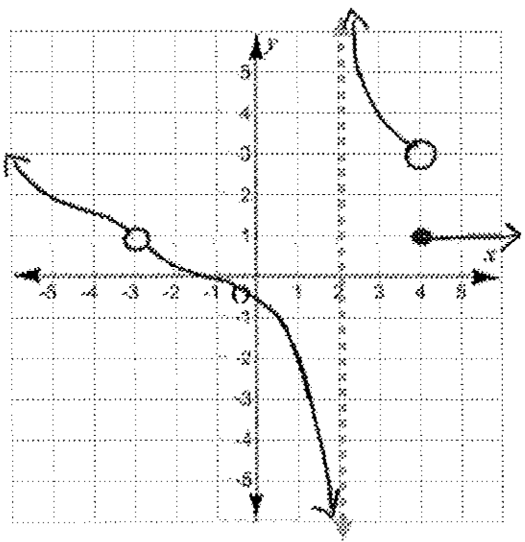
AP Calculus **Name** ______________________________

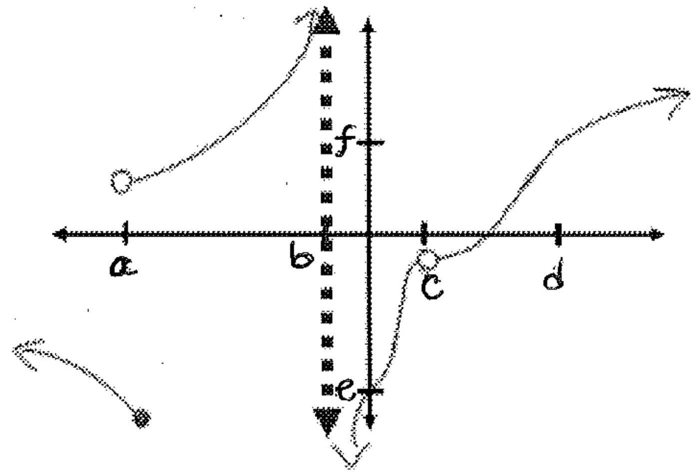
Guided Notes: Introduction to Graphical Limits **Date** ________

Warm-up: Use the graphs provided to evaluate each expression.

Graph of f(x):



Graph of g(x):



1. f(1)   2. f(4)   3. f(-3)   4. g(b)   5. g(d)   6. $(g \circ f)(-1)$

In general, what does it mean to say a graph is continuous? ______________________________

Types of Discontinuities

Removable Discontinuity: ______________________________

Non-removable Discontinuity: ______________________________

Infinite Discontinuity: ____________________ Jump Discontinuity: ____________________

Graphical Examples:

Big picture question: So, if a graph is not continuous and there is not a function value, is there at least a value that is being approached? Why should we care?

Example: Sketch a rough graph of $f(x) = \dfrac{x^2 - 3x + 2}{x - 2}$. Then complete the table that follows.

x	1.75	1.9	1.99	1.999	2	2.0001	2.01	2.1	2.25
f(x)									

Limits

In Math terms: ____

In other words: ____

Notation	Read as	Meaning
$\lim_{x \to k^-} f(x)$		
$\lim_{x \to k^+} f(x)$		
$\lim_{x \to k} f(x)$		

Important Note: ____

Use the calculator table displayed to evaluate each limit or function value.

a. f(1) b. $\lim_{x \to 1^-} f(x)$ c. $\lim_{x \to 1^+} f(x)$ d. $\lim_{x \to 1} f(x)$

1.

x	0.8	0.9	0.99	0.999	1	1.001	1.01	1.1	1.5
f(x)	3.7	3.88	3.92	3.957	4	4.03	4.1	4.54	5

a. ______ b. ______ c. ______ d. _____

2.

x	0.8	0.9	0.99	0.999	1	1.001	1.01	1.1	1.5
f(x)	10.3	78.54	212.9	782	0	-0.03	-0.4	-1.7	-2

a. ______ b. ______ c. ______ d. _____

3.

x	0.8	0.9	0.99	0.999	1	1.001	1.01	1.1	1.5
f(x)	4.835	4.91	4.952	4.993	error	4.993	4.969	4.94	4.72

a. ______ b. ______ c. ______ d. _____

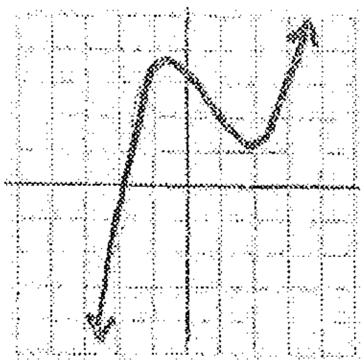
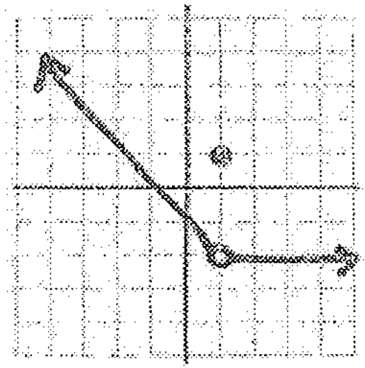
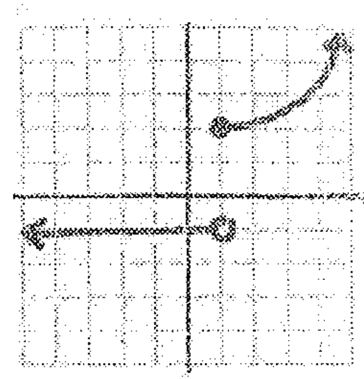
4.

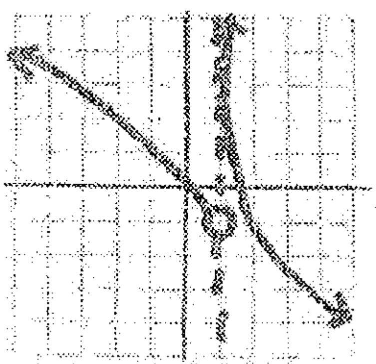
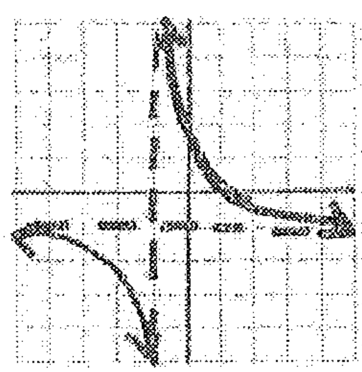
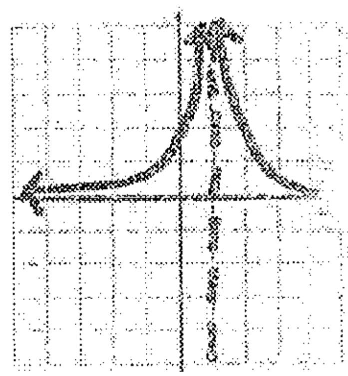
x	0.8	0.9	0.99	0.999	1	1.001	1.01	1.1	1.5
f(x)	-1.71	-0.54	-0.12	-0.041	error	-2.999	-2.991	-2.95	-2.9

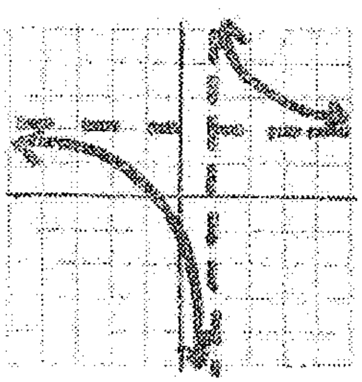
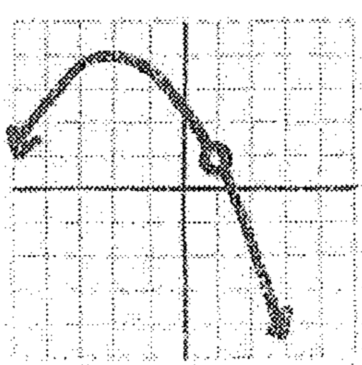
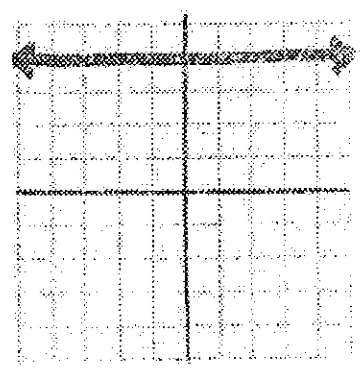
a. ______ b. ______ c. ______ d. _____

For each graph shown, determine:

