2.4 Investigating Symmetry

Essential Question: How do you determine whether a figure has line symmetry or rotational symmetry?

Explore 1 Identifying Line Symmetry

A figure has symmetry if a rigid motion exists that maps the figure onto itself. A figure has line symmetry (or reflectional symmetry) if a reflection maps the figure onto itself. Each of these lines of reflection is called a line of symmetry.

You can use paper folding to determine whether a figure has line symmetry.

1. Trace the figure on a piece of tracing paper.
2. If the figure can be folded along a straight line so that one half of the figure exactly matches the other half, the figure has line symmetry. The crease is the line of symmetry. Place your shape against the original figure to check that each crease is a line of symmetry.
3. Sketch any lines of symmetry on the figure. The figure has line of symmetry.

Common Core Math Standards

The student is expected to:

G-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Mathematical Practices

MP.7 Using Structure

Language Objective

Have students work with a partner to give clues about a figure, and identify whether figures have line symmetry, rotational symmetry, or both and draw the line(s) of symmetry.

Engage

Essential Question: How do you determine whether a figure has line symmetry or rotational symmetry?

Possible answer: To identify line symmetry, look for a line of reflection, which is a line that divides the figure into mirror-image halves. To identify rotational symmetry, think of the figure rotating around its center. The figure has rotational symmetry if a rotation of at most 180° produces the original figure.

Preview: Lesson Performance Task

View the online Engage. Discuss the photo of the flower with students. Consider whether you could turn the flower and have it still appear the same. Then preview the Lesson Performance Task.
Draw the lines of symmetry, if any, on each figure and tell the total number of lines of symmetry each figure has.

<table>
<thead>
<tr>
<th>Figure</th>
<th>How many lines of symmetry?</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>two</td>
</tr>
<tr>
<td></td>
<td>zero</td>
</tr>
<tr>
<td></td>
<td>one</td>
</tr>
</tbody>
</table>

Reflect

1. What do you have to know about any segments and angles in a figure to decide whether the figure has line symmetry? **Pairs of segments in the figure must have the same length and pairs of angles must have the same measure, so that one half of the figure will coincide with the other half when the figure is folded across a line of symmetry.**

2. What figure has an infinite number of lines of symmetry? A **circle**

3. Discussion A figure undergoes a rigid motion, such as a rotation. If the figure has line symmetry, does the image of the figure have line symmetry as well? Give an example. **Yes. The line of symmetry also undergoes the rigid motion. For example, if the L-shape in Step D is rotated into a V-shape, the line of symmetry is rotated the same way.**

**EXPLORE 2**

Identifying Rotational Symmetry

A figure has rotational symmetry if a rotation maps the figure onto itself. The angle of rotational symmetry, which is greater than 0° but less than or equal to 180°, is the smallest angle of rotation that maps a figure onto itself.

An angle of rotational symmetry is a fractional part of 360°. Notice that every time the 5-pointed star rotates \(\frac{360°}{5} = 72°\), the star coincides with itself. The angles of rotation for the star are 72°, 144°, 216°, and 288°. If a copy of the figure rotates to exactly match the original, the figure has rotational symmetry.

Trace the figure onto tracing paper. Hold the center of the traced figure against the original figure with your pencil. Rotate the traced figure counterclockwise until it coincides again with the original figure beneath.

By how many degrees did you rotate the figure? **120°**

What are all the angles of rotation? **120°, 240°**

**PROFESSIONAL DEVELOPMENT**

**Math Background**

A wallpaper pattern is a planar repeating pattern. Mathematicians classify wallpaper patterns based on the symmetries they exhibit, for example, only translation symmetry; translation and reflection symmetry; or all these plus rotational symmetry. Every wallpaper pattern can be classified by identifying its symmetries. Surprisingly, there are precisely 17 different classifications. That is, any repeating pattern that covers a plane can be reduced to one of 17 basic types. This unusual mathematical fact has had far-reaching applications in a number of fields, including chemistry and crystallography.

