine the balance in the account after five years.

9. How much would the same savings account be worth in 50 years if the interest is compounded quarterly?

10. **REFLECTION:** Is there a difference between the account balance in Question 8 and the account balance from the problem described in the table? If so, is the difference large or small? How might this difference influence your decision about investments?

11. **REFLECTION:** Is there a difference between the account balance in Question 9 and the account balance in Question 4? If so, is the difference large or small? How might this difference influence your decision about investments?

12. **EXTENSION:** Research interest rates for a savings account, checking account, and money market account at different financial institutions. Note the compounding period for each.

    How much would $10,000 be worth in each account in 50 years?

Decision Making in Finance: Present Value of an Investment

VI.B Student Activity Sheet 4: Road to $1 Million

In Student Activity Sheet 3, you analyzed the future value of an investment over time. You began with $2,600 invested in a savings account for 30 years. After 30 years, your initial investment would be worth $9.062.70. In this activity, you will look at the same investment in a different way. The question relates to the **time value of money** (TVM). *What is that $9,062.70 future value worth at various times in the 30-year investment?*

The following table lists the principal required to obtain the same future value of $9,062.70 for various investment lengths. So, in the table, the 30-year investment is the one you have already explored. The other values in the table show how much principal you would need to invest and the length of time of the investment for the same yield. This can be thought of as the **present value** of the investment.

Years Till Maturity	**Principal Required**
0	$9,062.70
5	$7,359.95
10	$5,977.16
15	$4,854.16
20	$3,942.20
25	$3,201.50
30	$2,600.00

1. Create a scatterplot of the given data. Label the axes and scales, and provide a title.

2. Calculate the regression equation for the given data. Graph the regression equation on the scatterplot. Explain why the function model you used makes sense in the problem situation.

Decision Making in Finance: Present Value of an Investment

VI.B Student Activity Sheet 4: Road to $1 Million

3. Josephine is 20 years old and wants to save $1 million for retirement in 50 years. Assume she invests in a savings account that earns at least the current rate of inflation. Determine how much Josephine must save today to reach her retirement goal.

Recall the future-value formula $FV = PV\left(1 + \frac{i}{n}\right)^{nt}$, using

- ***FV*** for future value
- ***t*** for time (years)
- ***i*** for interest rate (in decimal form)
- ***n*** for number of compound periods per year
- ***PV*** for the principal or present value

4. Suppose Josephine does not want to begin saving for her retirement immediately. Fill in the following table to show the amount of money that Josephine must invest to retire 50 years from now with $1,000,000 based on the number of years that she waits to start saving.

Years of Waiting to Save	Principal Required
0	
10	
20	
30	
40	
50	

5. REFLECTION: Suppose Josephine believes in spending now and saving later. How could you use the table from Question 4 to convince her otherwise?

6. Blaine wants to have $1,000 in 10 years. The following are the choices in which he can invest:

- a savings account earning 3% compounded quarterly,
- a checking account earning 1% compounded monthly, or
- a money market account earning 4.5% compounded semiannually.

Blaine plans on making no withdrawals or deposits for 10 years.

Rewrite the formula from Question 3 for present value and allow for any compounding period (***n***).

7. Rewrite the present-value formulas for each account that Blaine is considering. Make sure that the formulas include compounding periods other than annual and incorporate the different rates.

8. Graph the present-value formula for each account. Label the axes, scales, and curves, and provide titles.

Which factor has the most significant effect on the curve: the interest rate or compounding periods? Why?

9. **REFLECTION:** In which account should Blaine invest? Why?

10. **EXTENSION:** Locate an article about what investments financial experts are currently recommending for clients at various times of life (young, middle age, etc.). Prepare a short presentation to share with the class regarding your findings.

Student: ___________________________________ Class: ________________________ Date:_________________

Decision Making in Finance: Present Value of an Investment

VI.B Student Activity Sheet 5: A Cool Tool!

Vanessa is a financial planner specializing in retirement savings. She realizes the importance of using mathematical formulas and the appropriate tools to help her clients understand the reasoning behind the advice she is giving.

One of her favorite tools is a time-value-of-money (TVM) calculator. In Student Activity Sheet 4, you met Josephine, one of Vanessa's clients who wanted to retire with $1 million in savings.

1. In Josephine's initial situation, she plans to retire in 50 years with $1 million in savings. Vanessa advised her to find an account that earned at least the current rate of inflation. Use this information to complete the table below.

Variable	Definition of Variable	Value in Josephine's Situation
FV	future value, or value of the investment at maturity	
t	number of years of investment until maturity	
i	annual interest rate (as a decimal)	
PV	principal, or present value	
n	number of compounding periods per year	

Vanessa uses a TVM calculator to help Josephine understand how the different variables affect one another.

2. Identify the values in Josephine's situation for each variable that the TVM calculator uses.

Variable	Definition of Variable	Value in Josephine's Situation
N	number of compounding periods between the time of investment and the time of retirement	
I%	annual interest rate (as a percent)	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value, or value of the investment at maturity	
P/Y	number of payments per year (usually the same as the number of compounding periods per year, ***C/Y***)	
C/Y	number of compounding periods per year	

3. Use the TVM calculator to determine the present value (***PV***) of the investment required to meet Josephine's retirement goal. How does this amount compare to what you determined in Student Activity Sheet 4?

Use the TVM calculator to answer the following questions for some of Vanessa's other clients.

4. Reginald wants to find the future value of an investment of $6,000 that earns 6.25% compounded quarterly for 35 years.

Variable	Definition of Variable	Value in Reginald's Situation
N	number of compounding periods between the time of investment and the time of retirement	
I%	annual interest rate (as a percent)	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value, or value of the investment at maturity	
P/Y	number of payments per year (usually the same as the number of compounding periods per year, ***C/Y***)	
C/Y	number of compounding periods per year	

5. Hilda wants to have $10,000 in 10 years after investing in an account that earns 3.6% compounded monthly.

Variable	Definition of Variable	Value in Hilda's Situation
N	number of compounding periods between the time of investment and the time of retirement	
I%	annual interest rate (as a percent)	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value, or value of the investment at maturity	
P/Y	number of payments per year (usually the same as the number of compounding periods per year, ***C/Y***)	
C/Y	number of compounding periods per year	

Decision Making in Finance: Present Value of an Investment

VI.B Student Activity Sheet 5: A Cool Tool!

6. Juan wants to invest $1,250 in an account that earns 2.34% interest, compounded monthly. How many years will it take for the account to have a value of $5,000?

Variable	Definition of Variable	Value in Juan's Situation
N	number of compounding periods between the time of investment and the time of retirement	
I%	annual interest rate (as a percent)	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value, or value of the investment at maturity	
P/Y	number of payments per year (usually the same as the number of compounding periods per year, ***C/Y***)	
C/Y	number of compounding periods per year	

Student: ______________________________ Class: ____________________ Date:______________

Decision Making in Finance: Present Value of an Investment

VI.B Student Activity Sheet 5: A Cool Tool!

7. Another of Vanessa's clients, Ronnie, wants to save for retirement. Ronnie believes that he will need $2,000,000, since he is planning to be retired for 20 to 30 years. He can save in investments that have the following parameters:

- The number of years to save is 20 to 40.
- The number of compounding periods is annually, quarterly, monthly, weekly, and daily.
- The interest rate can be 2.77% to 5.23% or any rate between.

Ronnie wants to know the effect that each variable has on the present value. Select a variable, and use the following steps to complete the table below:

- Start with the minimum value of your variable.
- Use the average value of the other variables that have parameters.
- Calculate the present value of the investment.
- Decide the next value of your variable to test and repeat the process for a total of five different values.

Present-Value Analysis

Variable Value:	**Present Value (*PV*)**	**Percent Change in Present Value**
Minimum:		
Maximum:		

Student: ______________________________ Class: ____________________ Date:______________

Decision Making in Finance: Present Value of an Investment

VI.B Student Activity Sheet 5: A Cool Tool!

Present-Value Analysis

Variable Value:	Present Value (*PV*)	Percent Change in Present Value
Minimum:		
Maximum:		

8. Overall, what impact on the present value does each variable have?

9. **REFLECTION:** Of all the variables, which seems to have the greatest effect on lowering the present value of Ronnie's investment? Explain your reasoning.

10. **EXTENSION:** Prepare a short presentation of your findings for one of the following scenarios to share with the class.

- Sarah wants to save for a car. She has $4,250 in a savings account earning 1.49% compounded quarterly. If Sarah has four years until she gets her driver's license, will she have enough to buy a car? If not, what do you recommend that she do to reach her goal?
- Find the median price of a home in your area and the current annual rate of growth for home values. If you buy a home at the median price and expect it to increase in value at the current growth rate, what will the future value of your home be in 30 years? Would you buy the house knowing that the interest rate on the mortgage (that is, the loan needed to buy the house) is 6% and you must invest an additional 2% of the home's value in upkeep per year? Why or why not?
- Students often take out large loans to go to college. Currently, these loans have a payoff time of 25 years at an interest rate of about 7.5% compounded monthly. Suppose the remaining principal of Dexter's student loans is $33,760 and the remaining payoff time is 15 years at the 7.5% rate. Dexter recently inherited $40,000 and wants to know if he should pay off his student loans or invest the money. What do you recommend? Why?
- The state lottery offers to pay winnings in 25 annual payments or one lump sum, sometimes called a cash-out option. This week's lottery has a jackpot of $30 million and a cash-out value of $18.2 million. Granted that the odds are highly unlikely one would win, which option should a winner take—annual payments or a lump sum? Why?
- Pick an expensive item you want to buy within the next five years. Using current interest rates, find out how much you would have to save today. List the possible roadblocks to reaching the goal.

