LESSON 3.2

Proving Figures are Congruent Using Rigid Motions

Common Core Math Standards

The student is expected to:

COMMON CORE **G-CO.B.6**

... given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Also G-CO.A.5

Mathematical Practices

COMMON CORE **MP.3 Logic**

Language Objective

Have students work in pairs to label congruent and noncongruent figures.

ENGAGE

Essential Question: How can you determine whether two figures are congruent?

Possible answer: If one figure can be obtained from the other by a sequence of rigid motions, then they are congruent.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the photo and ask students to describe the pattern or patterns in the tile design. Then preview the Lesson Performance Task.

Name ______________ Class ______________ Date ______________

3.2 Proving Figures are Congruent Using Rigid Motions

Resource Locker

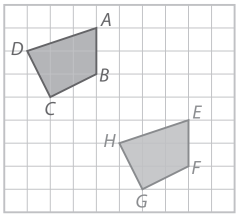
Essential Question: How can you determine whether two figures are congruent?

Explore Confirming Congruence

Two plane figures are congruent if and only if one can be obtained from the other by a sequence of rigid motions (that is, by a sequence of reflections, translations, and/or rotations).

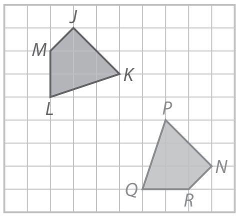
A landscape architect uses a grid to design the landscape around a mall. Use tracing paper to confirm that the landscape elements are congruent.

Ⓐ Trace planter *ABCD*. Describe a transformation you can use to move the tracing paper so that planter *ABCD* is mapped onto planter *EFGH*. What does this confirm about the planters?



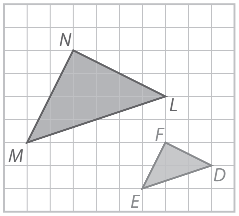
You can map *ABCD* to *EFGH* with a translation right 4 units and down 4 units. The planters are congruent because there is a rigid transformation that maps one to the other.

Ⓑ Trace pools *JKLM* and *NPQR*. Fold the paper so that pool *JKLM* is mapped onto pool *NPQR*. Describe the transformation. What does this confirm about the pools?



You can map *JKLM* to *NPQR* with a reflection over the fold line. The pools are congruent because there is a rigid transformation that maps one to the other.

Ⓒ Determine whether the lawns are congruent. Is there a rigid transformation that maps $\triangle LMN$ to $\triangle DEF$? What does this confirm about the lawns?

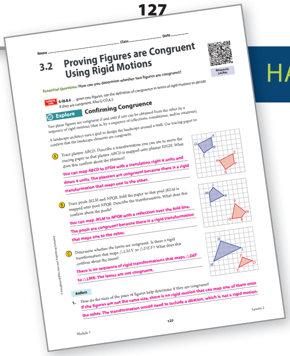


There is no sequence of rigid transformations that maps △*DEF* to △*LMN*. The lawns are not congruent.

Reflect

1. How do the sizes of the pairs of figures help determine if they are congruent?

If the figures are not the same size, there is no rigid motion that can map one of them onto the other. The transformation would need to include a dilation, which is not a rigid motion.



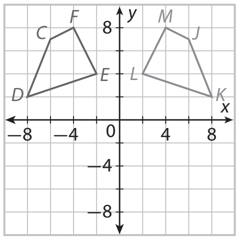
HARDCOVER PAGES 113–122

Turn to these pages to find this lesson in the hardcover student edition.

Explain 1 Determining if Figures are Congruent

Example 1 **Use the definition of congruence to decide whether the two figures are congruent. Explain your answer.**

(A)

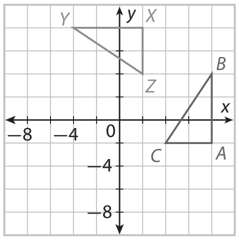


The two figures appear to be the same size and shape, so look for a rigid transformation that will map one to the other.