**EXPLORE 1**

Identifying Line Symmetry

INTEGRATE TECHNOLOGY

Have students use geometry software or cut out figures to examine the symmetry of regular polygons. Then have them use inductive reasoning to make conjectures about the number of lines of symmetry a regular \(n\)-gon has. **\(n\) lines of symmetry**

**QUESTIONING STRATEGIES**

What are the three rigid motions explained in this module? What does a rigid motion transformation preserve? **translation, reflection, rotation; shape and size**

**EXPLORE 2**

Identifying Rotational Symmetry

INTEGRATE TECHNOLOGY

Ask students to discuss the pros and cons of using geometry software to investigate properties of rotations and symmetry. Be sure students recognize that such software has the advantage of making it easy to change parameters (such as the angle of rotation) so that they can observe the effects of the changes.

**QUESTIONING STRATEGIES**

When you are testing a figure to see if it has rotational symmetry, where is \(P\), the center of rotation? **at the center of the figure**
**EXPLAIN 1**

**Describing Symmetries**

**INTEGRATE TECHNOLOGY**

Human faces appear to have symmetry, but most people’s faces aren’t perfectly symmetric. Photocopy a picture of a face onto two transparencies and cut each one down the center of the face. Flip the pieces of one transparency over and put the two left sides together and the two right sides together to create two different faces with perfect symmetry. Discuss with students how to tell if a figure has symmetry.

**QUESTIONING STRATEGIES**

How can you find the center point of a regular polygon? **The center is the point that is equidistant from each vertex or corner.**

**AVOID COMMON ERRORS**

Some students may think that any diagonal of a figure is a line of symmetry. Have them draw a rectangle that is not a square and one of the diagonals. Folding along this diagonal demonstrates that it is not a line of symmetry.

**Reflect**

4. **What figure is mapped onto itself by a rotation of any angle?** A circle

5. **Discussion** A figure is formed by line $l$ and line $m$, which intersect at an angle of $60^\circ$. Does the figure have an angle of rotational symmetry of $60^\circ$? If not, what is the angle of rotational symmetry?
   
   No, the angle of rotational symmetry for the figure is $180^\circ$. A rotation of $60^\circ$ about the intersection will only map one of the lines onto the other line.

**COLLABORATIVE LEARNING**

**Small Group Activity**

Have students identify common shapes that do not have line symmetry, for example, the capital letters F, G, and J. Have students name a letter of the alphabet with each type of symmetry:

- one line of symmetry (horizontal) **B C D E**
- one line of symmetry (vertical) **A M T U V W Y**
- two lines of symmetry **H I O X**
- rotational symmetry but not line symmetry **N S Z**
- no symmetry **F G J K L P Q R**
Step 2 Now look for other lines of symmetry. The two diagonals also describe matching halves. The figure has a total of 4 lines of symmetry.

Step 3 Next, look for rotational symmetry. Think of the figure rotated about its center until it matches its original position. The angle of rotational symmetry of this figure is $\frac{1}{4}$ of 360°, or 90°.

The other angles of rotation for the figure are the multiples of 90° that are less than 360°. So the angles of rotation are 90°, 180°, and 270°.

CONNECT VOCABULARY

Connect the idea of a reflection to a figure with line symmetry. If you identify the line of symmetry on the figure, and superimpose that line on the x- or y-axis on a coordinate plane, then the line of symmetry becomes the line of reflection, and you can see the image and preimage on either side.

LANGUAGE SUPPORT

Connect Vocabulary

In English and in Spanish, we usually add –s or –es to the end of a noun to form the plural, for example, triangles, points, figures. In English, some nouns are irregular and don't follow that convention. The plural form of the noun vertex is vertices. Notice that the x becomes a c and then –es is added to form this plural. In Spanish, the same thing happens with words that end in z. The z becomes a c and then –es is added to form the plural.
**ELABORATE**

**INTEGRATE MATHEMATICAL PRACTICES**

**Focus on Communication**

**MP.3** The number of angles of rotation less than $360^\circ$ is called the *order* of the rotational symmetry, so a square is of order 3 and an equilateral triangle is of order 2. A five-pointed star is of order 4.