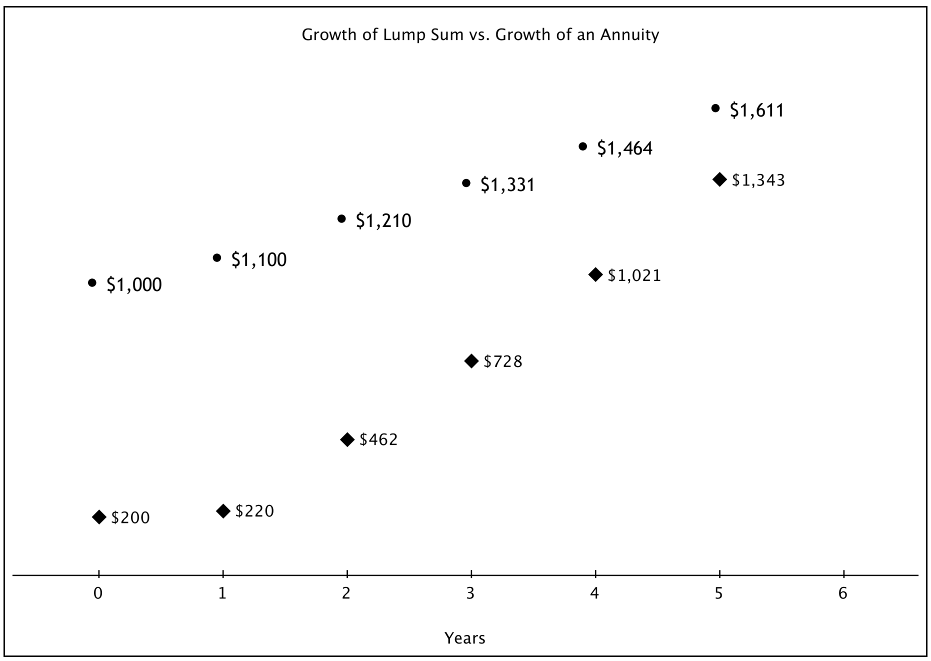
Student: ________________________________ Class: ______________________ Date:_______________

Decision Making in Finance: Building an Investment

VI.C Student Activity Sheet 6: Investing As You Go

An **annuity** is a financial product that accepts and grows funds and then, upon annuitization, pays out regular payments to the investor. Annuities are often used as retirement funds. Some annuities are funded with a lump-sum investment, while others are funded with an initial investment and additional regular deposits before retirement. What complicates the time value of money (TVM) of an annuity that you pay into is that the investment increases in value due to both compound interest and increasing principal.

The following graph shows the value of a lump-sum investment of $1,000 earning 10% compounded per year (•) versus an annuity with an initial investment of $200 earning 10% compounded per year with additional $200 deposits made each year (♦).



1. How is the process different for calculating the future value of each investment?

2. Refer to the future-value formula in Student Activity Sheet 3. How is the process different in calculating the future value of an annuity when compared to using the future-value formula?

3. An annuity can be thought of as a series of values connected by a common ratio. What common ratio connects the values of the annuity over time shown in the graph at the beginning of this activity sheet? How is the ratio related to the problem situation?

Decision Making in Finance: Building an Investment

VI.C Student Activity Sheet 6: Investing As You Go

4. The following formula can be used to calculate the sum of a series connected by a common ratio, such as the previous annuity example.

$$S_n = \frac{a_1(1 - r^n)}{(1 - r)}, \text{ where}$$

a_1 = the first term in the series, n = the number of terms in the series, and r = the common ratio.

Use the formula to calculate the value of the annuity described in the graph, and compare the results after five years.

5. In Student Activity Sheet 5, you learned to use a TVM calculator to determine different variables related to TVM. In your prior work with the TVM calculator, you only considered lump-sum investments (and the payment variable was always 0).

Explore using the TVM calculator to determine the future value of the $200 annuity over five years, and compare your answer with the known future value of $1,343.12. List the values you assigned to each variable and explain why.

(***Note:*** Interest is typically paid at the end of the compounding period. In this case, you make payments at the beginning of each period. Therefore, you must change appropriate variable from END to BEGIN.)

Variable	Definition of Variable	Value in This Situation
N	number of compounding periods between the time of investment and the time of retirement	
I%	annual interest rate (as a percent)	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value, or value of the investment at maturity	
P/Y	number of payments per year (usually the same as the number of compounding periods per year, ***C/Y***)	
C/Y	number of compounding periods per year	

Decision Making in Finance: Building an Investment

VI.C Student Activity Sheet 6: Investing As You Go

6. Amy is 25 years old and has attended some retirement planning seminars at work. Knowing she should start thinking about retirement savings early, Amy plans to invest in an annuity earning 5% interest compounded annually. She plans to save $100 from her monthly paychecks so that she can make annual payments of $1,200 into the annuity. Use the TVM calculator to determine the future value of the investment after 35 years.

Variable	Definition of Variable	Value in This Situation
N	number of compounding periods between the time of investment and the time of retirement	
I%	annual interest rate (as a percent)	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value, or value of the investment at maturity	
P/Y	number of payments per year (usually the same as the number of compounding periods per year, ***C/Y***)	
C/Y	number of compounding periods per year	

7. Amy seeks the advice of a financial planner, who recommends $850,000 for retirement. Will Amy's annuity plan provide the necessary funds for her retirement? If not, what could she do to increase the value of the investment at retirement? Of those actions, which does she have relative control over?

Decision Making in Finance: Building an Investment

VI.C Student Activity Sheet 6: Investing As You Go

8. Amy finds another annuity that accounts for **monthly** compounding and **monthly** payments. The annuity pays 6% annual interest, compounded monthly. Use the TVM calculator to determine the monthly payments Amy needs to make over 40 years to have $850,000 at the time of her retirement.

Variable	Definition of Variable	Value in This Situation
N	number of compounding periods between the time of investment and the time of retirement	
I%	annual interest rate (as a percent)	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value, or value of the investment at maturity	
P/Y	number of payments per year (usually the same as the number of compounding periods per year, ***C/Y***)	
C/Y	number of compounding periods per year	

9. REFLECTION: What recommendations would you make to Amy about her retirement goals and using an annuity to financially support those goals?

10. EXTENSION: Contact a financial planner or conduct research via the Internet to determine what recommendations might be available for a client such as Amy in today's financial environment. Prepare a report of your findings to share with the class.

Student: ______________________________ Class: ____________________ Date:______________

Decision Making in Finance: Building an Investment

VI.C Student Activity Sheet 7: Investment Probability

Interest rates are a measure, among many other factors, of risk. The more risky an investment is in actuality and perception, the higher the rate of return. In general, stocks (an investment security that gives you ownership in a company) are riskier than bonds (a security in which you actually lend money to a company). Thus, the rate of return is much higher for stocks than bonds; on average, stocks have a rate of return of 10% annually and bonds 5% annually.

Use the following information when working through these activities:

- All investments have a rate of return (which sometimes can be negative).
- The rate of return on stocks is a percentage called a *return on investment* (ROI) that compounds not from interest payments but from an overall annual increase based on a price per share that changes daily.
- The rate of return on bonds is an actual interest rate percentage that is assumed to compound (much like a certificate of deposit), but may not if you decide not to reinvest the interest.
- Financial analysts use the time value of money (TVM) based on risk, rate of return, and the relationship it has with other investments to determine the market value or price of a share of stock or bond.
- Although interest rates are used in bonds, financial experts use *interest* as the lending rate that the Federal Reserve sets for banks. This may not seem related to stock prices or bonds, but the interest rate set by the Federal Reserve affects the value of all investments.

1. Stock Texas is worth $14.92 per share on Monday. The interest rate drops on Tuesday, and Stock Texas is worth $15.04 per share. What type of relationship can you assume that Stock Texas has with interest rates? Why?

What does this relationship imply about the risk of stocks compared to bonds? Explain your reasoning.

2. On Wednesday, Bond Austin has the best risk rating, *Aaa,* at a price of $72. On Thursday, the risk rating drops to a lower rating of *Aa,* and the price drops to $64. What type of relationship can you assume that the price of Bond Austin has with its risk ratings? Why?

Do you think that this is a reasonable assumption about the relationship between bonds and risk ratings? Why or why not?

3. Assume losing a letter is considered one unit of risk and you assign the highest (meaning better) rating a 9. What does the price of Bond Austin drop to if the risk rating suddenly becomes *Bb* (a risk rating of 5)?

Decision Making in Finance: Building an Investment

VI.C Student Activity Sheet 7: Investment Probability

4. Stock Texas has a price of $156 per share when Bond Austin has a price of $23 per bond. Use an equation modeling the inverse variation between the stock and bond prices to predict the price of Stock Texas when Bond Austin is worth $75.

   What is the bond price if the stock price is $71.76?

5. **REFLECTION:** How certain is this prediction? What other factors could affect the price of either investment?

6. **EXTENSION:** Emily, who is 25 years old, has $25,000 to invest. She wants to invest in stocks, bonds, and/or cash accounts (collectively called an investment **portfolio**). Currently interest rates (and inflation) are relatively low, but seem to be on the rise. Decide the percentage and amount that Emily should invest in each category.

   Suppose interest rates go up, but overall risk in investments increases. Should Emily consider adjusting her portfolio? Explain your reasoning.

   Emily will keep her investment for 35 years, which is the time of her retirement. Using the portfolio you developed, find the future value of each category if stocks have an average annual rate of increase of 12%, bonds an average annual rate of increase of 6%, and cash an average annual rate of increase of 3%.

   What is the expected value of each category if the probability of realizing the average rate for stocks is 0.65, bonds 0.8, and cash 0.95?

7. **EXTENSION:** Create your own portfolio and explain what factors influence its expected value. Prepare a report of your information and predictions to share with the class.

Student: ______________________________ Class: ____________________ Date:______________

Decision Making in Finance: Using Credit

VI.D Student Activity Sheet 8: Making Sense of Credit

Anatomy of a Credit Card Statement

The following is a monthly statement from a typical credit card company. Parts left out intentionally are denoted by *???* and highlighted in gray.

TEXAS CREDIT

OPENING/CLOSING DATE:	7/19/08 – 08/18/08
PAYMENT DUE DATE:	9/12/08
MINIMUM PAYMENT DUE:	$93.30

CARD SUMMARY — ACCOUNT NUMBER 5555 5555 5555 5555

PREVIOUS BALANCE	$2,342.51	TOTAL CREDIT LINE	$3,000
PAYMENT, CREDITS	-$150.21	AVAILABLE CREDIT	$376
PURCHASES, CASH, DEBITS	$410.89	CASH ACCESS LINE	$500
FINANCE CHARGES	???	AVAILABLE FOR CASH	$376
NEW BALANCE	???		