You can map *CDEF* onto *JKLM* by reflecting *CDEF* over the *y*-axis. This reflection is a rigid motion that maps *CDEF* to *JKLM*, so the two figures are congruent.

The coordinate notation for the reflection is $(x, y) \to (-x, y)$.

(B)



The two figures appear to be the same/different.

You can map $\triangle ABC$ to $\triangle XYZ$

by **a counter-clockwise rotation of 90° around the origin**.

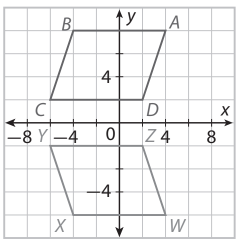
This is/is not a rigid motion that maps $\triangle ABC$ to $\triangle XYZ$, so the two figures are/are not congruent.

The coordinate notation for the rotation is **$(x, y) \to (-y, x)$**.

Your Turn

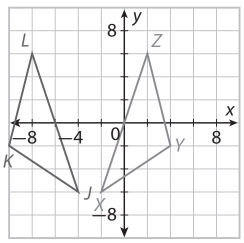
Use the definition of congruence to decide whether the two figures are congruent. Explain your answer.

2.



You can map *ABCD* to *WXYZ* with a reflection across the *x*-axis, so the figures are congruent. The coordinate notation for the reflection is $(x, y) \to (x, -y)$.

3.



You can map $\triangle JKL$ to $\triangle XYZ$ with a reflection across the *y*-axis, followed by a horizontal translation, so the figures are congruent. The coordinate notation for the reflection is $(x, y) \to (-x, y)$ and for the translation is $(x, y) \to (x - 6, y)$.

PROFESSIONAL DEVELOPMENT

Learning Progressions

COMMON CORE

In this lesson students learn that two figures are congruent if and only if there is a sequence of rigid motions that maps one figure to the other. That means if they can find the sequence of rigid motions that maps one figure to the other, then they can confirm that the preimage and image figures are congruent. It also means, if the figures are known to be congruent, that there is a sequence of rigid motions that maps one figure to the other. In upcoming lessons, students will use this transformations-based definition to develop congruence criteria for triangles.

EXPLORE

Confirming Congruence

INTEGRATE TECHNOLOGY

Students have the option of confirming congruence using the rigid motion activity either in the book or online.

QUESTIONING STRATEGIES

Do the figures appear to be congruent? Why or why not? **yes, because they have the same size and shape**

Can either figure be considered to be the preimage? Why or why not? **Yes, if *ABCD* is congruent to *EFGH*, then the reverse is true.**

EXPLAIN 1

Determining if Figures are Congruent

CONNECT VOCABULARY EL

Define *congruence.* Ask students to give examples of congruent figures in the classroom. Students might mention floor tiles with the same size and shape, or desktops that are rectangles of the same size and shape. Tell students that “the same size and shape” is an informal way of deciding whether two figures may be congruent, but a formal definition of congruence is based on rigid motions.

QUESTIONING STRATEGIES

How can a rigid motion be used to determine if two figures are congruent? **Each rigid motion preserves size and shape, so if a sequence of rigid motions can be found to map one figure to the other, then the preimage and image figure are congruent.**

EXPLAIN 2

Finding a Sequence of Rigid Motions

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Relate congruence to rigid motion by comparing the size and shape of the preimage and image. Point out that the sequence of rigid motions that maps one figure to another may not be unique. Encourage students to look for alternate sequences that work.

QUESTIONING STRATEGIES

? For each pair of figures, how do you know that a sequence of rigid motions that maps one figure to the other must exist? **If the figures are known to be congruent—by the definition of congruent, there is a sequence of rigid motions that maps one to the other.**

? What would the notation $(x, y) \rightarrow (-x, y + 2)$ mean? **The transformation reflects each *x*-coordinate across the *x*-axis and raises the figure by 2 units.**

AVOID COMMON ERRORS

Some students may think that if two figures are congruent, then there is one rigid motion that can map one figure to the other. Explain that it may take a sequence of rigid motions to map a figure to a congruent figure. Ask students to find examples of when this may be true.