**INTEGRATE MATHEMATICAL PRACTICES**

**Focus on Reasoning**

**MP.2** Give students pictures of figures or objects that have line symmetry, rotational symmetry, both, or neither. Students get two sets of each figure. One set of pictures is placed face up between the pair of students and one set is face down. The first student draws a card, such as the picture of a square, and gives oral clues such as, “This figure has both rotational symmetry and line symmetry. Its angle of rotation is 90 degrees; it has 4 sides and 4 angles; and has 4 lines of symmetry.” The second student picks the picture that matches the clues. They switch roles and repeat the process.

**SUMMARIZE THE LESSON**

How do you determine whether a figure has line symmetry or rotational symmetry? A figure has line symmetry if the figure can be reflected across a line so that the image coincides with the preimage. A figure has rotational symmetry if the figure can be rotated about a point by an angle greater than $0^\circ$ and less than or equal to $180^\circ$ so that the image coincides with the preimage.

**DIFFERENTIATE INSTRUCTION**

**Modeling**

Bring in books or suggest websites that show examples of mandalas. Have students find and describe examples of rotational symmetry in each. Have them create an original mandala.

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**Your Turn**

Describe the type of symmetry for each figure. Draw the lines of symmetry, name the angles of rotation, or both if the figure has both.

6. Figure $ABCD$

- Types of symmetry: line, rotational
- Number of lines of symmetry: 4
- Angles of rotation: $90^\circ$, $180^\circ$, $270^\circ$

7. Figure $EFGHI$

- Types of symmetry: line
- Number of lines of symmetry: 1
- Angles of rotation: none

8. Figure $KLNPR$

- Types of symmetry: rotational
- Number of lines of symmetry: 0
- Angles of rotation: $72^\circ$, $144^\circ$, $216^\circ$ and $288^\circ$

9. Figure $TUVW$

- Types of symmetry: none
- Number of lines of symmetry: 0
- Angles of rotation: none

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**Elaborate**

10. How are the two types of symmetry alike? How are they different?

Both types of symmetry show how a figure can be mapped onto itself by a rigid motion. In line symmetry, the figure is mapped onto itself by reflection, and in rotational symmetry, the mapping is by rotation.

11. Essential Question Check-In How do you determine whether a figure has line symmetry or rotational symmetry?

Possible answer: To identify line symmetry, look for a line of reflection that divides the figure into mirror-image halves. To identify rotational symmetry, think of the figure rotating around its center. The figure has rotational symmetry if a rotation of at most $180^\circ$ produces the original figure.
Evaluate: Homework and Practice

Draw all the lines of symmetry for the figure, and give the number of lines of symmetry. If the figure has no line symmetry, write zero.

1.  
   ![Line of Symmetry Image]  
   Lines of symmetry: 1

2.  
   ![Line of Symmetry Image]  
   Lines of symmetry: 8

3.  
   ![Line of Symmetry Image]  
   Lines of symmetry: 1

For the figures that have rotational symmetry, list the angles of rotation less than 360°. For figures without rotational symmetry, write “no rotational symmetry.”

4.  
   ![Figure]  
   Angles of rotation: no rotational symmetry

5.  
   ![Figure]  
   Angles of rotation: 45°, 90°, 135°, 180°

6.  
   ![Figure]  
   Angles of rotation: 225°, 270°, 315°

7.  
   ![Tile Design Image]  
   rotational symmetry

8.  
   ![Tile Design Image]  
   both line and rotational symmetry

In the tile design shown, identify whether the pattern has line symmetry, rotational symmetry, both line and rotational symmetry, or no symmetry.

9.  
   Reflection across CF
   Figure EDCBAF

10. Rotation of 240° clockwise, or 120° counterclockwise
    Figure CDEFAB

11. Reflection across the line that connects the midpoint of BC and the midpoint of EF
    Figure DCBAFE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

**MP.1** It is also possible to define a symmetry based on translations. A pattern has translation symmetry if it can be translated along a vector so that the image coincides with the preimage. Tiled floors may be examples of this.