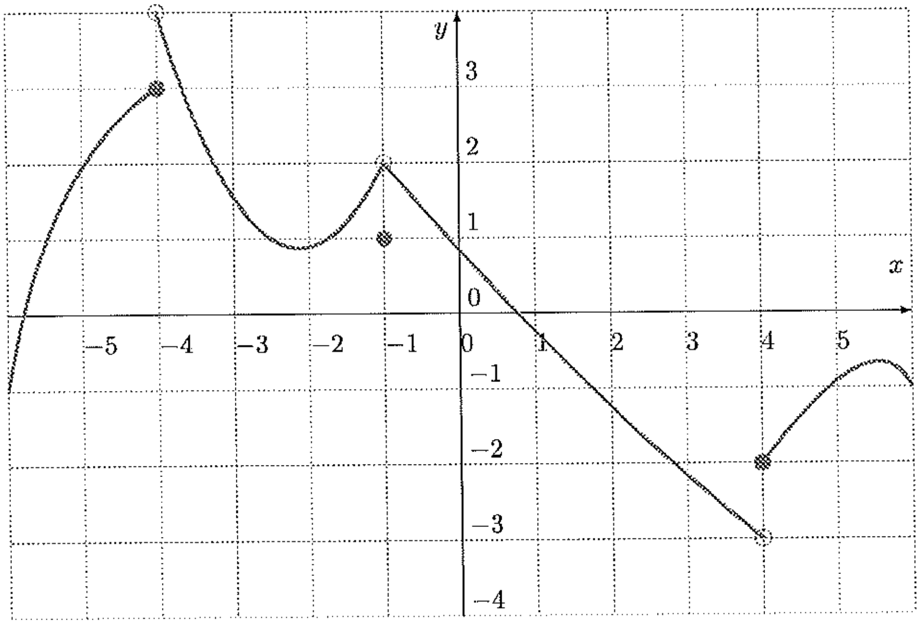
a. f(1) b. $\lim_{x \to 1^-} f(x)$ c. $\lim_{x \to 1^+} f(x)$ d. $\lim_{x \to 1} f(x)$

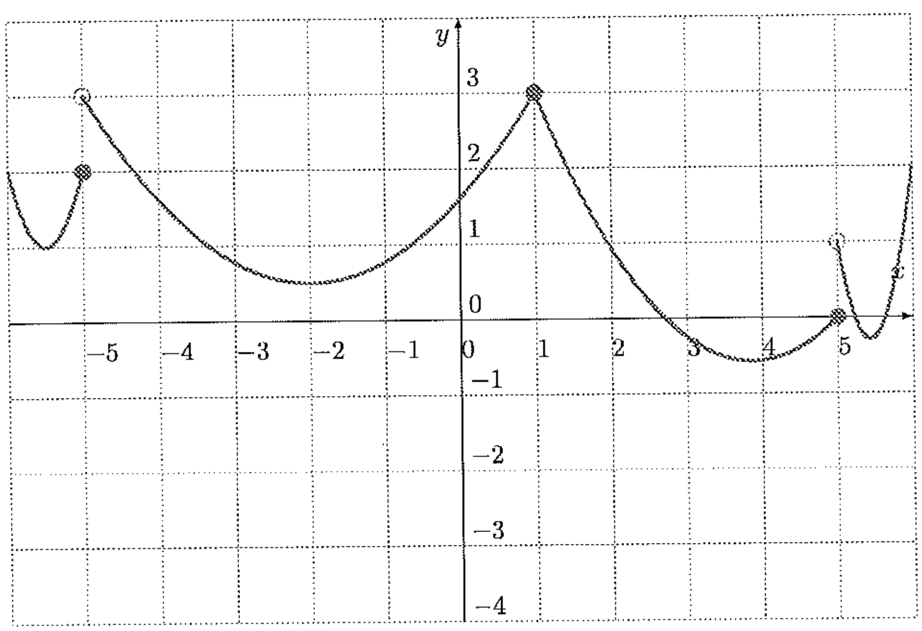
1. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x\to -1^-} f(x)$ b) $\lim_{x\to -1^+} f(x)$ c) $\lim_{x\to -1} f(x)$ d) $\lim_{x\to -4} f(x)$ e) $\lim_{x\to 4} f(x)$

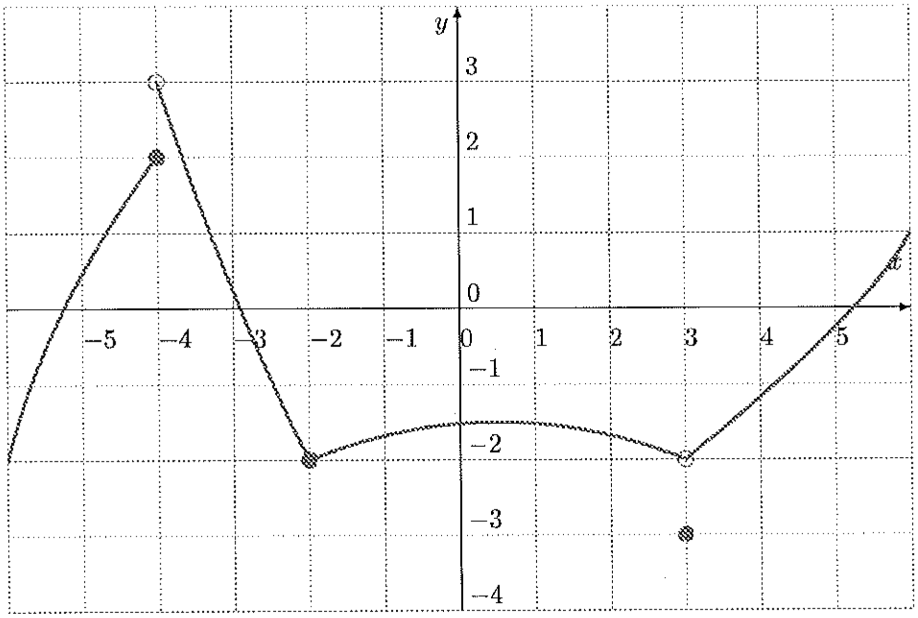
2. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x\to 1^-} f(x)$ b) $\lim_{x\to 1^+} f(x)$ c) $\lim_{x\to 1} f(x)$ d) $\lim_{x\to -5} f(x)$ e) $\lim_{x\to 5} f(x)$

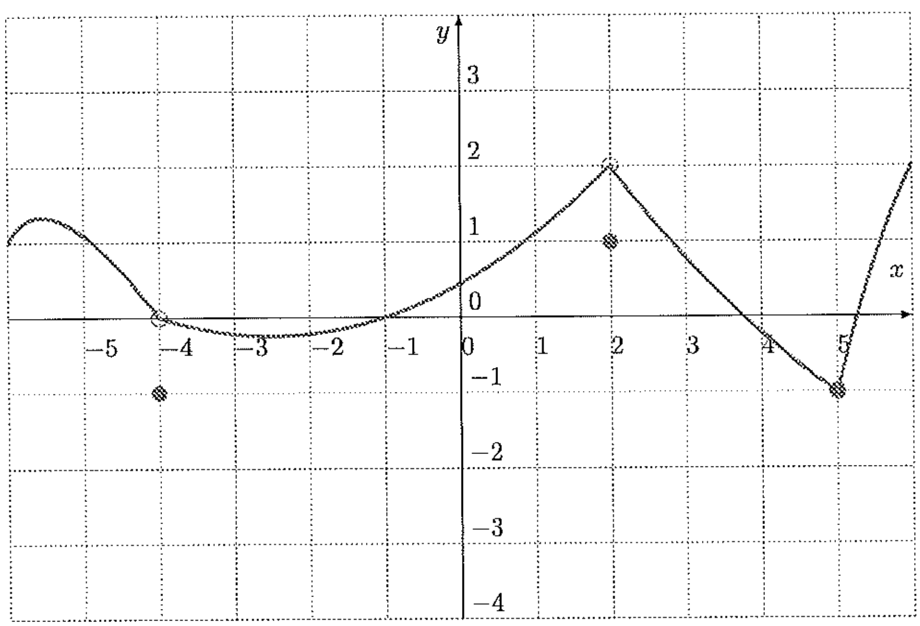
3. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x\to -2^-} f(x)$ *b)* $\lim_{x\to -2^+} f(x)$ *c)* $\lim_{x\to -2} f(x)$ *d)* $\lim_{x\to -4} f(x)$ *e)* $\lim_{x\to 3} f(x)$

4. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x\to 2^-} f(x)$ *b)* $\lim_{x\to 2^+} f(x)$ *c)* $\lim_{x\to 2} f(x)$ *d)* $\lim_{x\to -4} f(x)$ *e)* $\lim_{x\to 5} f(x)$

A

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Refer to the graph below in order to answer the following questions. If a limit doesn't exist explain why.