CONNECT VOCABULARY EL

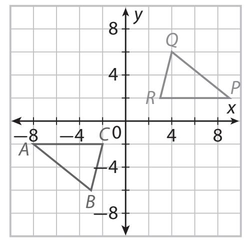
Help students understand how *congruence* is related to rigid motions by pointing out how a single rigid motion can produce a congruent figure. Therefore, a sequence of rigid motions must also produce a congruent figure. Point out that this is true both in a plane and on a coordinate plane.

Explain 2 Finding a Sequence of Rigid Motions

The definition of congruence tells you that when two figures are known to be congruent, there must be some sequence of rigid motions that maps one to the other.

Example 2 **The figures shown are congruent. Find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.**

Ⓐ $\triangle ABC \cong \triangle PQR$

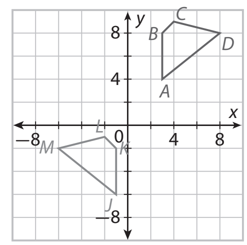


Map $\triangle ABC$ to $\triangle PQR$ with a rotation of 180° around the origin, followed by a horizontal translation.

Rotation: $(x, y) \rightarrow (-x, -y)$

Translation: $(x, y) \rightarrow (x + 1, y)$

Ⓑ $ABCD \cong JKLM$



Map $ABCD$ to $JKLM$ with a **reflection across the y-axis**, followed by a **translation**.

Reflection: $(x, y) \rightarrow$ **$(-x, y)$**

Translation: $(x, y) \rightarrow$ **$(x + 2, y - 10)$**

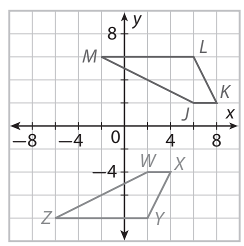
Reflect

4. How is the orientation of the figure affected by a sequence of transformations?
If the transformations include a reflection, then the orientation will change. A translation or rotation will preserve the original orientation.

Your Turn

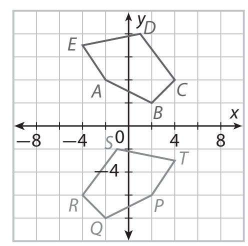
The figures shown are congruent. Find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.

5. $JKLM \cong WXYZ$



Reflect *JKLM* across the *x*-axis: $(x, y) \rightarrow (x, -y)$. Then translate the image: $(x, y) \rightarrow (x - 4, y - 2)$.

6. $ABCDE \cong PQRST$



Reflect *ABCDE* across the *y*-axis: $(x, y) \rightarrow (-x, y)$. Then translate the image: $(x, y) \rightarrow (x, y - 10)$.

COLLABORATIVE LEARNING

Small Group Activity

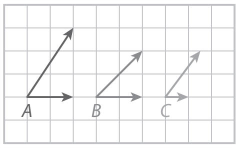
Give students the coordinates of a pair of congruent figures in the coordinate plane. Have each student describe a sequence of rigid motions that will map one figure to the other. Instruct them to switch papers and use another student's sequence of rigid motions to confirm that the given figures are congruent. Have them use geometry software to do the rigid motions and check the results against their own sequences.

Explain 3 Investigating Congruent Segments and Angles

Congruence can refer to parts of figures as well as whole figures. Two angles are congruent if and only if one can be obtained from the other by rigid motions (that is, by a sequence of reflections, translations, and/or rotations.) The same conditions are required for two segments to be congruent to each other.

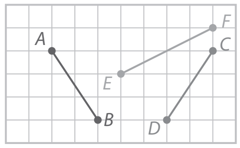
Example 3 **Determine which angles or segments are congruent. Describe transformations that can be used to verify congruence.**

Ⓐ



$\angle A$ and $\angle C$ are congruent. The transformation is a translation. There is no transformation that maps $\angle B$ to either of the other angles.