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<th>Mathematical Practices</th>
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<td>1 Recall of Information</td>
<td>MP.4 Modeling</td>
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<td>2</td>
<td>2 Skills/Concepts</td>
<td>MP.4 Modeling</td>
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<tr>
<td>3–4</td>
<td>1 Recall of Information</td>
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<tr>
<td>5</td>
<td>2 Skills/Concepts</td>
<td>MP.4 Modeling</td>
</tr>
<tr>
<td>6</td>
<td>1 Recall of Information</td>
<td>MP.4 Modeling</td>
</tr>
<tr>
<td>7–8</td>
<td>2 Skills/Concepts</td>
<td>MP.4 Modeling</td>
</tr>
</tbody>
</table>
AVOID COMMON ERRORS
Some students may stop when they have found one line of symmetry or one angle of rotation. Remind them to reread the directions to see if they are asked to find all lines of symmetry or all angles of rotation.

JOURNAL
Have students create four different, simple logos. For the first logo, there should be no rotations that map the logo onto itself. For the second, a rotation of 180° should map the logo onto itself; for the third, 120°; and for the fourth, 90°.

Exercise Depth of Knowledge (D.O.K.) COMMON Mathematical Practices

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<td>2 Skills/Concepts</td>
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<td></td>
</tr>
<tr>
<td>16</td>
<td>3 Strategic/Thinking</td>
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In the space provided, sketch an example of a figure with the given characteristics. Possible answers are shown.

12. no line symmetry; angle of rotational symmetry: 180°  

13. one line of symmetry; no rotational symmetry

14. Describe the line and rotational symmetry in this figure.

four lines of symmetry; angle of rotational symmetry: 90°

H.O.T. Focus on Higher Order Thinking

15. Communicate Mathematical Ideas How is a rectangle similar to an ellipse? Use concepts of symmetry in your answer.

Both have two perpendicular lines of symmetry, and both have 180° rotational symmetry.

16. Explain the Error A student was asked to draw all of the lines of symmetry on each figure shown. Identify the student's work as correct or incorrect. If incorrect, explain why.

a.  
Incorrect; the two diagonals are not lines of symmetry.

b.  
Incorrect; the figure has no lines of symmetry.

c.  
Incorrect; the figure has three more lines of symmetry, each connecting the remaining pairs of opposite vertices.
Lesson Performance Task

Use symmetry to design a work of art. Begin by drawing one simple geometric figure, such as a triangle, square, or rectangle, on a piece of construction paper. Then add other lines or two-dimensional shapes to the figure. Next, make identical copies of the figure, and then arrange them in a symmetric pattern.

Evaluate the symmetry of the work of art you created. Rotate it to identify an angle of rotational symmetry. Compare the line symmetry of the original figure with the line symmetry of the finished work.

Answers will vary. Students’ responses should identify all lines of symmetry (horizontal, vertical, and diagonal) as well as all angles of rotational symmetry ($90^\circ$, $180^\circ$, and $270^\circ$).

AVOID COMMON ERRORS

In evaluating rectangular shapes for symmetries, students sometimes identify the diagonals as lines of symmetry. Unless a rectangle is a square, its diagonals are not lines of symmetry.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 Students should recognize that while single elements of their designs might exhibit symmetries, those symmetries might not extend to the entire design.

For example, in a design composed of a square and four isosceles triangles, each of those shapes contains lines of symmetry. None of those lines, however, is a line of symmetry for the entire design.

EXTENSION ACTIVITY

The flags of many nations have rotational symmetry or line symmetry. The flags of a few nations, such as Jamaica, have both. Research the flags of the nations of the world to find examples of symmetry. If you wish, disregard color and concentrate only on the designs of the flags. Draw or print out examples of designs you find especially interesting or attractive.

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.
1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0 points: Student does not demonstrate understanding of the problem.