1. $\lim_{x \to \infty} g(x) =$

2. $\lim_{x \to \infty} g(x) =$

3. $\lim_{x \to a^+} g(x) =$

4. $\lim_{x \to a^-} g(x) =$

5. $\lim_{x \to a} g(x) =$

6. $\lim_{x \to 0} g(x) =$

7. $\lim_{x \to b^+} g(x) =$

8. $\lim_{x \to b^-} g(x) =$

9. $\lim_{x \to b} g(x) =$

10. $\lim_{x \to c} g(x) =$

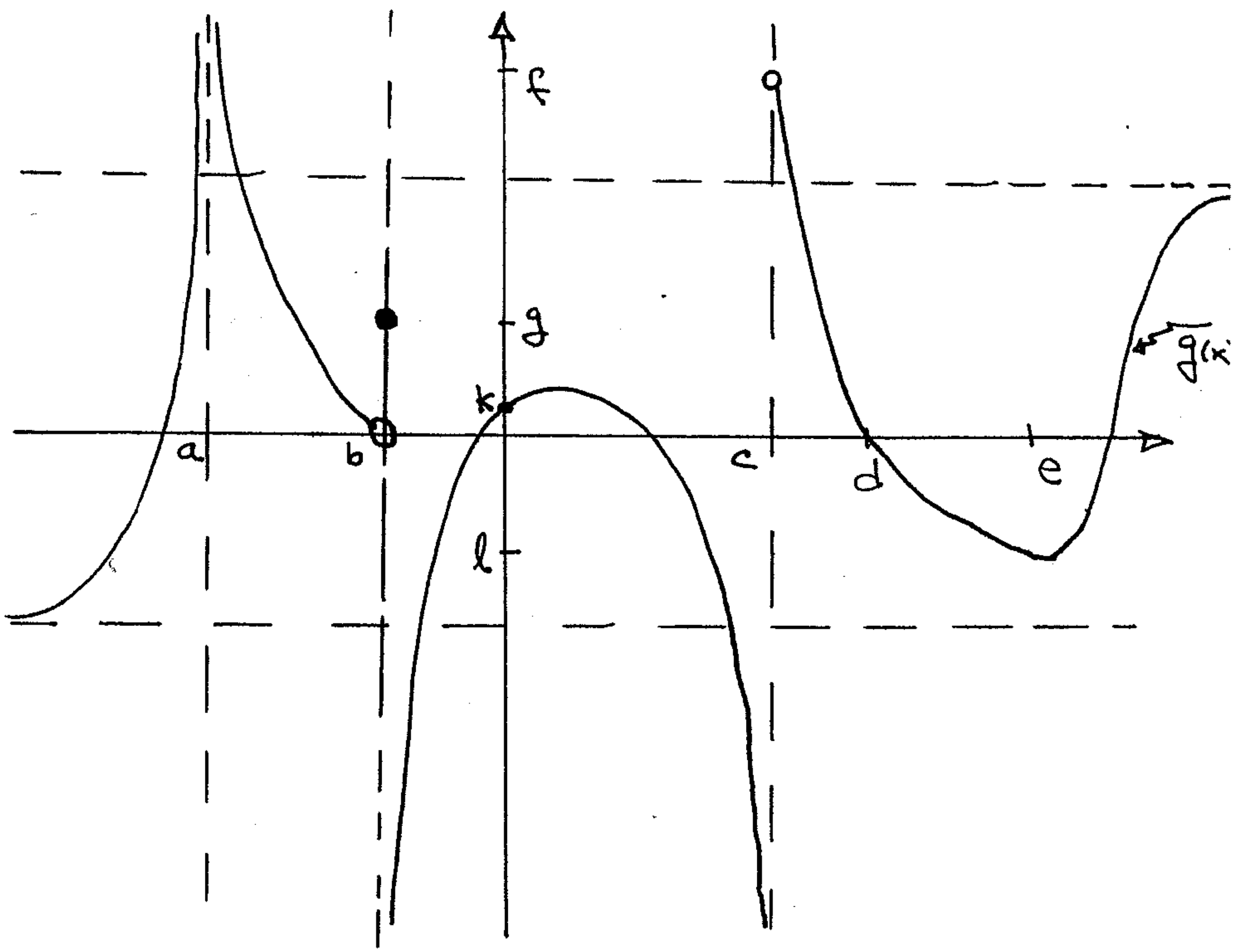
11. $\lim_{x \to d} g(x) =$

12. $\lim_{x \to e} g(x) =$

13. g(e) =

14. g(0) =

14. g(b) =



B

Calculus Term I 1998 Mr. Stadler

Refer to the graph below in order to answer the following questions. If a limit doesn't exist explain why.