TRANSACTIONS

DATE	DESCRIPTION	CREDIT	DEBIT
7/23	GAS		$70.61
7/24	PAYMENT – THANK YOU	$100	
7/24	HARDWARE STORE		$139
7/28	FLOWERS		$24.95
8/03	GROCERIES		$176.33
8/18	HARDWARE STORE RETURN	$50.21	

FINANCE CHARGES

TYPE	DAILY PERIODIC RATE 31 DAYS IN CYCLE	APR	AVERAGE DAILY BALANCE	FINANCE CHARGE DUE TO PERIODIC RATE
PURCHASES	???	28.99%	???	???
CASH	???	28.99%	$0	$0

1. Use the information in the statement to determine the balances throughout the month and then calculate the average daily balance for these purchases.

2. The **daily periodic rate** describes the interest you are paying on your credit every day.
   - Use the following formula to calculate the daily periodic rate to five decimal points.
   - Use this rate to determine the finance charge to the nearest cent. (***Note:*** *APR* stands for *annual percentage rate*.)

$$\text{daily periodic rate} = \frac{\text{APR}}{\text{days in year}}$$

3. Calculate the new balance, considering credits, debits, and finance charges.

Decision Making in Finance: Using Credit

VI.D Student Activity Sheet 8: Making Sense of Credit

4. What percentage is the minimum payment to the new balance before interest?

5. Marley has a credit card with an APR of 22.75% and a current balance of $14,677.90. If Marley uses the same percentages from the previous questions, what is her minimum payment (to the nearest cent)?

6. Using the minimum payment from Question 5, how long will it take Marley to pay off the current balance, assuming she does not add any more charges to her credit card? How much in interest would paying only the minimum every month cost her?

Variable	Definition of Variable	Value in Marley's Situation
N	number of compounding periods	
I%	annual interest rate (as a percent)	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value	
P/Y	number of payments per year	
C/Y	number of compounding periods per year	

7. Suppose Marley makes $2,500 per month. Create a budget for Marley to find how much she has left over to pay the minimum on her credit card. (Remember to consider the taxes taken out of her paycheck: Social Security—6.2%, Medicare—1.45%, and federal income tax—15%.)

Can Marley afford the minimum payment? If so, how much more than the minimum can she pay? If not, what do you recommend she do to afford the payment and pay off the credit card?

Decision Making in Finance: Using Credit

VI.D Student Activity Sheet 8: Making Sense of Credit

8. The credit statement shows the APR. However, most credit card companies compound interest more often than annually. The actual interest rate you pay each year, taking into account compounding, is called the **effective annual rate** (EAR). It can be calculated with the following formula:

$$\text{EAR} = \left(1 + \frac{\text{APR}}{\boldsymbol{n}}\right)^{n} - 1,$$ where ***n*** is the number of compounding periods per year.

Benny's credit card APR is 26.55% compounded daily. What is his actual interest rate per year—that is, his EAR?

9. **REFLECTION:** Is the EAR higher than the APR? Why or why not?

10. **EXTENSION:** Research nonprofit consumer debt counseling sites that explain the elements of a credit card statement, some misconceptions about credit, and the pitfalls that get credit card users in trouble.

Student: ______________________ Class: ________________ Date:____________

Decision Making in Finance: Using Credit

VI.D Student Activity Sheet 9: Understanding Credit Card Debt

J.R. owes $9,000 on a credit card charging a 16.8% annual percentage rate (APR). He stopped using the card and has a debt plan to pay $319.97 per month to pay off the balance in 36 months.

1. Create an amortization table for the 36 months of J.R.'s debt plan.

2. Graph the principal and interest portions as separate bar graphs for the 36 months.

3. **REFLECTION:** Compare and contrast the two graphs.

4. Will the payment in the 36th month be the same as all the rest? Why or why not?

5. **EXTENSION:** Prepare a short presentation of your findings for one of the following scenarios to share with the class.

   - Phoenix has gotten herself in a bit of trouble with credit cards. The following are the current balances and interest rates on her credit cards:
     - Visa: $6,750 at 19.8% APR
     - MasterCard: $8,267 at 16.5% APR
     - Gas card: $1,579 at 22.65% APR
     - Department store card: $3,345 at 21.99% APR

     Phoenix earns $3,000 per month as a painter. Can she afford this debt? Develop a debt plan so that her credit cards are paid off in two years.
   - Horace paid for a $0.79 pack of gum with a credit card. Due to his revolving balance, he will end up paying 23.49% interest on that pack of gum for 10 years. How much did it really cost Horace to charge that pack of gum? How much would a $1,000 couch really cost him?
   - Neeraj will pay $350 per month toward his credit card debt for five years. Create a report that demonstrates how Neeraj could have used that money differently had he not used his credit cards.
   - You want to buy $10,000 in furniture and electronics for your new home. Research different credit card offers and, assuming you qualify for the full amount, choose the card(s) on which you will charge this purchase and explain your choice.

Student: ______________________ Class: ____________________ Date:______________

Decision Making in Finance: Using Credit
VI.D Student Activity Sheet 10: Buying a Losing Investment

1. Christina is considering buying a new car with a sticker price of $23,599. Her credit union offers her a three-year car loan at 5.99% annual percentage rate (APR) with 10% as a down payment. Find the monthly payment.

Variable	Definition of Variable	Value in Christina's Loan Situation
N	number of compounding periods	
I%	annual interest rate	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value	
P/Y	number of payments per year	
C/Y	number of compounding periods per year	

2. Christina's car will be worth $14,250 in three years. What will the total cost of the car be at the end of the loan?

What is the benefit of this type of financing? What is the cost of this type of financing?

3. Christina considers a different option. The dealership offers 0% down and 0% APR for two years. The car will be worth $17,629 in two years.

What will the monthly payments be under these conditions? How much will the total cost of the car be if Christina takes this loan?

Which loan should Christina take? Why?

4. Christina has an offer to lease the same car for three years at $349 per month. The lease has a balloon payment of $1,200 at the end of three years. What is the total cost of the lease?

5. What interest rate is Christina being charged for leasing the car?

Variable	Definition of Variable	Value in Christina's Leasing Situation
N	number of compounding periods	
I%	annual interest rate	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value	
P/Y	number of payments per year	
C/Y	number of compounding periods per year	

Should Christina take the lease? Why or why not?

6. The car manufacturer offers a lease-to-purchase option at 1.9% APR for three years. At the end of this option, Christina can keep the vehicle by paying the depreciated value or walk away for a fee of $150. What is the monthly payment of the lease-to-purchase option? What is the total cost of the purchase option if she walks away?

Variable	Definition of Variable	Value in Christina's Lease-to-Purchase Situation
N	number of compounding periods	
I%	annual interest rate	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value	
P/Y	number of payments per year	
C/Y	number of compounding periods per year	

Decision Making in Finance: Using Credit

VI.D Student Activity Sheet 10: Buying a Losing Investment

7. **REFLECTION:** Which alternative should Christina choose: the loan, the lease, or the purchase option? Why?

8. Christina works for a law firm and makes $42,350 a year. Based on standard budgeting used in Student Activity Sheet 8 and using your choice in Question 7, can she afford the car? Explain your answer.

9. **EXTENSION:** Wanda wants to buy a new car for $34,650. The bank will give her a car loan for five years at 4.5% APR with $0 down payment. What will her monthly payment be?

Variable	Definition of Variable	Value in Wanda's Loan Situation
N	number of compounding periods	
I%	annual interest rate	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value	
P/Y	number of payments per year	
C/Y	number of compounding periods per year	

   a. Wanda's car will be worth $18,935 in five years. The manufacturer offers a lease-to-purchase option at 7% APR. At the end of the purchase option, Wanda can keep the vehicle by paying the depreciated value or walk away for a fee of $180. What will her monthly payment be?

Variable	Definition of Variable	Value in Wanda's Lease-to-Purchase Situation
N	number of compounding periods	
I%	annual interest rate	
PV	principal, or present value	
PMT	amount of each regular payment	
FV	future value	
P/Y	number of payments per year	
C/Y	number of compounding periods per year	

b. What is the total cost for the loan? What is the total cost for the purchase option if Wanda walks away for $180? Which alternative should Wanda choose: the loan or the purchase option? Why?

10. EXTENSION: Research websites that calculate and compare all three methods of financing vehicles. Select a vehicle, determine the monies involved in each type of financing, and make a decision regarding which is the best option. Prepare a short presentation to share with the class.

Advanced Mathematical Decision Making

In Texas, also known as

Advanced Quantitative Reasoning

Unit VII: Networks and Graphs

This course is a project of
The Texas Association of Supervisors of Mathematics and
The Charles A. Dana Center at The University of Texas at Austin
With support from the Greater Texas Foundation

2010

Advanced Mathematical Decision Making

In Texas, also known as

Advanced Quantitative Reasoning

Student Materials

These student materials are excerpted from one of seven units that make up the 2010 AMDM/AQR instructional materials (developed under the name Advanced Mathematical Decision Making).

Unit I: Analyzing Numerical Data
Unit II: Probability
Unit III: Statistical Studies
Unit IV: Using Recursion in Models and Decision Making
Unit V: Using Functions in Models and Decision Making
Unit VI: Decision Making in Finance
Unit VII: Networks and Graphs

Table of Contents

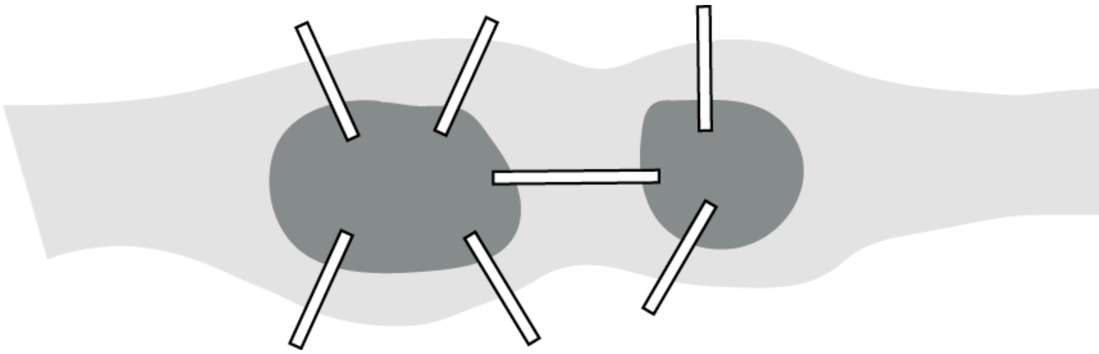
Student: ___________________________ Class: ___________________ Date:_____________

Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 1: Euler Circuits and Paths

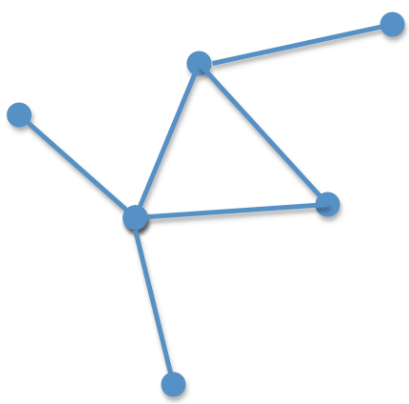
The Königsberg Bridge Problem

The following figure shows the rivers and bridges of Königsberg. Residents of the city occupied themselves by trying to find a walking path through the city that began and ended at the same place and crossed every bridge **exactly once**.