Ⓑ



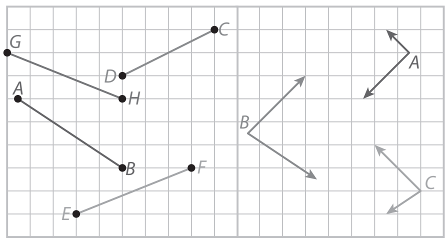
$\overline{AB}$ and **$\overline{CD}$** are congruent. A sequence of transformations is a **reflection** and a translation.

There is no transformation that maps **$\overline{EF}$** to either of the other segments.

Your Turn

7. Determine which segments and which angles are congruent. Describe transformations that can be used to show the congruence.

$\angle B$ and $\angle C$ are congruent. $\overline{EF}$ and $\overline{GH}$ are congruent. In both cases, a sequence of transformations is a reflection and a translation.



Elaborate

8. Can you say two angles are congruent if they have the same measure but the segments that identify the rays that form the angle are different lengths?
Yes, the definition of congruence for angles requires only that the angle between the rays be the same. The lengths of the segments does not matter.

9. **Discussion** Can figures have congruent angles but not be congruent figures?
Yes, two figures can have congruent angles but not be congruent figures. They could appear to be different sized versions of the same figure.

10. **Essential Question Check-In** Can you use transformations to prove that two figures are not congruent?
Maybe. If a dilation with scale factor $\neq 1$ maps one figure onto the other, then the figures cannot not be mapped using only rigid motions, so they cannot be congruent.

EXPLAIN 3

Investigating Congruent Segments and Angles

QUESTIONING STRATEGIES

How does the congruence of angles and segments relate to the congruence of two figures? Why? **Since rigid motions preserve angle measure and distance, verifying that corresponding angles and corresponding segments have the same measure determines whether two figures are congruent.**

AVOID COMMON ERRORS

Students may believe that two angles cannot be congruent if the rays forming the angles have different lengths. Remind students that rays continue forever in one direction, so the length representing a ray in a diagram is arbitrary. Draw two congruent angles, one with longer rays. Discuss why the angles are congruent even though one appears to be larger.

ELABORATE

QUESTIONING STRATEGIES

Can you say two angles are congruent if they have the same measure but the segments that identify the rays that form the angle are different lengths? Explain. **Yes. The angle measures determine if the two angles are congruent, not the rays or parts of the rays that make up their sides.**

Can you say two segments are congruent if their orientation is different? Explain. **Yes. The orientation of the segments does not affect their lengths, and therefore does not affect their congruence.**

SUMMARIZE THE LESSON

How are congruent figures related to transformations? **Two figures are congruent if one can be mapped to the other by a rigid transformation (rotation, reflection, or translation) or by a sequence of rigid transformations.**

DIFFERENTIATE INSTRUCTION

Visual Cues

Draw two congruent triangles and label the vertices. Highlight one side of one triangle blue. Have students name the corresponding side of the other triangle and highlight that side blue as well. Repeat with the other two pairs of corresponding sides using green and purple. Then shade one of the angles red. Have students name the corresponding angle of the other triangle and shade that red as well. Repeat with the other two pairs of corresponding angles using yellow and orange.

EVALUATE

Personal Math Trainer

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Confirming Congruence	
Example 1 Determining if Figures are Congruent	Exercises 1–5, 14–15, 17
Example 2 Finding a Sequence of Rigid Motions	Exercises 6–9, 16, 18–21, 24, 26–31
Example 3 Investigating Congruent Segments and Angles	Exercises 10–13, 23, 25

INTEGRATE MATHEMATICAL PRACTICES

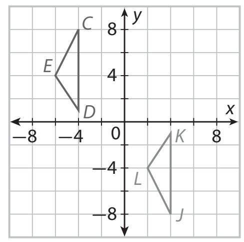
Focus on Technology

MP.5 Students can verify that two figures are congruent by using geometry software to do a sequence of rigid motions. Remind students to use the measuring features to show that angle measures and segment lengths are preserved.