1. $\lim_{x \to \infty} g(x) =$

2. $\lim_{x \to -\infty} g(x) =$

3. $\lim_{x \to a^+} g(x) =$

4. $\lim_{x \to a^-} g(x) =$

5. $\lim_{x \to a} g(x) =$

6. $\lim_{x \to 0} g(x) =$

7. $\lim_{x \to b^+} g(x) =$

8. $\lim_{x \to b^-} g(x) =$

9. $\lim_{x \to b} g(x) =$

10. $\lim_{x \to c} g(x) =$

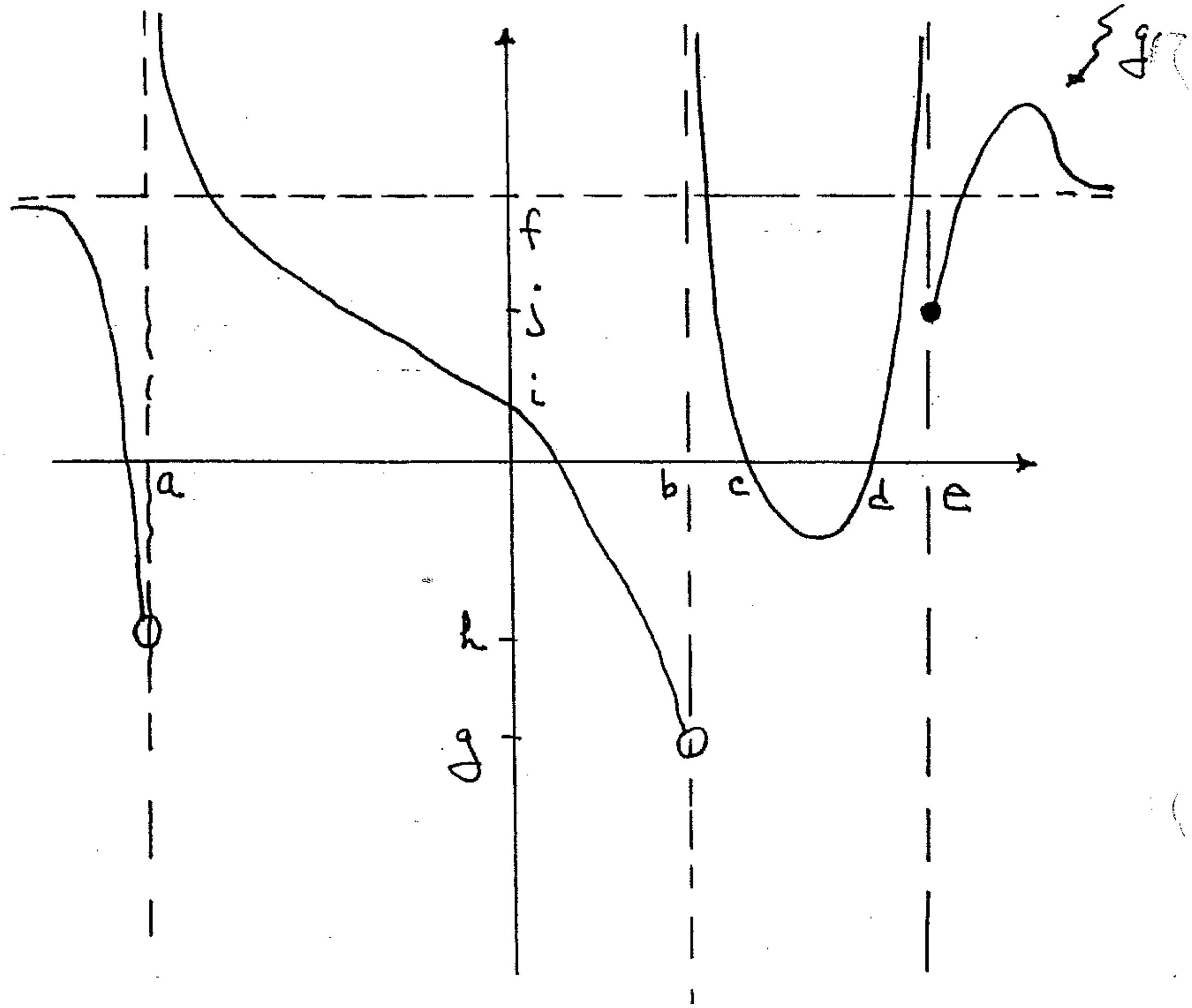
11. $\lim_{x \to d} g(x) =$

12. $\lim_{x \to e} g(x) =$

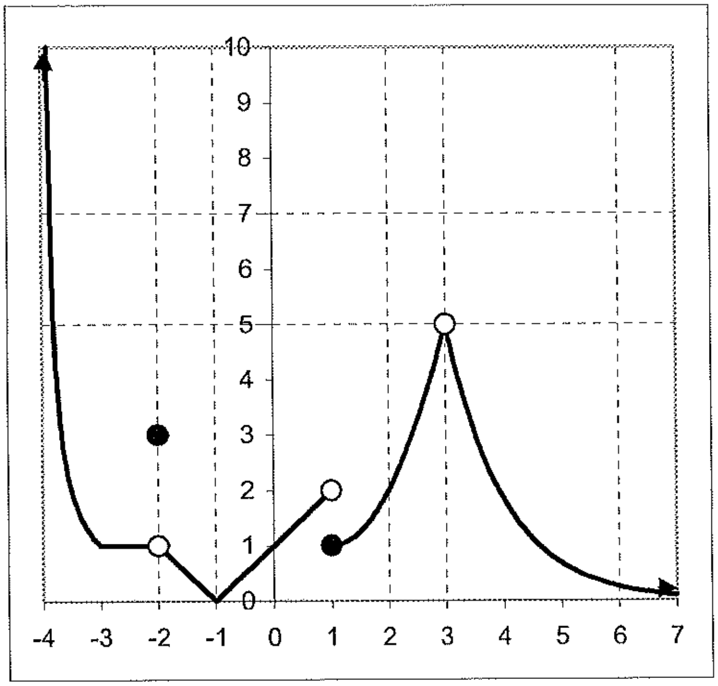
13. g(e) =

14. g(0) =

14. g(b) =



MATH 1205: Limits In-Class Worksheet



Using the above graph, find each of the following (You should assume that y=0 is a horizontal asymptote and x = -4 is a vertical asymptote):

1) $f(-2) =$ ______ 2) $\lim_{x \to -2^+} f(x) =$ ______ 3) $\lim_{x \to -2} f(x) =$ ______

4) $\lim_{x \to -1^+} f(x) =$ ______ 5) $\lim_{x \to -1^-} f(x) =$ ______ 6) $\lim_{x \to -1} f(x) =$ ______

7) $\lim_{x \to 1^+} f(x) =$ ______ 8) $\lim_{x \to 1^-} f(x) =$ ______ 9) $\lim_{x \to 1} f(x) =$ ______

10) $f(3) =$ ______ 11) $\lim_{x \to 3^+} f(x) =$ ______ 12) $\lim_{x \to 3^-} f(x) =$ ______

13) $\lim_{x \to 3} f(x) =$ ______ 14) $\lim_{x \to -4^+} f(x) =$ ______ 15) $\lim_{x \to \infty} f(x) =$ ______

16) $f(1) =$ ______ 17) $\lim_{x \to -3} f(x) =$ ______ 18) $f(-4) =$ ______

AP Calculus
Guided Notes- Evaluating Limits

Name ______________________________
Date ____________________

WARM-UP
Given $f(x) = 2x^2 + 3x - 5$, find each of the following.

1. $f(0)$
2. $f(1)$
3. $f(-4)$
4. $f(x + \Delta x)$

Use your graphing calculator to graph $f(x) = 2x^2 + 3x - 5$. Based on the graph, answer the following questions.

5. $f(0)$
6. $\lim_{x\to 0} f(x)$
7. $\lim_{x\to 1} f(x)$
8. $\lim_{x\to -4} f(x)$

When does a limit not exist?
1. ______________________________
2. ______________________________
3. ______________________________

Evaluating Limits

Strategy 1: ______________________________

Ex. 1 $\lim_{x\to 2}\left(4x^2 - 3x - 7\right)$

Ex. 2 $\lim_{x\to 4}\frac{x^2 - 1}{x^2 - 3x + 2}$

Some Properties of Limits

1. ____________________
2. ____________________
3. ____________________

Ex. 3 $\lim_{x\to 2} 3$

Ex. 4 $\lim_{x\to -4} x$

Ex. 5 $\lim_{x\to 2} x^2$

Strategy 2; ______________________________

Ex. 6 $\lim_{x\to 1}\frac{x^2 - 1}{x^2 - 3x + 2}$

Ex. 7 $\lim_{x\to 0}\frac{\sqrt{2+x} - \sqrt{2}}{x}$

Ex. 8 $\lim_{\Delta x\to 0}\frac{2(x + \Delta x) - 2x}{\Delta x}$

Ex. 9 $\lim_{x\to 0}\frac{\frac{1}{2+x} - \frac{1}{2}}{x}$

More Properties of Limits

Let lim f(x) = L and lim g(x) = K

1. Scalar Multiple: ________________________________

2. Sum or Difference: ______________________________

3. Product: ____________________________________

4. Quotient: ___________________________________

5. Power: _____________________________________

Ex. 10 Given that $\lim_{x\to k} f(x) = 9$ and $\lim_{x\to k} g(x) = -4$, evaluate each of the following limits.

a. $\lim_{x\to k}[3f(x) - g(x)]$

b. $\lim_{x\to k}\sqrt{f(x)}$

c. $\lim_{x\to k}\frac{g^2(x)}{f(x)}$

ONE FOR THE ROAD:

$$\lim_{x\to 2}\begin{cases} 3x-7, x<2 \\ x^2-5, x\geq 2 \end{cases}$$

Kuta Software - Infinite Calculus

Name___________________________________

Evaluating Limits

Date________________ Period____

Evaluate each limit.

1) $\lim_{x \to -1} 5$

2) $\lim_{x \to -\frac{5}{2}} (-x + 2)$

3) $\lim_{x \to 2} (x^3 - x^2 - 4)$

4) $\lim_{x \to 1} \left(-\frac{x^2}{2} + 2x + 4\right)$

5) $\lim_{x \to 3} -\sqrt{x + 3}$

6) $\lim_{x \to \frac{3}{2}} -\sqrt{2x + 4}$

7) $\lim_{x \to 1} -\frac{x - 4}{x^2 - 6x + 8}$

8) $\lim_{x \to \frac{3}{2}} \frac{-x - 3}{x^2 + x + 1}$

9) $\lim_{x \to \pi} \sin(x)$

10) $\lim_{x \to \frac{3\pi}{4}} 2\cos(x)$

Critical thinking questions:

11) Give an example of a limit that evaluates to 4.

12) Give an example of a limit of a quadratic function where the limit evaluates to 9.

For each of the following problems, find the requested limit.

1) $\lim_{x \to 2} 7 =$

2) $\lim_{x \to 5} \sqrt{x-2} =$

3) $\lim_{x \to -5} \frac{x^2-25}{x+5} =$

4) $\lim_{x \to -3} \frac{x}{x+3} =$

5) $\lim_{x \to \infty} \frac{3x^4-x^3+5}{10-2x^4} =$

6) $\lim_{x \to \infty} \frac{3x^3-x+1}{5x^3-7x^4} =$

7) $\lim_{x \to 2} \frac{x^2-x-6}{x+2} =$

8) $\lim_{x \to \infty} \frac{x^2-x-6}{x+2}$

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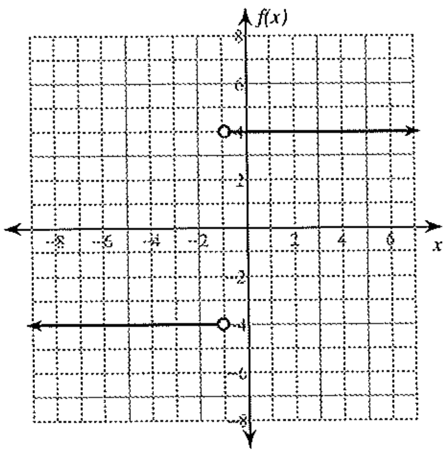
Name________________________________

Evaluating Limits

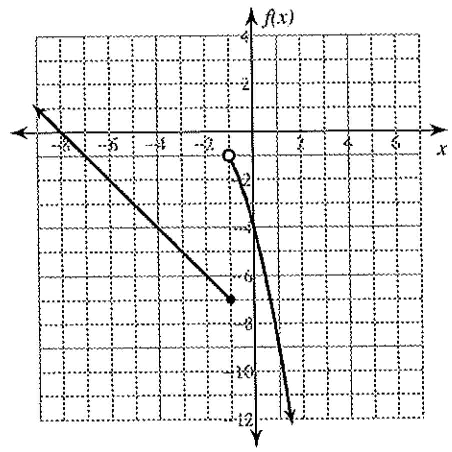
Date________________ Period____

Evaluate each limit.