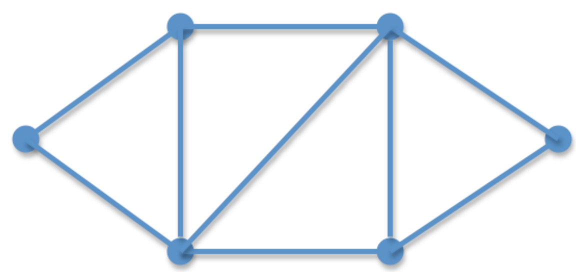


1. If you were a resident of Königsberg, where would you start your walk and what path would you choose?

2. What about when you visit the Eastern and Western wildflower gardens that have fabulous sculptures in addition to beautiful flowers along the walkways. You want to see each display without backtracking (seeing something you have already seen). Where would you start your walk and what path would you choose?



Western garden



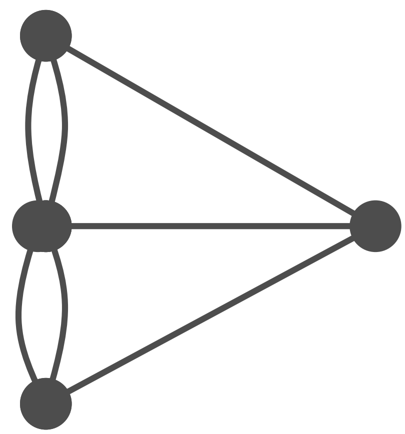
Eastern garden

Student: ____________________________ Class: ____________________ Date:______________

Networks and Graphs: Circuits, Paths, and Graph Structures

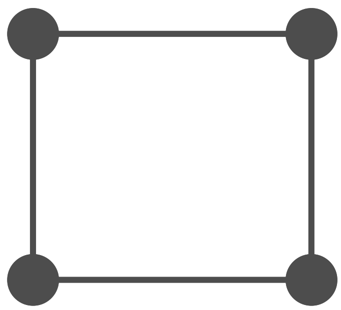
VII.A Student Activity Sheet 1: Euler Circuits and Paths

When Leonhard Euler, a famous mathematician, turned his attention to the Bridge problem, his first step was to **model** the bridges of Königsberg with a simple graph. The points, or **vertices**, represented land and the **edges** represented the bridges connecting them. Euler's map of Königsberg, while much simpler, conveyed all the necessary information about which parts of land were connected by which bridges. It looked something like the following:

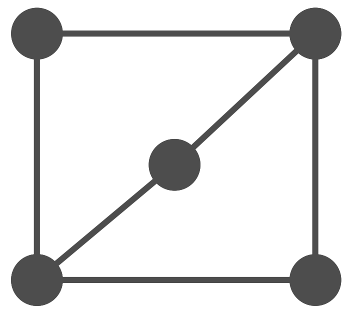


The Bridge problem is now stated: *Given a graph, find a path through the vertices (points) that uses every edge exactly once*. Such a path is called a *Euler path*. If a Euler path begins and ends at the same vertex, it is called a *Euler circuit*.

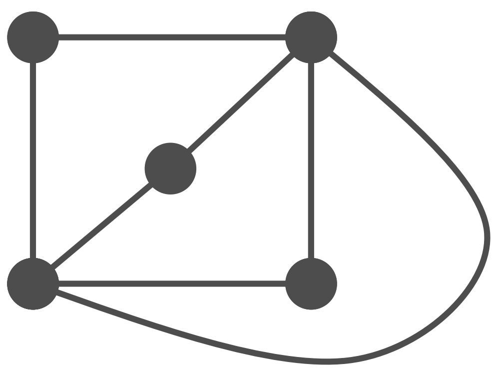
3. For the following graphs, decide which have Euler circuits and which do not.



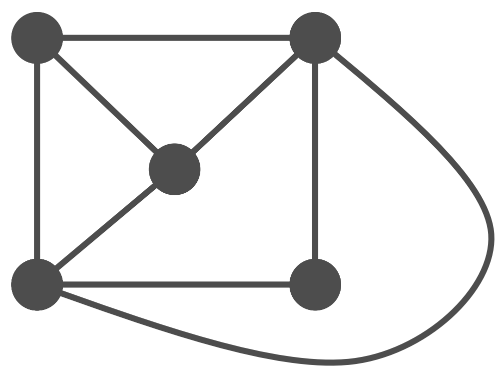
Graph I



Graph II



Graph III



Graph IV

4. The *degree* of a vertex is the number of edges that meet at the vertex. Determine the degree of each vertex in Graphs I–IV.

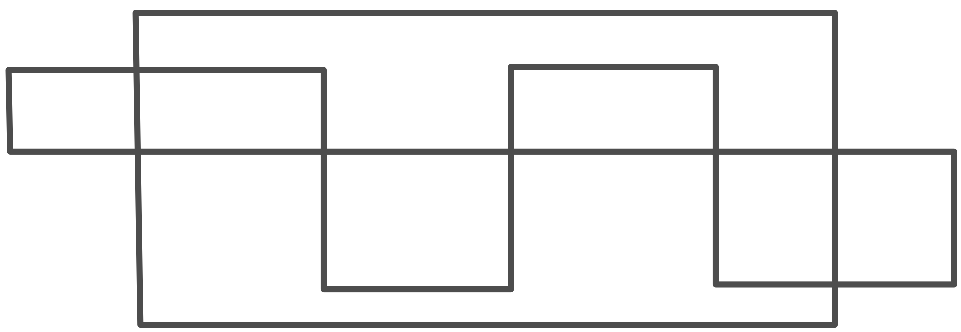
5. For the graphs from Question 3 that have Euler circuits, how many vertices have an odd degree?

6. For the graphs from Question 3 that have Euler circuits, how many vertices have an even degree?

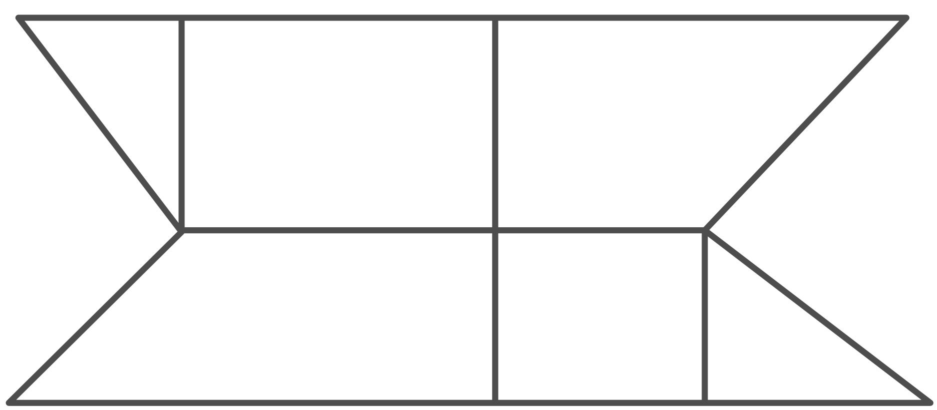
7. Form a **conjecture** about how you might quickly decide whether a graph has a Euler circuit, and explain why your conjecture seems reasonable.

8. What does your conjecture tell you about the Königsberg Bridge problem and the garden scenario?

9. Your friend Chet calls you on his cell phone and tells you that he has discovered a large rock embedded with gems! He is somewhere in your favorite hiking area, which has many interconnected paths, as shown below. Chet does not know exactly where he is, but he needs your help to carry the rock. To find him, you decide it would be most efficient to jog along all the paths in such a way that no path is covered twice. Find this efficient route on the map below or explain why no such route exists.



10. You have been hired to paint the yellow median stripe on the roads of a small town. Since you are being paid by the job and not by the hour, you want to find a path through the town that traverses each road only once. In the map of the town's roads below, find such a path or explain why no such path exists.



11. **REFLECTION:** For what situation(s) is it satisfactory to have only a *path* exist and not a *circuit*?

12. **EXTENSION:** Determine some other real-world problems whose solutions may involve finding Euler circuits or paths in graphs. There are a variety of road-traversing problems: delivering mail, garbage/recyclable collecting in a city, sweeping/cleaning streets, and so on.

    For each situation, describe what real-world complications exist that might make the problem more difficult. For example, when delivering mail, most streets have houses on either side of the street and the postal worker may decide to go up one side of the street and down the other. If the city has alleyways, perhaps the garbage collectors just need to travel down the alleys.

    Be prepared to make a short presentation of your findings to the class.

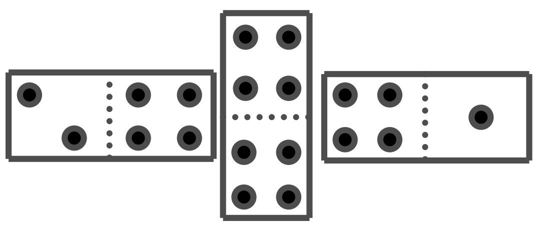
Student: ______________________ Class: ________________ Date: ____________

Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 2: Dominoes

Dominoes are rectangular tiles divided into two squares. Each square has a number (usually represented by a series of dots) from 0 to 6. A double-six set of dominoes has tiles of every possible combination of these numbers, from 0–0 to 6–6. Each possible combination of numbers appears only once in a set, so a complete set of dominoes contains 28 tiles. Two tiles from a double-six set of dominoes are shown below:

In many games, you must place the dominoes next to each other in such a way that squares with identical numbers are placed next to each other. Doubles are traditionally rotated before they are placed. Adhering to these rules, the following shows a legal placement of three dominoes:



Domino Placement Problem

Can all the dominoes in a double-six set be placed in a single line of tiles adhering to the placement rules previously described? Can they all be placed so that the single line loops back to the first domino? The remainder of this activity sheet will help you answer these questions.