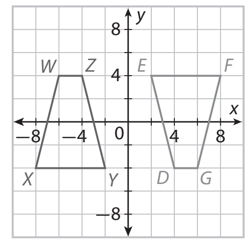
Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

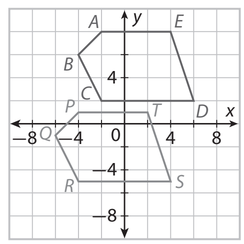
Use the definition of congruence to decide whether the two figures are congruent. Explain your answer. Give coordinate notation for the transformations you use.

1. 

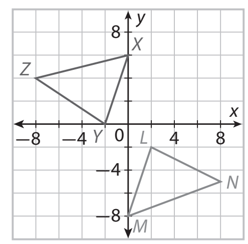
You can map $\triangle CDE$ to $\triangle JKL$ by reflecting $\triangle CDE$ over the x-axis, followed by a horizontal translation. So, the two figures are congruent. reflection: $(x, y) \rightarrow (x, -y)$; translation: $(x, y) \rightarrow (x + 8, y)$.

2. 

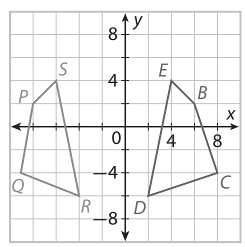
You can map $WXYZ$ to $DEFG$ with a reflection across the x-axis, followed by a horizontal translation. So, the two figures are congruent. reflection: $(x, y) \rightarrow (x, -y)$; translation: $(x, y) \rightarrow (x + 10, y)$.

3. 

You can map $ABCDE$ to $PQRST$ with a translation. So, the figures are congruent. translation: $(x, y) \rightarrow (x - 2, y - 7)$.

4. 

There is no sequence of rigid transformations that will map one figure onto the other, so they are not congruent.

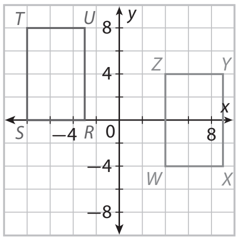
5. 

There is no sequence of rigid transformations that will map one figure onto the other, so they are not congruent.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1–5	**1** Recall of Information	**MP.5** Using Tools
6–9	**1** Recall of Information	**MP.4** Modeling
10–23	**1** Recall of Information	**MP.5** Using Tools
24–29	**2** Skills/Concepts	**MP.5** Using Tools

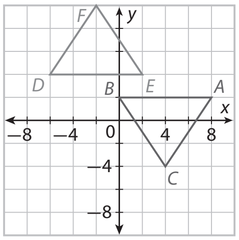
The figures shown are congruent. Find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.

6. *RSTU* ≅ *WXYZ*



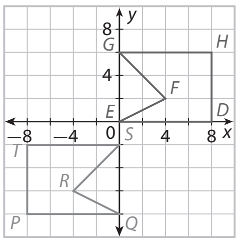
Map *RSTU* to *WXYZ* with a reflection across the *y*-axis, followed by a translation. reflection: $(x, y) \rightarrow (-x, y)$; translation: $(x, y) \rightarrow (x + 1, y - 4)$

7. △*ABC* ≅ △*DEF*



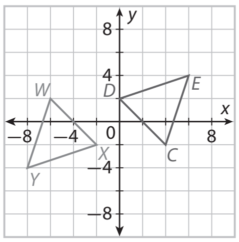
Map △*ABC* to △*DEF* with a rotation of 180° around the origin, followed by a translation. rotation: $(x, y) \rightarrow (-x, -y)$; translation: $(x, y) \rightarrow (x + 2, y + 6)$

8. *DEFGH* ≅ *PQRST*



Map *DEFGH* to *PQRST* with a reflection across the *y*-axis, followed by a vertical translation. reflection: $(x, y) \rightarrow (-x, y)$; translation: $(x, y) \rightarrow (x, y - 8)$