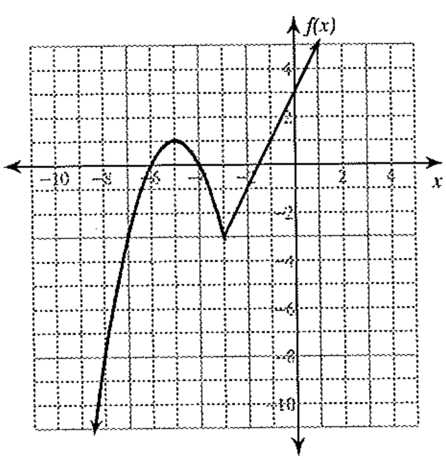
1) $\lim_{x \to -1^+} \dfrac{4x+4}{|x+1|}$



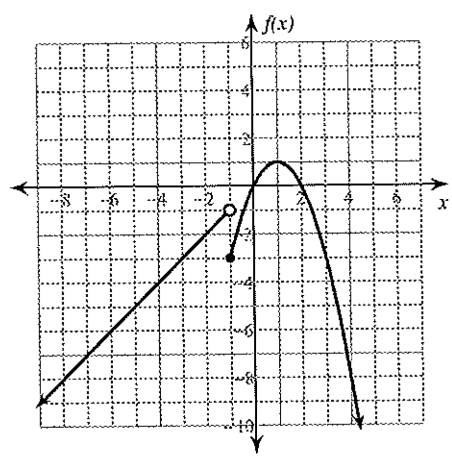
2) $\lim_{x \to -1^-} f(x),\ f(x) = \begin{cases} -x-8, & x \le -1 \\ -x^2-4x-4, & x > -1 \end{cases}$



3) $\lim_{x \to -3} f(x),\ f(x) = \begin{cases} -x^2-10x-24, & x \le -3 \\ 2x+3, & x > -3 \end{cases}$

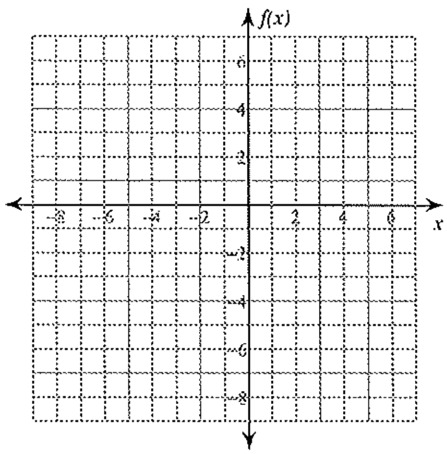


4) $\lim_{x \to -1} f(x),\ f(x) = \begin{cases} x, & x < -1 \\ -x^2+2x, & x \ge -1 \end{cases}$

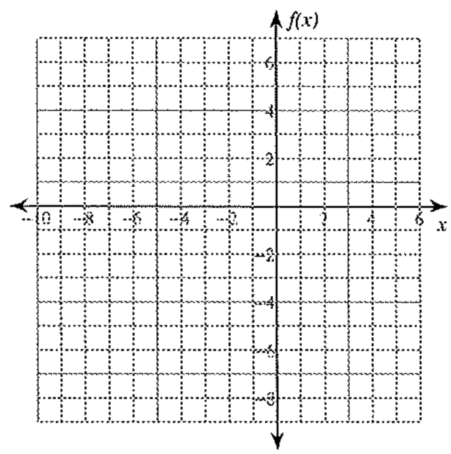


Evaluate each limit. You may use the provided graph to sketch the function.

5) $\lim_{x \to -1^-} f(x),\ f(x) = \begin{cases} -x - 3, & x \le -1 \\ x + 1, & x > -1 \end{cases}$



6) $\lim_{x \to -2} f(x),\ f(x) = \begin{cases} -x^2 - 4x - 5, & x \le -2 \\ -1, & x > -2 \end{cases}$



Evaluate each limit.

7) $\lim_{x \to 0^+} f(x),\ f(x) = \begin{cases} 1, & x \le 0 \\ -x^2 + 4x - 3, & x > 0 \end{cases}$

8) $\lim_{x \to 0^-} \dfrac{|x|}{x}$

9) $\lim_{x \to 0^+} \lfloor -2x + 1 \rfloor$

10) $\lim_{x \to 1} f(x),\ f(x) = \begin{cases} \dfrac{x}{2} + \dfrac{9}{2}, & x < 1 \\ x^2 - 6x + 10, & x \ge 1 \end{cases}$

11) $\lim_{x \to -1} \dfrac{3|x + 1|}{x + 1}$

12) $\lim_{x \to -2} f(x),\ f(x) = \begin{cases} x^2, & x \le -2 \\ -\dfrac{x}{2} + 3, & x > -2 \end{cases}$

Critical thinking questions:

13) Give an example of a two-sided limit of a piecewise function where the limit does not exist.

14) Given an example of a two-sided limit of a function with an absolute value where the limit does not exist.

Calculus Name________________________________ ID: 1

Limits Practice

Date________________ Period____

Evaluate each limit.

1) $\lim_{x \to 1} -\frac{x^2 - 4x + 3}{x - 1}$

2) $\lim_{x \to -1} -\frac{x^2 - 1}{x + 1}$

3) $\lim_{x \to 3} \frac{x - 3}{x^2 - 4x + 3}$

4) $\lim_{x \to 1} -\frac{x^2 + x - 2}{x - 1}$

5) $\lim_{x \to 3} f(x),\ f(x) = \begin{cases} x^2 - 6x + 8, & x \neq 3 \\ -4, & x = 3 \end{cases}$

6) $\lim_{x \to -2} f(x),\ f(x) = \begin{cases} x - 2, & x \neq -2 \\ 1, & x = -2 \end{cases}$

7) $\lim_{x \to 2} f(x),\ f(x) = \begin{cases} x^2 - 6x + 7, & x \neq 2 \\ -4, & x = 2 \end{cases}$

8) $\lim_{x \to -2} f(x),\ f(x) = \begin{cases} x + 2, & x \neq -2 \\ -2, & x = -2 \end{cases}$

9) $\lim_{x \to -2} \frac{x}{\frac{1}{2 + x} - \frac{1}{2}}$

10) $\lim_{x \to 0} \frac{\frac{1}{3 + x} - \frac{1}{3}}{x}$

11) $\lim_{x \to 0} \frac{\sin^2(x)}{4x^2}$

12) $\lim_{x \to 0} \frac{\sin(3x)}{\sin(x)}$

13) $\lim_{x \to -2^-} f(x),\ f(x) = \begin{cases} x^2 + 4x + 5, & x < -2 \\ -\frac{x}{2}, & x \geq -2 \end{cases}$

14) $\lim_{x \to 2^-} f(x),\ f(x) = \begin{cases} \frac{x}{2}, & x \leq 2 \\ x - 1, & x > 2 \end{cases}$

15) $\lim_{x \to 0^+} f(x),\ f(x) = \begin{cases} -2x - 1, & x < 0 \\ \frac{x}{2} - 1, & x \geq 0 \end{cases}$

16) $\lim_{x \to -2^-} f(x),\ f(x) = \begin{cases} -\frac{x}{2} - \frac{5}{2}, & x < -2 \\ 2x, & x \geq -2 \end{cases}$

17) $\lim_{x \to 1^+} \frac{2x}{x - 1}$

18) $\lim_{x \to 3^+} -\frac{3}{x^2 - 9}$

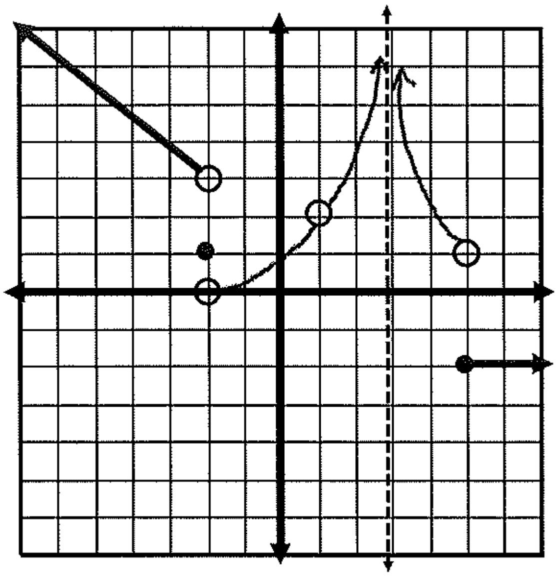
19) $\lim_{x \to 3^+} \frac{x - 3}{x^2 - 2x - 3}$

20) $\lim_{x \to 3^+} -\frac{x - 2}{x^2 - 5x + 6}$

AP Calculus
Limits Quiz

Name: ______________________
Date: __________ Period: ________

Find the limit given the graph below.