1. Decide how to use a graph to model the Domino Placement problem. Carefully define what your vertices represent and how you know when two vertices are connected by an edge.

2. Based on your model, restate the Domino Placement problem.

3. What type of previously solved problem in this unit is this problem related to?

4. Solve the Domino Placement problem.

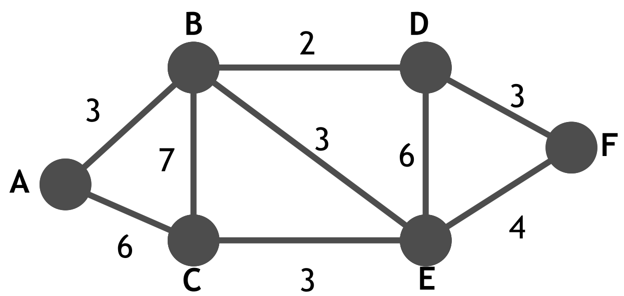
Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 3: Weighted Graphs

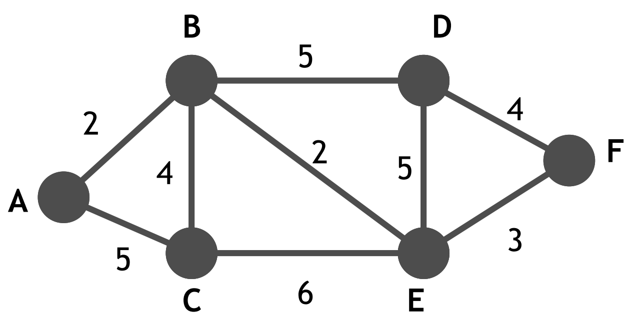
The Snowplow Problem

As the new snowplow operator, you must decide the best route through three cities. In each city, you need to plow all the roads and return to your starting place, but you must also keep from backtracking as much as possible.

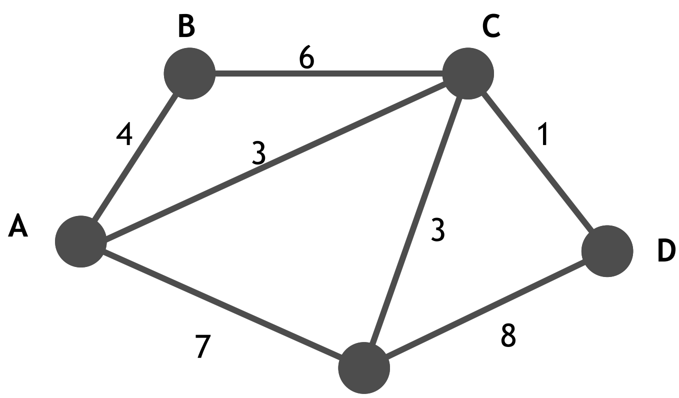
1. Construct two snowplow routes through each of the following cities and indicate the time it will take to travel each route. The time it takes to traverse each road (in hours) is indicated in the graph.



City I



City II



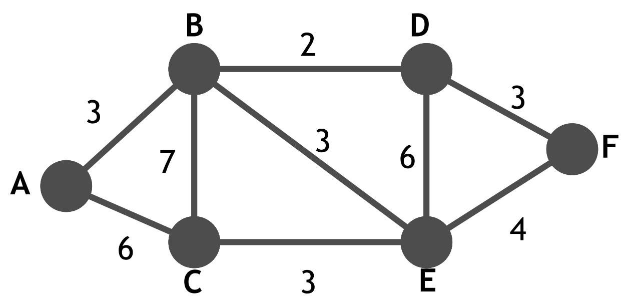
City III

Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 3: Weighted Graphs

2. **REFLECTION:** How would you solve the Snowplow problem for a graph that has no vertices of an odd degree?

3. City I has two vertices of an odd degree.

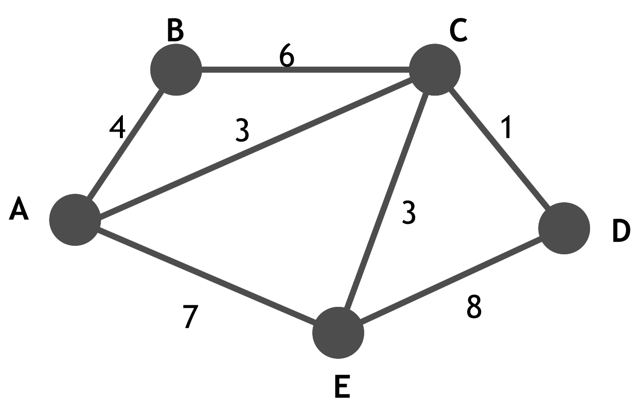


   a. Find these vertices and the shortest path between them.

   b. For each edge in this shortest path, put in a second copy of the edge.

   c. At this point, your graph should have no vertices of an odd degree. Find a Euler circuit and compare this path with the paths you found for City I.

4. Follow the procedure outlined in Question 3 to find a solution to the Snowplow problem for City III.



Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 3: Weighted Graphs

5. **EXTENSION:** Prior to the beginning of school, a huge task occurs at almost every school in the nation—cleaning the floors! Whether it be waxing, steam cleaning, or mopping, it is critical that the floors be ready for the first day of school. Your task is to design the plan for this cleaning project.

   - Draw a graph of the hallways in your school or portion of your school (at least six edges).
   - Assign weights to each edge according to the width and length of the corresponding hallway and how long it will take to complete that edge or hallway.

   Design a path through your school or portion of your school and determine the total number of hours required to complete the cleaning of the floors. Be prepared to share your design with the class.

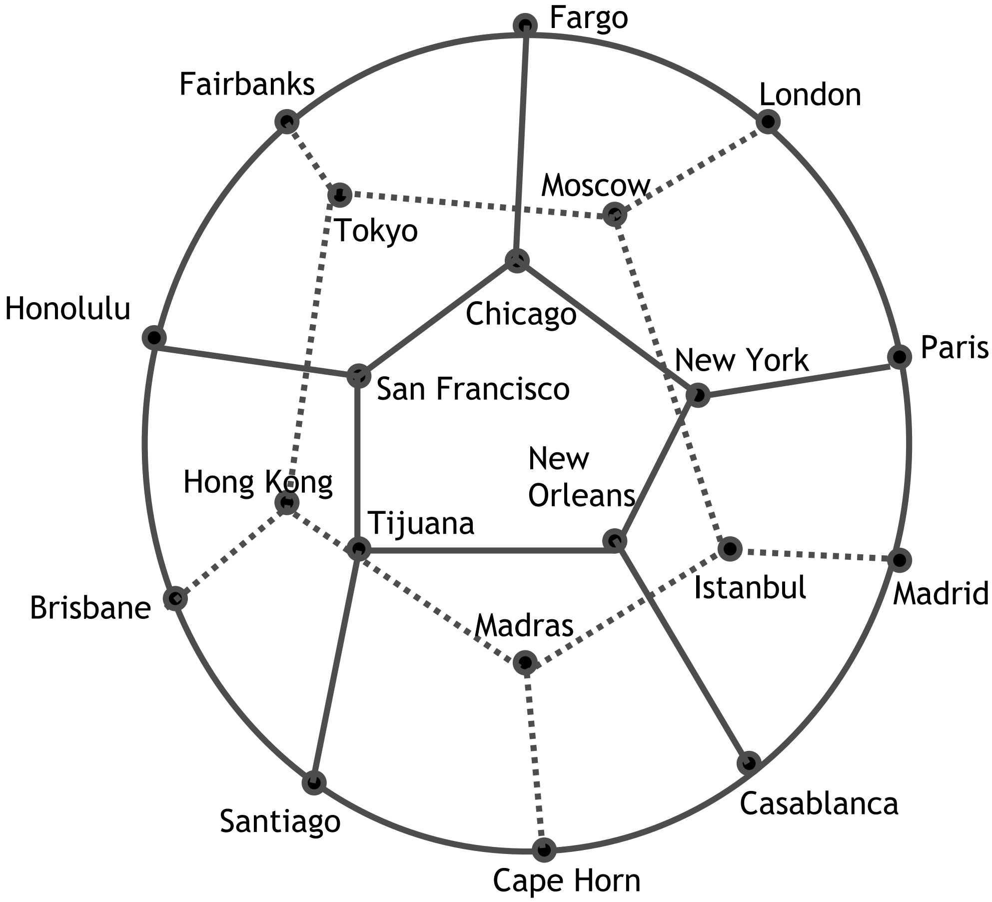
Student: ___________________________________ Class: ________________________ Date:_________________

Networks and Graphs: Circuits, Paths, and Graph Structures

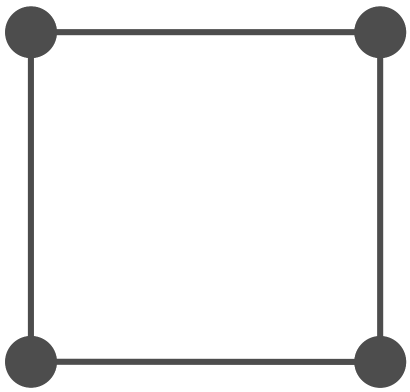
VII.A Student Activity Sheet 4: Hamiltonian Circuits and Paths

A Voyage Around the World

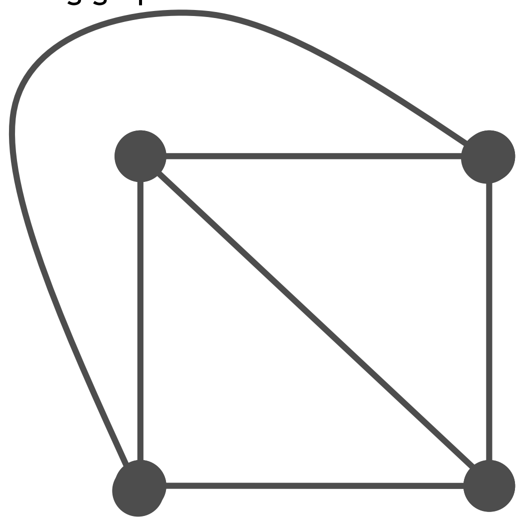
1. Plan a trip around the world by visiting each city exactly once and using only the identified routes to travel from city to city. The dashed lines represent routes on the opposite side of the globe.