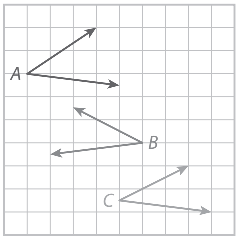
9. △*CDE* ≅ △*WXY*



Map △*CDE* to △*WXY* with a rotation of 180° around the origin, followed by a horizontal translation. rotation: $(x, y) \rightarrow (-x, -y)$; translation: $(x, y) \rightarrow (x - 2, y)$

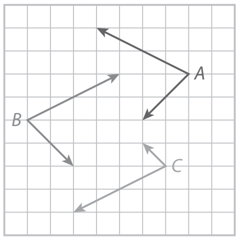
Determine which of the angles are congruent. Which transformations can be used to verify the congruence?

10.



None of the angles are congruent. There is no transformation that maps one of the angles to another.

11.



∠*A*, ∠*B* and ∠*C* are all congruent. The sequence of transformations is a reflection and a translation.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 Suggest that students use tracing paper to investigate which corresponding segments for two figures are congruent and which pairs of corresponding angles are congruent. Have them fold the tracing paper to see if the figures are coincident and, if they are, then write a sequence of rigid motions that can map one figure to the other. Have them also use the tracing paper to draw the figure and its congruent image on graph paper and then give the algebraic rules that map one figure to the other.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
30–33	**3** Strategic Thinking	**MP.2** Reasoning
34–35	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning
36	**3** Strategic Thinking H.O.T.	**MP.4** Modeling

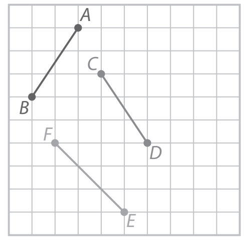
INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 When examining congruent figures on graph paper, students can see how each vertex is mapped to its corresponding vertex by circling corresponding vertices in the same color, using a different color for each pair of corresponding vertices. Students can also highlight pairs of corresponding sides in the same color, using a different color for each pair.

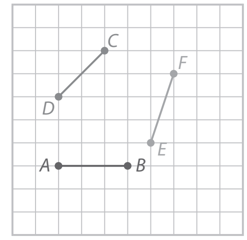
Determine which of the segments are congruent. Which transformations can be used to verify the congruence?

12.



$\overline{AB}$ and $\overline{CD}$ are congruent; reflection, then translation. There is no transformation that maps $\overline{EF}$ to either of the other segments.

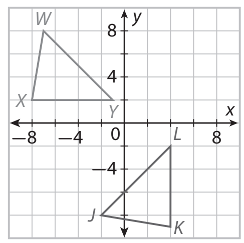
13.



None of the segments are congruent. There is no rigid transformation that maps one of them to another.

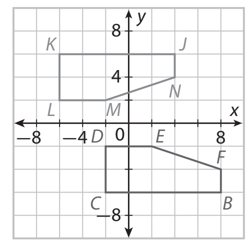
Use the definition of congruence to decide whether the two figures are congruent. Explain your answer. Give coordinate notation for the transformations you use.

14.



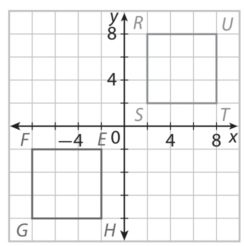
Yes. Map $\triangle JKL$ to $\triangle WXY$ with a clockwise rotation of 90° around the origin, followed by a translation. rotation: $(x, y) \to (y, -x)$ translation: $(x, y) \to (x + 1, y + 6)$.

15.



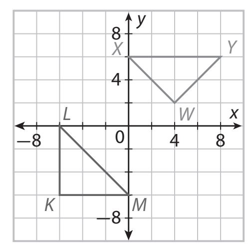
Yes. Map $BCDEF$ to $JKLMN$ with a reflection across the x-axis, followed by a horizontal translation. reflection: $(x, y) \to (x, -y)$ translation: $(x, y) \to (x - 4, y)$.

16.