1.) $\lim_{x\to 3} f(x)$ 1. __________

2.) $\lim_{x\to -2} f(x)$ 2. __________

3.) $\lim_{x\to 1} f(x)$ 3. __________

4.) $\lim_{x\to -2^-} f(x)$ 4. __________

5.) $\lim_{x\to 5^+} f(x)$ 5. __________

6.) $\lim_{x\to \infty} f(x)$ 6. __________

Find the limit analytically.

7.) $\lim_{x\to 2} x^2+4x-3$

8.) $\lim_{x\to 1} \frac{x^2-1}{x+1}$

7. ______________

8. ______________

9.) $\lim_{x\to \pi} \tan x$

10.) $\lim_{x\to -5} \sqrt[3]{x+4}$

9. ______________

10. ______________

11.) $\lim_{x\to 0} \frac{\sin\frac{x}{5}}{x}$

12.) $\lim_{x\to 0} \frac{\frac{1}{x+2}-\frac{1}{2}}{x}$

11. ______________

12. ______________

13.) $\lim_{x \to 3} \frac{\sqrt{x+1}-2}{x-3}$

14.) $\lim_{x \to 0} \frac{2-2\cos x}{x}$

13. ________________

14. ________________

15.) $\lim_{x \to \infty} \frac{10x^3-7x+3}{5x^3+4x^2}$

16.) $\lim_{x \to 0} \frac{\tan 7x}{\sin 3x}$

15. ________________

16. ________________

17. Determine whether the limit exists. If so, state the limit. If not, tell why not. Show all work leading to your answer.

$$\lim_{x \to 2} \begin{cases} 4x^2-3x+1, x \le 2 \\ 9-3x, x > 2 \end{cases}$$

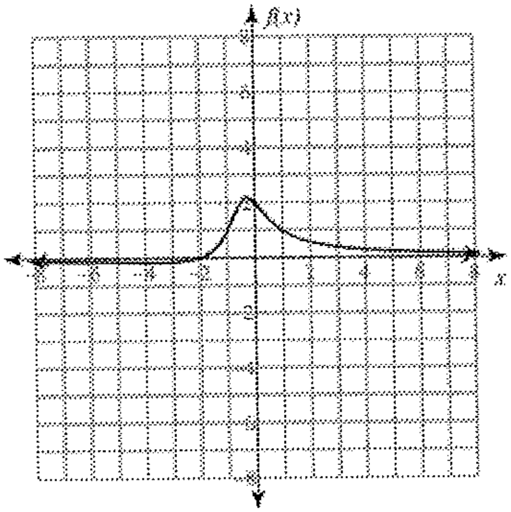
BONUS: $\lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{\tan x - 1}$

AP Calculus Name ____________________

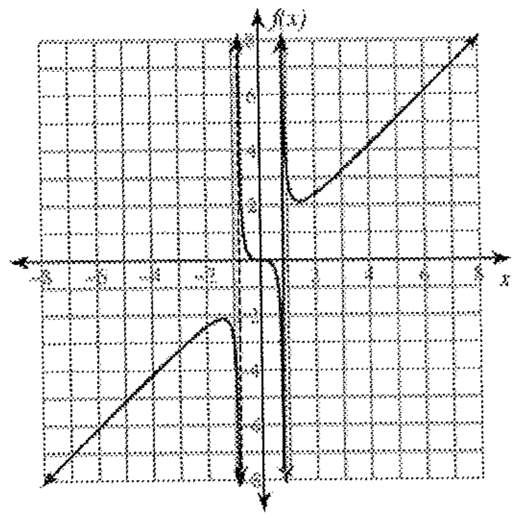
Limits at Infinity, cont. Date ______________

Part 1: Evaluate each limit. The graph is provided on the first two to help you.

1) $\lim_{x\to\infty} \frac{x+2}{x^2+x+1}$



2) $\lim_{x\to\infty} \frac{3x^3}{3x^2-2}$



3. $\lim_{x\to\infty} \frac{2x}{x-4}$

4. $\lim_{x\to-\infty} \frac{2x}{x-4}$

5. $\lim_{x\to\infty} \frac{2x}{x^2-4}$

6. $\lim_{x\to-\infty} \frac{2x}{x^2-4}$

7. $\lim_{x\to\infty} \frac{2x^2}{x-4}$

8. $\lim_{x\to-\infty} \frac{2x^3}{x-4}$

9. $\lim_{x\to\infty} \frac{3x-5-4x^3}{3x^3-4}$

10. $\lim_{x\to-\infty} \frac{2x^4-3x^2-1}{5-x}$

11. $\lim_{x\to\infty} \frac{2x^4-3x^2-1}{5-x}$

12. $\lim_{x\to\infty} \frac{2x^4-3x^2-1}{5-x^2}$

13. $\lim_{x\to-\infty} \frac{2x^2-7x}{x^3-4x}$

14. $\lim_{x\to-\infty} \frac{2x^4-3x^2-1}{5-7x^4}$

15. $\lim_{x\to\infty} -\frac{3x^2}{4x+4}$

16. $\lim_{x\to-\infty} (x^3-4x^2+5)$

17. $\lim_{x\to-\infty} (x^2-4x+5)$

18. $\lim_{x\to\infty} \frac{\sqrt{2x^4-3x^2-1}}{5+x^2}$

19. $\lim_{x\to\infty} \frac{\sqrt{2x^6-3x^2-1}}{5+x^3}$

20. $\lim_{x\to-\infty} \frac{\sqrt{2x^6-3x^2-1}}{5+x^3}$

Part 2: Sketch a possible graph of a function that meets all the given criteria.

1. a. $\lim_{x\to 2^-} f(x) = 2$ b. $\lim_{x\to 2^+} f(x) = 2$ c. f(2) = -3

2. a. $\lim_{x\to 2^-} f(x) = 2$ b. $\lim_{x\to 2^+} f(x) = -5$ c. f(2) is undefined

3. a. $\lim_{x\to 2} f(x) = 2$ b. $\lim_{x\to \infty} f(x) = 4$ c. $\lim_{x\to -\infty} f(x) = -5$

4. a. $\lim_{x\to \infty} f(x) = 4$ b. $\lim_{x\to -\infty} f(x) = -2$ c. $\lim_{x\to 3^-} f(x) = \infty$

d. $\lim_{x\to 3^+} f(x) = -\infty$ e. f(5) = 0 f. $\lim_{x\to -3} f(x) = 0$

AP Calculus

Practice Problem Set 1- Limits

Name ______________________

Date ____________ Period ___

1. $\lim_{x\to 8}\left(x^2-5x-11\right)=$

2. $\lim_{x\to 5}\left(\frac{x+3}{x^2-15}\right)=$

3. $\lim_{x\to 0}\pi^2=$

4. $\lim_{x\to 3}\left(\frac{x^2-2x-3}{x-3}\right)=$

5. $\lim_{x\to\infty}\left(\frac{10x^2+25x+1}{x^4-8}\right)=$

6. $\lim_{x\to\infty}\left(\frac{x^4-8}{10x^2+25x+1}\right)=$

7. $\lim_{x\to\infty}\left(\frac{x^4-8}{10x^4+25x+1}\right)=$

8. $\lim_{x\to\infty}\left(\frac{\sqrt{5x^4+2x}}{x^2}\right)=$

9. $\lim_{x\to 6^+}\left(\frac{x+2}{x^2-4x-12}\right)=$

10. $\lim_{x\to 6^-}\left(\frac{x+2}{x^2-4x-12}\right)=$

11. $\lim_{x\to 6}\left(\frac{x+2}{x^2-4x-12}\right)=$

12. $\lim_{x\to 0^+}\left(\frac{x}{|x|}\right)=$

13. $\lim_{x\to 0^-}\left(\frac{x}{|x|}\right)=$

14. $\lim_{x\to 7^+}\left(\frac{x}{x^2-49}\right)=$

15. $\lim_{x\to 7}\left(\frac{x}{x^2-49}\right)=$

16. $\lim_{x\to 7}\frac{x}{(x-7)^2}=$

17. Let $f(x)=\begin{cases} x^2-5, x\le 3 \\ x+2, x>3 \end{cases}$

Find: (a) $\lim_{x\to 3^-} f(x)$; (b) $\lim_{x\to 3^+} f(x)$; and (c) $\lim_{x\to 3} f(x)$