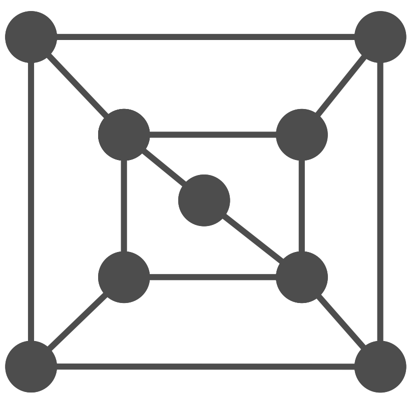
2. A path on a graph that goes through each vertex once is called a *Hamiltonian path*. A path that starts and stops at the same vertex and goes through each vertex once is called a *Hamiltonian circuit*. Which of the following graphs have a Hamiltonian circuit?



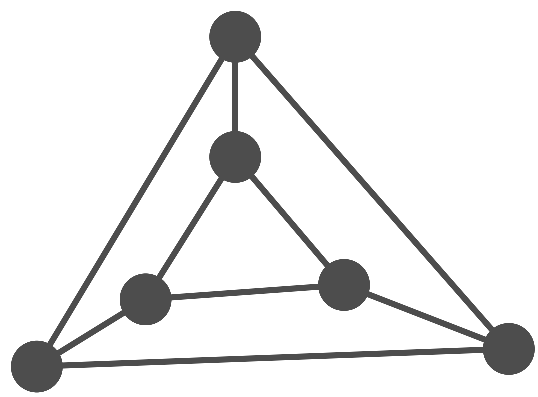
Graph I



Graph II



Graph III



Graph IV

3. Form a conjecture about when you think a graph might have a Hamiltonian circuit.

4. Share your conjecture with others and try to find examples of graphs that disprove your conjecture. These are called *counterexamples*.

5. **REFLECTION:** Compare and contrast a Euler circuit and a Hamiltonian circuit.

6. **EXTENSION:** Describe a situation (other than travel) that requires a Hamiltonian circuit exist, but not a Euler circuit. Include either a diagram and graph or similar diagrams that show the connection of the graph to the real situation. Provide any details necessary to connect to the real-world application of this learning.

Student: ______________________ Class: ______________________ Date: ______________

Networks and Graphs: Circuits, Path, and Graph Structures

VII.A Student Activity Sheet 5: Knight's Tour

One game piece in chess is called a *knight,* and it is usually represented by a horse-like figure. This is the only chess piece that can jump over other pieces. An allowable move for a knight consists of moving two squares in one direction, followed by turning 90º, and then moving one square. A knight cannot move diagonally. The following figure shows a portion of a chessboard and all the possible squares (**X**) the knight (**K**) could go to in one move.

	X		X	
X				X
		K		
X				X
	X		X	

<u>Knight's Tour Problem</u>

A traditional chessboard consists of an 8-by-8 grid with 64 squares. For the purposes of this activity, only a 4-by-4 grid is considered.

If a knight is placed in the upper left-hand square, is there a sequence of moves that allows the knight to visit every square on the chessboard exactly once and then return in the last move to its starting place? The remainder of this activity sheet will help you answer this question.

1. Decide how to use a graph to model the Knight's Tour problem. Carefully define what your vertices represent and how you know when two vertices are connected by an edge.

2. Based on your model, restate the Knight's Tour problem.

3. What type of previously encountered problem in this unit is this problem related to?

4. Solve the Knight's Tour problem on a variety of smaller chessboards: 4 by 4, 5 by 5, or 3 by 6.

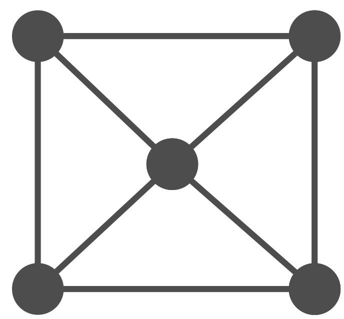
Student: ________________________________ Class: ______________________ Date:______________

Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 6: High-speed Internet

Your company must run Ethernet cables to five different offices so that all five offices have high-speed Internet access. For each computer to be on the office network, there must be a way to get from each computer to the other computers by following the cable.

1. One worker proposed running cable between the five offices as illustrated in the following diagram. The vertices represent the offices, and the edges represent segments of cable.



How many lengths of cable (edges) are used? Explain why this an inefficient way to run the cable.

2. Design a more efficient network and indicate how many lengths of cable are used.

3. Compare your efficient network with others in the class.

a. Did everyone use the same number of cable lengths?

b. Did everyone's network have the same shape?

4. A *cycle* in a graph is a path that starts and ends at the same vertex and does not use any edge more than once.

a. Identify two cycles in the graph from Question 1.

b. Does your network from Question 2 have any cycles? Should it?

c. What does the existence of cycles tell you about the efficiency of a network?

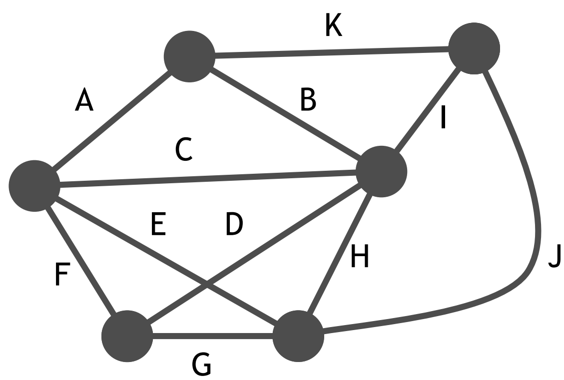
5. **REFLECTION:** Describe how a cycle is similar to a Euler circuit.

6. Write a set of step-by-step instructions to form an algorithm for converting an inefficient network into an efficient network.

Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 6: High-speed Internet

7. Have another student apply your algorithm to the following inefficient network and indicate the number of edges in the final efficient network.



8. **EXTENSION:** Research other situations that might be modeled with graphs in such a way that cycles become important. Draw an efficient network for the situation (not a cable connection, of course!). Prepare a short presentation for the class.

Use the following questions as needed:

- Do all graphs have cycles?
- Are the cycles unique?

Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 7: Minimal Spanning Trees

A railroad system connecting five cities is being planned. The goal is to build this system using the least amount of money, while ensuring that each city can be reached by any other city in the system. Based on the distance and terrain, the following chart gives estimates for the cost, in hundreds of thousands of dollars, to build a railroad between any two cities.

	City A	**City B**	**City C**	**City D**	**City E**
City A		10	5	4	2
City B			7	9	11
City C				5	10
City D					12

1. Create a graph to model the costs contained in the table.

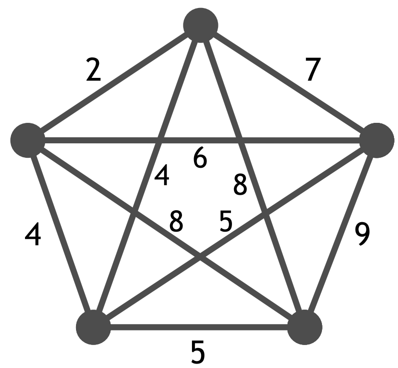
2. Restate the railroad problem using the terminology associated with graphs (for example, *vertices, edges, paths, cycles*).

3. Construct two different graphs that represent possible railroad networks, and calculate the total cost to build each rail system.

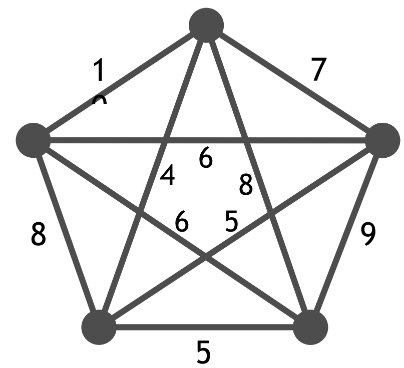
4. What rail system leads to the lowest construction cost?

5. How did you arrive at the most efficient rail system? State your answer in a step-by-step algorithm that someone else could follow.

6. The numbers in the following graphs depict the cost associated with building a railroad between cities represented by vertices. For each graph, test your algorithm. Does it yield the most efficient network? If not, try modifying your algorithm.



Network I



Network II

7. **REFLECTION:** Given any weighted graph (like the ones from this activity), does an efficient network of minimal cost always exist? Why or why not?

8. **EXTENSION:** What other real-world problems might be solved by creating and analyzing graphs with weighted edges? Conduct research to respond to this question, and prepare a short presentation for the class.

Student: ______________________________ Class: ____________________ Date:______________

Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 8: Kruskal's Algorithm

A graph whose edges are given numerical values is called a *weighted graph*. Keeping all the vertices connected by a path resulting in a minimum total weight is called *finding a minimal spanning tree*. The word *spanning* means that each vertex remains connected to the graph, and the word *tree* indicates that there are no cycles.

The following procedure, known as Kruskal's Algorithm, can be used to find a minimal spanning tree in a weighted graph.

Kruskal's Algorithm

Assume that you start with a table of the weights associated with each edge (just like the Railroad problem in Student Activity Sheet 7).

Step 1: Put all of the weights in a list from smallest to largest.

Step 2: Find the smallest weight in the list and include the associated edge and two vertices, as long as that does not create a cycle.

Step 3: Remove this weight from the list.

Step 4: Repeat Steps 2 and 3 until all vertices are connected.

1. A series of bridges will be constructed to connect a group of seven islands. The highway department wants to make sure that a vehicle can be driven from one island to the others in this new network of bridges. The cost of building a bridge is directly proportional to the length of the bridge. The following table provides the distances in miles between each pair of islands.

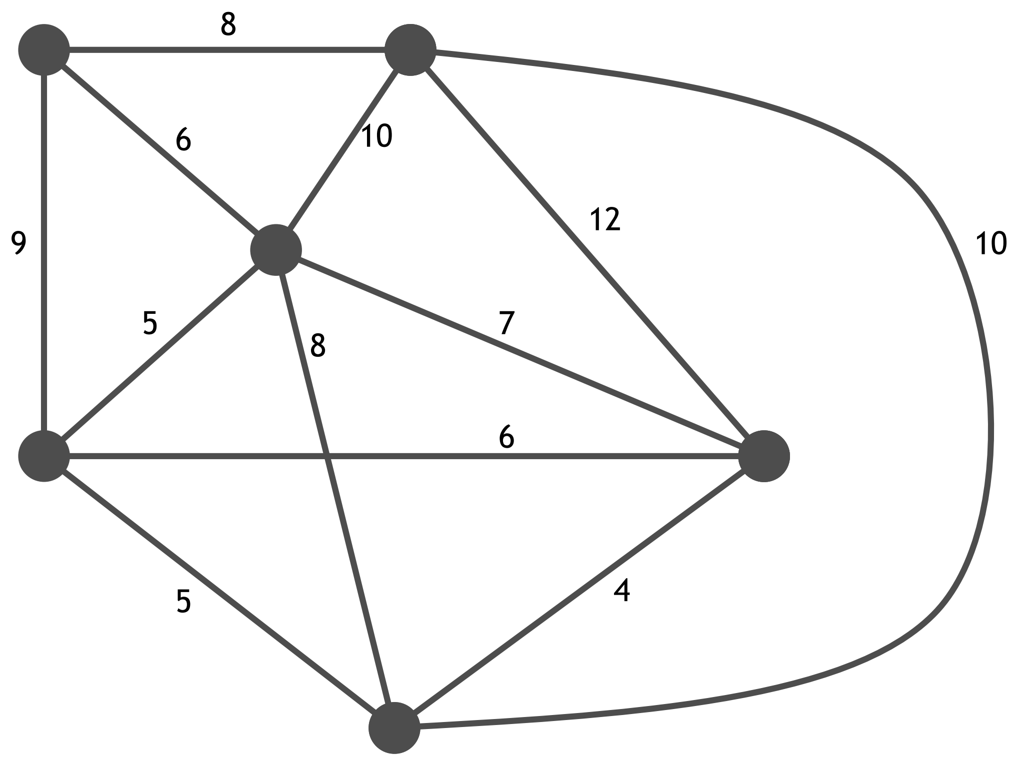
	Island A	Island B	Island C	Island D	Island E	Island F	Island G
Island A		10	8	8	7	10	9
Island B			4	9	13	3	7
Island C				12	11	5	9
Island D					9	10	6
Island E						6	11
Island F							8

Use Kruskal's Algorithm to determine which islands should be connected by bridges. Draw a graph that represents the seven islands with the bridges that will be constructed.

Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 8: Kruskal's Algorithm

2. Use Kruskal's Algorithm to find a minimal spanning tree in the following graph.



3. REFLECTION: Do all graphs have spanning trees? Are spanning trees unique?

4. EXTENSION: Other algorithms exist for arriving at a minimal spanning tree. Conduct research to find one and share it with your class through a brief presentation. Compare and contrast your findings with Kruskal's Algorithm as appropriate.

Student: ______________________________ Class: ____________________ Date:______________

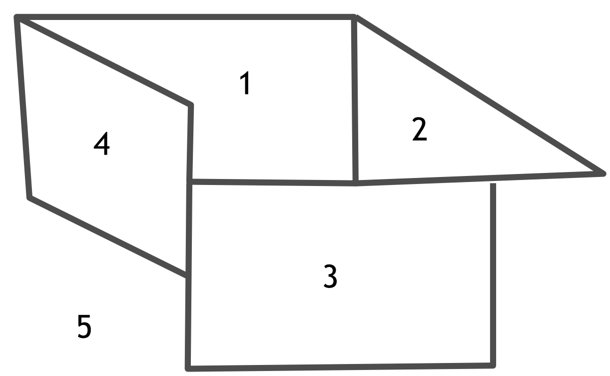
Networks and Graphs: Graph Coloring

VII.C Student Activity Sheet 9: Map Coloring

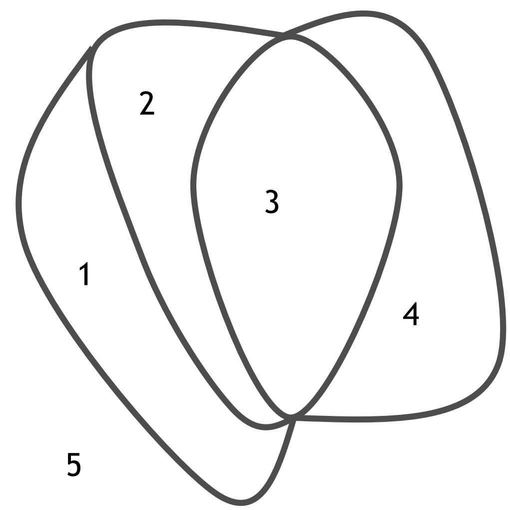
Map Coloring Problem

You are the publisher of a new edition of the world atlas. As you prepare the different maps for printing, you need to make sure that countries adjacent to each other (sharing a common border) are given different colors.

1. For the following two maps, decide how to color each of the five countries (regions) so that no two adjacent countries are colored the same. Treat the outside region as a single country (perhaps it represents an ocean colored blue). Assume that every country is composed of a single contiguous region (for example, you treat Alaska and Hawaii as separate regions when constructing a map of the world).



Map I



Map II

2. How many colors did you use to color each map?

3. **REFLECTION:** Did you use fewer colors than anyone else? If not, describe how you can adjust your map to use fewer colors. If yes, how are you confident that the fewest colors have been used that can be?

4. If you want to color each map using the least number of colors (still keeping adjacent regions separate colors), how many colors are needed for each map?

5. Create a map that **requires** the use of three colors.

6. Create a map with at least four different regions that **could be** colored with two colors.

7. **EXTENSION:** Create a map that needs five colors. What is the largest number of colors required to color any map, that keeps adjacent regions separate? Justify your response.

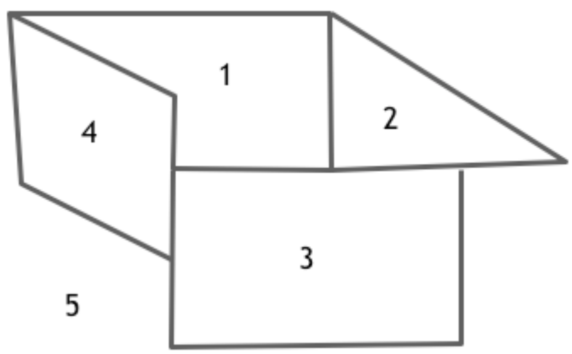
Networks and Graphs: Graph Coloring

VII.C Student Activity Sheet 10: Coloring Maps and Scheduling

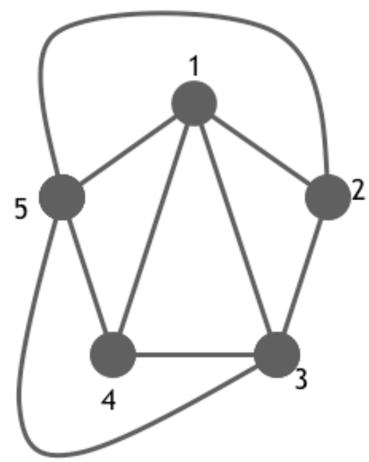
Creating Graphs from Maps

1. Revisit the map coloring exercises from Student Activity Sheet 9 in terms of graphs. For example, Map I can be represented by the following graph. The graph should include a vertex for each country (or region) in your map. If two countries share a border and need to be colored differently, the graph shows an edge between the vertices that represent them.

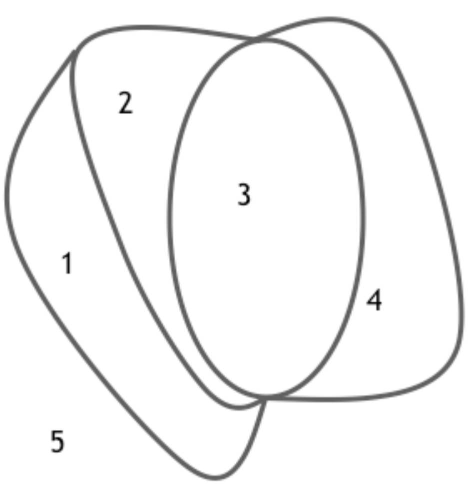
After studying the relationship between Map I and the graph for Map I, create a graph that represents Map II.



Map I



Graph for Map I



Map II

2. Restate the Map Coloring problem from Student Activity Sheet 9 in terms of a Graph Coloring problem.

   You are the publisher of a new edition of the world atlas. As you prepare the different maps for printing, you need to make sure that countries adjacent to each other (sharing a common border) are given different colors.

3. Create a graph that **requires** three colors.

4. Create a graph that **could be** colored with two colors.

5. What types of graphs can always be colored with two colors?

6. **EXTENSION:** Create a graph that needs five colors, and then draw the associated map.

7. **REFLECTION:** When might a graph not correspond to a map?

8. The *chromatic number* of a graph is the minimum number of colors needed to color each vertex in such a way that any two vertices sharing an edge are a different color. Provide examples of graphs that have chromatic numbers of 3 and 4.

9. Give an example of a graph with 20 vertices that has a chromatic number of 2. Does your graph have any cycles? (***Recall:*** A cycle is a path through the graph that starts and ends at the same vertex and does not reuse any edges.)

Scheduling Problem

Mrs. Jacobs, the new principal at Riverdale High School, wants to make a good impression by offering a lot of new exciting classes for her students. The principal plans to use her knowledge of graph theory to determine when each class will be offered.

Since she is trying to make her students happy, Mrs. Jacobs does not want to offer two different classes at the same time if there are students wanting to take both. She decides to construct a graph in the following way: Each class is represented by a vertex and if there is a student interested in two classes, those two vertices are connected by an edge.

10. Suppose there are five classes (A, B, C, D, and E) and only five students wishing to take the following classes:

- Jason wants to take Classes A and E.
- Emory want to take Classes B, C, and E.
- Felicity wants to take Classes A and D.
- Geoff wants to take Classes B and C.
- Hilary wants to take Classes D and E.

Construct the graph for the principal.

11. Find the chromatic number of the graph, and color the graph using the least number of colors.

12. How can the graph coloring solution help the principal with her scheduling problem?

Networks and Graphs: Graph Coloring

VII.C Student Activity Sheet 10: Coloring Maps and Scheduling

13. EXTENSION: Select another situation that might be modeled with colored graphs. Several suggestions are described to stimulate your research. Prepare a short presentation of your findings to share with the class.