18. Let $f(x)=\begin{cases} x^2-5, x\le 3 \\ x+1, x>3 \end{cases}$

Find: (a) $\lim_{x\to 3^-} f(x)$; (b) $\lim_{x\to 3^+} f(x)$; and (c) $\lim_{x\to 3} f(x)$

19. Find $\lim_{x\to \frac{\pi}{4}} 3\cos x$.

20. Find $\lim_{x\to 0} 3\frac{x}{\cos x}$.

21. Find $\lim_{x\to 0} 3\frac{x}{\sin x}$.

22. Find $\lim_{x\to 0} \frac{\sin 3x}{\sin 8x}$.

23. Find $\lim_{x\to 0} \frac{\tan 7x}{\sin 5x}$.

24. Find $\lim_{x\to \infty} \sin x$.

25. Find $\lim_{x\to \infty} \sin\frac{1}{x}$.

26. Find $\lim_{x\to 0} \frac{x^2 \sin x}{1-\cos^2 x}$.

27. Find $\lim_{x\to 0} \frac{\sin^2 7x}{\sin^2 11x}$.

28. Find $\lim_{h\to 0} \frac{(3+h)^2-9}{h}$.

29. Find $\lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h}$.

30. Find $\lim_{h\to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h}$.

AP Calculus
Guided Notes- Continuity

Name ______________________________
Date ____________ Period _____

Warm-up: Sketch a possible graph of the function that meets all the given conditions.

1. f(2) = 3, $\lim_{x \to 2} f(x) = -1$

2. f(1) = 5, $\lim_{x \to 1^-} f(x) = 0$, $\lim_{x \to 1^+} f(x) = 5$

3. f(-3) is undefined, $\lim_{x \to -3} f(x)$ exists

Continuity at a Point (By Calculus)

1. ____

2. ____

3. ____

Continuity on an Interval

A function is continuous on an interval if it is continuous at every point in that interval. A function can be:

1. Continuous on ________________.

2. Continuous on ________________.

3. Continuous __________.

Types of Discontinuities

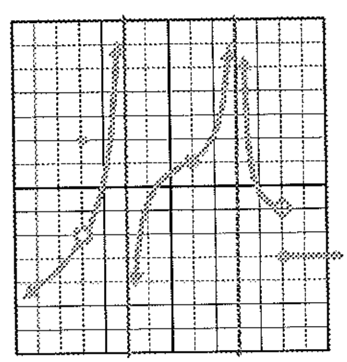
1. ________________ 2. ________________

Use the graph to answer each question below.

a. Find all values at which x is not continuous.

b. Classify each discontinuity by type.

c. Find the intervals for which the function is continuous.



Determine the x-values at which f is not continuous, and classify each by type.

Ex. 1 $f(x) = 4x^2 + 2x - 3$

Ex. 2 $f(x) = \dfrac{x}{x+5}$

Ex. 3 $f(x) = \dfrac{x}{x^2 + 5x}$

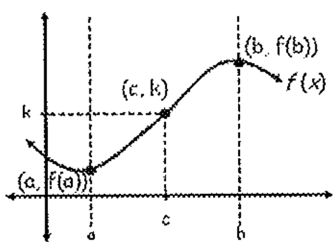
Ex. 4 $f(x) = \tan x$

Find the constant q such that the function is continuous on the entire real number line.

$$f(x) = \begin{cases} x^3, x \le 2 \\ qx^2, x > 2 \end{cases}$$

Continuity of Compound Functions: ____

Intermediate Value Theorem
If f is continuous [a, b], and k is any number between f(a) and f(b), then there is at least one number c in [a, b] such that f(c) = k.



Application: Use the IVT to show that the polynomial function $f(x) = x^2 + 2x - 1$ has a zero on [0, 1].

Does the IVT guarantee that the following functions have a zero on [-1, 1]? Explain your answer.

Ex. 1 $f(x) = \dfrac{x^2 - 1}{x + 1}$

Ex. 2 $f(x) = \sin x$

PRACTICE PROBLEM SET 2

Now try these problems. The answers are in Chapter 21.

1. Is the function $f(x)=\begin{cases} x+7, \ x<2 \\ 9, \ x=2 \\ 3x+3, \ x>2 \end{cases}$ continuous at $x = 2$?

2. Is the function $f(x)=\begin{cases} 4x^2-2x, \ x<3 \\ 10x-1, \ x=3 \\ 30, \ x>3 \end{cases}$ continuous at $x = 3$?

3. Is the function $f(x)=\begin{cases} 5x+7, \ x<3 \\ 7x+1, \ x>3 \end{cases}$ continuous at $x = 3$?

4. Is the function $f(x) = \sec x$ continuous everywhere?

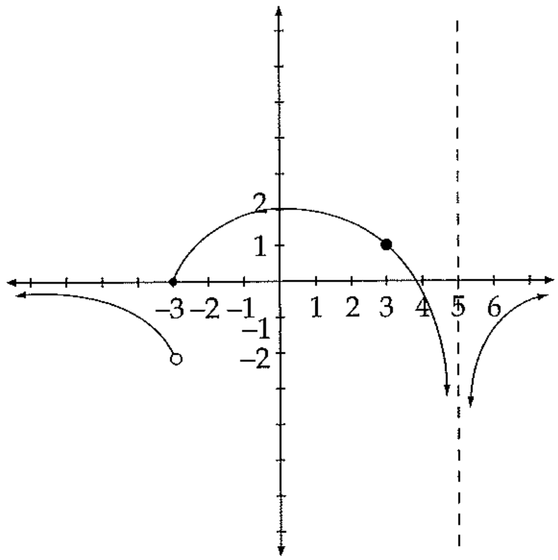
5. Is the function $f(x) = \sec x$ continuous on the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$?

6. Is the function $f(x) = \sec x$ continuous on the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$?

7. For what value(s) of k is the function $f(x) = \begin{cases} 3x^2-11x-4, \ x\le 4 \\ kx^2-2x-1, \ x>4 \end{cases}$ continuous at $x = 4$?

8. For what value(s) of k is the function $f(x) = \begin{cases} -6x-12, & x<-3 \\ k^2-5k, & x=-3 \\ 6, & x>-3 \end{cases}$ continuous at $x = -3$?

9. At what point is the removable discontinuity for the function $f(x) = \dfrac{x^2+5x-24}{x^2-x-6}$?



10. Given the graph of $f(x)$ above, find:

(a) $\lim_{x\to -\infty} f(x)$

(b) $\lim_{x\to \infty} f(x)$

(c) $\lim_{x\to 3^-} f(x)$

(d) $\lim_{x\to 3^+} f(x)$

(e) $f(3)$

(f) Any other discontinuities.

Name ______________________ Date________ Per_____

Review of Limits and Continuity

			MATCHING
1.	$\lim_{x\to 0}\frac{\sin 2x}{3x}=$	A.	2
2.	Test for Continuity at x =1: $f(x)=\begin{cases}\frac{x^3-1}{x-1}, & x\neq 1\\ 3, & x=1\end{cases}$	C.	-2.5
3.	$\lim_{x\to\infty}\frac{11}{x^2}=$	D.	0
4.	$\lim_{x\to 1}11=$	E.	4
5.	$\lim_{x\to 0}\frac{3}{x}=$	F.	$\frac{4}{5}$
6.	$\lim_{x\to 7}\frac{x^2-49}{x-7}=$	H.	36
7.	Test for continuity at x = 2: $f(x)=\begin{cases}\frac{x^2-4}{x-2}, & x\neq 2\\ 4, & x=2\end{cases}$	I.	$\frac{1}{2}$
8.	$\lim_{x\to 0}\frac{x^2-5x}{x^2+2x}=$	J.	Continuous @ x =1
9.	$\lim_{x\to 1}(x+1)=$	K.	Not Continuous @ x =1
10.	$\lim_{x\to\infty}\frac{5x-8x^2}{3+x^2}=$	L.	DNE
11.	$\lim_{x\to\infty}\frac{12x^3-7}{15x^3-2x}=$	M.	-8
12.	$\lim_{x\to -3}\frac{x^2+2x-3}{x^2-2x-15}=$	N.	Continuous @ x = 5
13.	$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}$	O.	Continuous @ x = 2
14.	$\lim_{x\to 4}\frac{x^2-4x}{x-4}=$	P.	Not Continuous @ x = 5
15.	Test for continuity at x = 2: $f(x)=\frac{x^2}{x-2}$	R.	Not Continuous @ x = 2
16.	Test for continuity at x = 5:	S.	14

	$f(x)=\dfrac{3}{x-1}$		
17.	$\lim_{x\to 6} x^2 =$	T.	$\dfrac{-12}{145}$
18.	$\lim_{x\to -12} \dfrac{x}{x^2+1} =$	U.	11
		V.	$\dfrac{1}{4}$
		W.	3
		Y.	$\dfrac{2}{3}$
		Z.	NONE OF THE ABOVE