The notion of coloring graphs can be used to solve a variety of problems involving various types of conflicts over space or time. Some examples include the following:

- **Conflict over time:** Virtually any type of scheduling problem such as appointments or job duties based on an individual's qualifications.
- **Conflict over space:** ***1)*** Create several terrariums to display a variety of plants and reptiles. Certain reptiles may not get along with others, and certain plants should not be placed in terrariums with certain reptiles. Based on a set of compatibility conditions, you could decide the minimum number of terrariums necessary for the exhibit. ***2)*** Radio stations that are within a certain distance of each other cannot be assigned the same broadcasting frequency. Given several radio stations and the distances between each pair, determine the minimum number of distinct frequencies necessary to allow all stations to operate.
- **Other conflicts:** Put a roomful of people into small working groups. Each individual may have a list of others with whom he/she does not work well, thereby disallowing them to share a group. Given each person's "Cannot Work With" list, how many groups are necessary?
- **Chemistry:** Certain chemicals cannot be stored with other chemicals. For example, to answer the question regarding how many storage facilities are required to house the following chemicals, graph coloring can be helpful.

Chemicals	Cannot be stored with
1	2, 5, 7
2	1, 3, 5
3	2, 4
4	3, 7
5	1, 2, 6, 7
6	5
7	1, 4, 5

Student: ___________________________ Class: ____________________ Date:______________

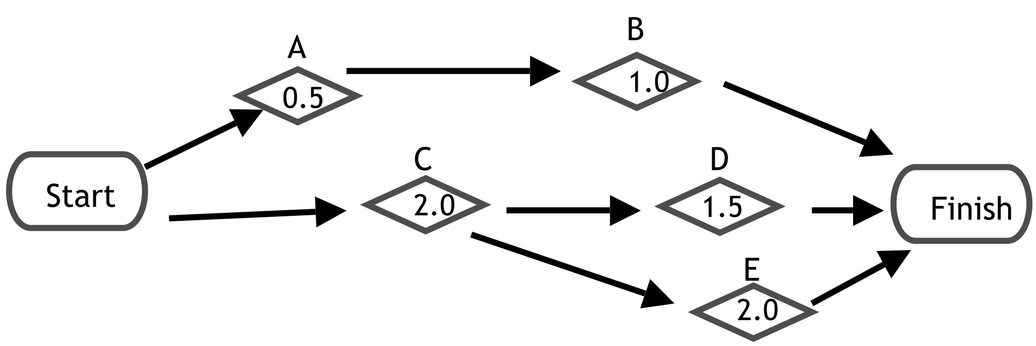
Networks and Graphs: Program Evaluation and Review Technique (PERT) Charts

VII.D Student Activity Sheet 11: Activity Graphs

You are in charge of organizing the senior class party. The following is your estimate of the time required to perform all the necessary activities:

Activity	Time (in hours)
A. Plan music playlist	0.5
B. Download music	1.0
C. Buy groceries and decorations	2.0
D. Bake cake and prepare food	1.5
E. Set up	2.0

Since there are several classmates helping, some of these tasks can be performed at the same time. For instance, people can begin setting up the decorations while the cake is baking. However, you cannot bake the cake or set up until after the shopping has taken place. And you cannot begin downloading music until you know what songs you want to download. The following *activity graph* can be used to help organize this information:



1. What do the numbers in this graph represent?

2. Why is there an arrow going from Activity A to Activity B, but not from Activity A to Activity C?

3. Beginning at Start, there are several paths through the graph (following the arrows) that end at Finish. For each path, calculate the total time required to perform all the activities along the path.

Networks and Graphs: Program Evaluation and Review Technique (PERT) Charts

VII.D Student Activity Sheet 11: Activity Graphs

4. What is the *minimum* amount of time required to perform all five activities?

5. Which path corresponds to this *minimum* time? Which activities are along this path?

6. Which activities could take a little longer to complete *without* affecting the total completion time?

Scheduling classes in college can be very similar to the previous scenario. Over four years, there are certain classes that you must take, and many classes have prerequisites—classes that must be taken first. Suppose you need to take the following classes with the identified prerequisites.

Class	**Prerequisite**
Calculus I	None
Calculus II	Calculus I
Physics I	Calculus I
Physics II	Physics I
Psychology I	None
Speech	None
Argument and Debate	Speech

7. Construct an activity graph for this situation using the following rules:

- Create Start and Finish squares.
- Any activity that can be performed right away is connected to Start.
- Activity A is connected to Activity B by an arrow **only when** Activity A needs to be performed directly before Activity B.
- Any activity that does not precede any other activity can be connected to Finish.

8. Identify the longest path from Start to Finish. How long is this path?

9. If each class is a semester long, how many semesters are needed to take all these classes?

Networks and Graphs: Program Evaluation and Review Technique (PERT) Charts

VII.D Student Activity Sheet 11: Activity Graphs

10. If you want to finish these classes as soon as possible, which classes should you not delay taking?

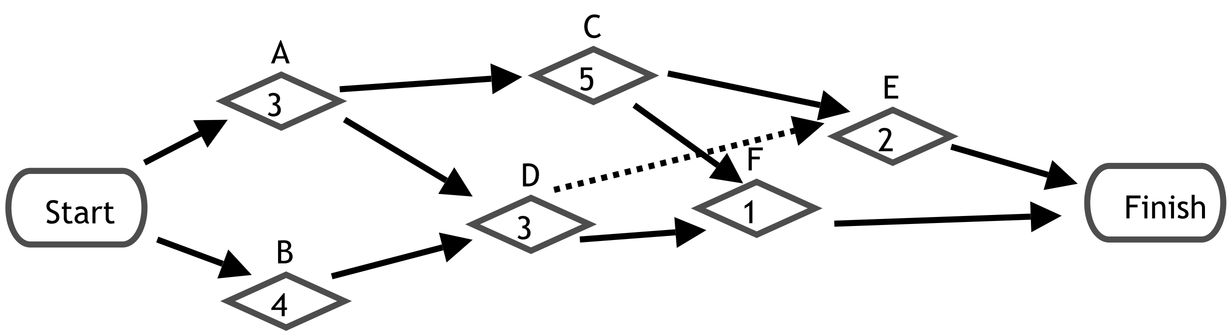
11. How long could you wait to take Psychology I without delaying your overall program of classes?

12. How long could you wait to take Speech without delaying your overall program of classes?

13. Given any activity graph like the previous ones, explain how you would determine the minimum time required to perform all activities.

14. Activities that cannot be delayed without increasing the minimum time for completion are called *critical activities*. Given any activity graph like the previous ones, explain how you would determine which activities are critical activities.

For the following activity graph, the times given for each activity are in hours.



15. Determine the minimum time required to complete all the activities shown in the graph.

16. Which activities are critical activities?

17. How long could Activity F be delayed without affecting the overall completion time?

18. How long could Activity D be delayed without affecting the overall completion time?

Networks and Graphs: Program Evaluation and Review Technique (PERT) Charts

VII.D Student Activity Sheet 11: Activity Graphs

19. What if Activities F and D were both delayed?

20. **REFLECTION:** Can you find a formula that determines how long an individual activity could be delayed without affecting the total completion time for all the activities?

Sometimes the time to complete an activity is given by two numbers: the estimate for a minimum completion time and the estimate for a maximum completion time. How would having two possible completion times affect your analysis?

21. **EXTENSION:** Design a chart to represent the planning and design of a particular event at your school, (for example, Project Graduation, prom, fundraiser, community project). Prepare a short presentation including appropriate visuals to share with the class.

Student: ___________________________________ Class: ________________________ Date:_________________

Networks and Graphs: Program Evaluation and Review Technique (PERT) Charts

VII.D Student Activity Sheet 12: Building a Robot

You are leading a group that is designing and building a robot; the group is divided into several teams. The following table indicates the different activities that go into this complex process, which teams are in charge of which activities, the number of individuals from that team dedicated to that activity, how long the activity will likely take, and which activities must be completed before an activity can be started.

Activity	Time to Complete (in Weeks)	Must First Finish Activity…	Team	No. of Members
Sensory Program (SP)	4	None	Computer programmers	3
Artificial Intelligence (AI)	5	None	Computer programmers	3
Motion (M)	10	SP, AI	Computer programmers	3
Voice System (VS)	3	AI	Computer programmers	3
Eye Design (ED)	5	SP	Engineers	3
Arm and Leg Design (ALD)	3	M	Engineers	3
Body Design (BD)	4	None	Engineers	3
Head Assembly (HA)	5	VS, ED	Technicians	3
Torso Assembly (TA)	2	BD	Technicians	3
Arm and Leg Assembly (ALA)	8	ALD	Technicians	3
Appearance (A)	2	TA, ALA	Technicians	3
Final Assembly (FA)	4	HA, TA, ALA	Technicians	3

Student: ________________________________ Class: ______________________ Date:_______________

Networks and Graphs: Program Evaluation and Review Technique (PERT) Charts

VII.D Student Activity Sheet 12: Building a Robot

1. Using the information in the first three columns of the table, build an activity graph. Include Start and Finish boxes.

2. Assuming the times given for each activity are accurate, what is the minimum time required to design and build the robot?

3. At what point in the timeline does each activity (for the completion of the entire robot) begin and end?

4. Which of the 12 activities are critical activities?

5. **EXTENSION:** Since any delay in the completion time for critical activities results in a longer total completion time, these activities may need extra people assigned to them. Suppose you can reassign team members to an activity according to the following guidelines:

- No one can work on an activity outside of his/her team. For example, a computer programmer must be assigned to Activity 1, 2, 3, or 4 and cannot be assigned to any of the other activities.
- Every activity must have at least one person assigned to it at all times.
- An activity that receives extra help can be completed 1 week earlier for each additional person assigned to it.
- An activity cannot be completed in less than 1 week, even if more people are assigned to it.
- An activity takes 1 week longer to complete for each person removed from the original group.

a. If you could reassign one person, how would you do it? How does the reassignment affect the total completion time?

b. If you could reassign two people, how would you do it? How do the reassignments affect the total completion time?

c. If you could reassign any number of people, how would you do it? How do the reassignments affect the total completion time?

6. **REFLECTION:** Could the total completion time be further improved by allowing people to work on activities outside of their official team designation? Justify your response with appropriate reasoning.