1. Match each problem on the left (above) with an answer from the right-hand column.

2. Now write the corresponding letter to each problem in the spaces below.

___ ___ ___   ___ ___ ___ ___
7 5 3   10 9 18 17

___ ___ ___ ___ ___ ___ ___ ___
18 14 9 8 17 14 15 6

___ ___ ___ ___ ___   ___ ___ ___;
16 14 13 14 15   3 12 14

___ ___ ___ ___   ___ ___ ___ ___
18 17 14 1   2 4 6 18

___ ___ ___ ___   ___ ___
18 14 16 3   18 7

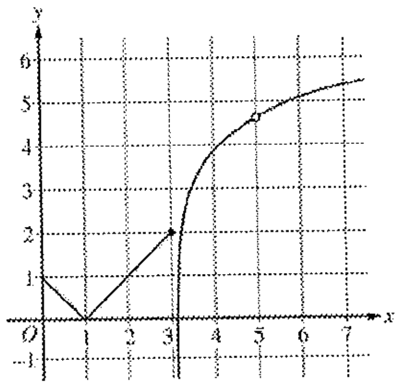
___ ___ ___ ___ ___ ___ ___ ___
12 16 11 12 16 12 18 1

AP Calculus 2018-2019 **Name ______________________________**

AP Packet for Unit 1- Limits and Continuity **Period_______**

Multiple Choice. Write the letter of the best answer choice in the blank provided. Capital letters please! ☺

__________ 1. The graph of the function f is shown below. Which of the following limits does not exist?



Graph of f

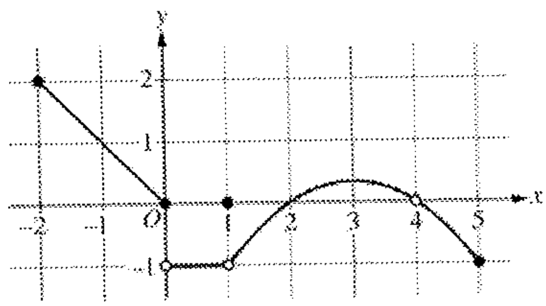
A. $\lim_{x\to 0} f(x)$

B. $\lim_{x\to 3} f(x)$

C. $\lim_{x\to 3^-} f(x)$

D. $\lim_{x\to 5} f(x)$

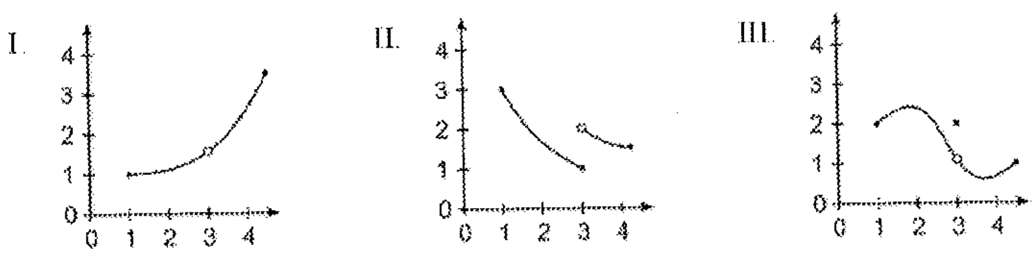
__________ 2. The graph of the function *f* is shown below. For what values of ***a*** does $\lim_{x\to a} f(x) = 0$?



Graph of f

A. 2 only
B. 2 and 4
C. 0 and 2 only
D. 0, 1, and 2

__________ 3. For which of the graphs shown does the $\lim_{x\to 3} f(x)$ exist?



A. I only B. III only C. I and II only D. I and III only

__________ 4. $\lim_{x\to -3}\left(\frac{x^2-9}{x^2-2x-15}\right)$

A. 0 B. 3/5 C. ¾ D. 9/4

__________ 5. What is $\lim_{h\to 0}\frac{2\left(\frac{1}{2}+h\right)^3-2\left(\frac{1}{2}\right)^3}{h}$?

A. 0 **B. 3/2** **C.** 1 **D.** 6

__________ 6. Evaluate the limit, if it exists: $\lim_{x\to 25}\left(\frac{\sqrt{x}-5}{x-25}\right)$

A. 0 B. $\frac{1}{25}$ C. $\frac{1}{10}$ D. $\frac{1}{5}$

__________ 6. $\lim_{x\to 0}(x\csc x)$ is

A. $-\infty$ B. -1 C. 0 D. 1

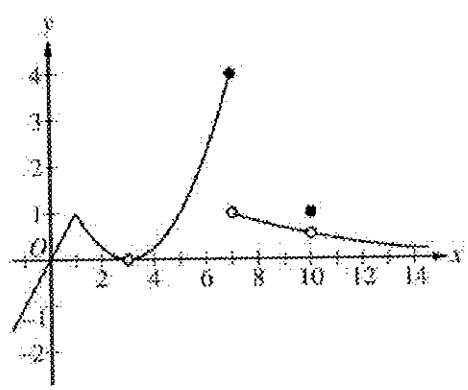
__________ 7. $\lim_{x\to\frac{\pi}{4}}\frac{\sin\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}$ is

A. 0 B. $\frac{1}{\sqrt{2}}$ C. $\frac{\pi}{4}$ D. 1

__________ 8. $\lim_{\theta\to 0}\frac{1-\cos\theta}{2\sin^2\theta}$ is

A. 0 B. $\frac{1}{8}$ C. $\frac{1}{4}$ D. 1

__________ 9. The graph of the function f is shown below. At what value of x does f have a jump discontinuity?



Graph of f

A. 1
B. 3
C. 7
D. 10

_________ 10. If f is a continuous function such that f(2) = 6, which of the following statements must be true?

A. $\lim_{x\to 1} f(2x) = 3$

B. $\lim_{x\to 2} f(x^2) = 36$

C. $\lim_{x\to 2} f(2x) = 12$

D. $\lim_{x\to 2} (f(x))^2 = 36$

_________ 11. $\lim_{x\to\infty} \frac{2x^3 + 3x^2 - 4x + 2}{4x^3 - x^2 + 5x - 3}$

A) $-\frac{1}{2}$ B) 0 C) $\frac{1}{2}$ D) 1 E) 2

_________ 12. $\lim_{x\to\infty} \frac{\sqrt{9x^4 + 1}}{4x^2 + 3}$ is

(A) $\frac{1}{3}$ (B) $\frac{3}{4}$ (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) infinite

_________ 13. Let f be a function that is continuous on the closed interval $[1, 3]$ with $f(1) = 10$ and $f(3) = 18$. Which of the following statements must be true?

(A) $10 \le f(2) \le 18$

(B) f is increasing on the interval $[1, 3]$.

(C) $f(x) = 17$ has at least one solution in the interval $[1, 3]$.

(D) $f'(x) = 8$ has at least one solution in the interval $(1, 3)$.

(E) $\int_1^3 f(x)\,dx > 20$

_________ 14.

$$f(x) = \begin{cases} x^2 \sin(\pi x) & \text{for } x < 2 \\ x^3 + cx - 18 & \text{for } x \ge 2 \end{cases}$$

Let f be the function defined above, where c is a constant. For what value of c, if any, is f continuous at $x = 2$?

(A) 2 (B) 7 (C) 9 (D) $4\pi - 4$ (E) There is no such value of c.

_________ 15. Function f is defined such that for all x ≥ 0, the line y = 3 is a horizontal asymptote. Which of the following must be true?

A. F(3) is undefined B. f(x) ≠ 3 for all x ≥ 0 C. $\lim_{x\to\infty} f(x) = 3$ D. None of these

______ 16. The vertical line $x = 2$ is an asymptote for the graph of the function f. Which of the following statements must be false?

(A) $\lim_{x \to 2} f(x) = 0$

(B) $\lim_{x \to 2} f(x) = -\infty$

(C) $\lim_{x \to 2} f(x) = \infty$

(D) $\lim_{x \to \infty} f(x) = 2$

(E) $\lim_{x \to \infty} f(x) = \infty$

Free Response. Write a complete answer in the space provided.

1. Let f be the function defined below. Is f continuous at x = 1?

$$f(x) = \begin{cases} 10 - 2x - x^2 & \text{for } x \le 1 \\ 3 + 4e^{x-1} & \text{for } x > 1 \end{cases}$$

2. The function *f* and *g* are continuous for all real numbers. The table below gives values of the functions and their first derivatives at selected values of *x*. The function h is given by ***h(x) = f(g(x)) – 6***. Explain why there must be a value of *r* on [1, 3] for which ***h(r) = -5***.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	−4	4	2
4	−1	3	6	7