Yes. Map $EFGH$ to $RSTU$ with a counter-clockwise rotation of 90° around the origin, followed by a vertical translation. rotation: $(x, y) \to (-y, x)$ translation: $(x, y) \to (x, y + 10)$.

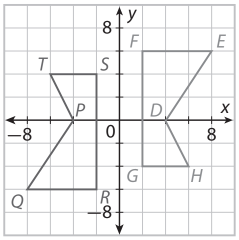
17.



No, the figures are not congruent. There are no transformations to map $\triangle KLM$ to $\triangle WXY$.

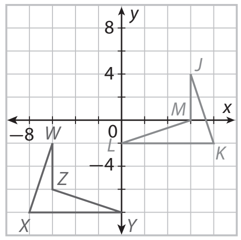
The figures shown are congruent. Find a sequence of transformations for the indicated mapping. Give coordinate notation for the transformations you use.

18. Map *PQRST* to *DEFGH*.



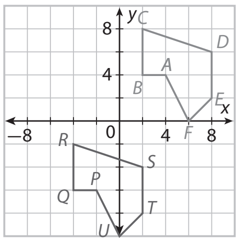
Map *PQRST* to *DEFGH* with a rotation of 180° around the origin. The coordinate notation for the rotation is $(x, y) \to (-x, -y)$.

19. Map *WXYZ* to *JKLM*.



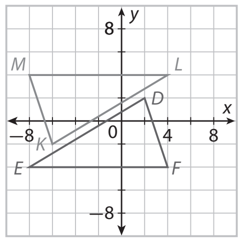
Map *WXYZ* to *JKLM* with a reflection across the *y*-axis, followed by a vertical translation. reflection: $(x, y) \to (-x, y)$; translation: $(x, y) \to (x, y + 6)$.

20. Map *PQRSTU* to *ABCDEF*.



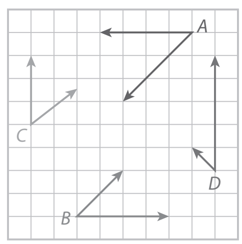
Map *PQRSTU* to *ABCDEF* with a combined translation. The coordinate notation for the translation is $(x, y) \to (x + 6, y + 10)$.

21. Map $\triangle DEF$ to $\triangle KLM$.



Map $\triangle DEF$ to $\triangle KLM$ with a rotation of 180° about the origin, followed by a horizontal translation. Rotation: $(x, y) \to (-x, -y)$; translation: $(x, y) \to (x - 4, y)$.

22. Determine whether each pair of angles is congruent or not congruent. Select the correct answer for each lettered part.



a. $\angle A$ and $\angle B$ ● Congruent ○ Not congruent

b. $\angle A$ and $\angle C$ ○ Congruent ● Not congruent

c. $\angle B$ and $\angle C$ ○ Congruent ● Not congruent

d. $\angle B$ and $\angle D$ ● Congruent ○ Not congruent

e. $\angle C$ and $\angle D$ ○ Congruent ● Not congruent

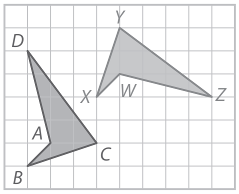
AVOID COMMON ERRORS

Students may make an error when using computations to determine if a transformed figure is congruent or not congruent. Emphasize that a resulting figure with the sides crossing each other is an indication of an error, not necessarily a noncongruent figure.

COLLABORATIVE LEARNING

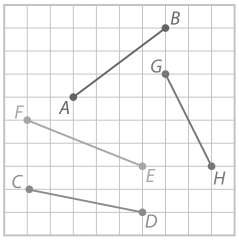
Give each student a sheet of graph paper. On the top half, have students draw ΔABC. Then ask them to perform a sequence of two or three rigid motions to draw $\Delta A'B'C'$. They may use each transformation only once. On the bottom half, have them write the sequence of rigid motions using precise mathematical language or symbols, then cut the paper in half. Collect the half sheets, making one pile of drawings and one pile of descriptions. Randomly pass out the papers so that each student receives one from each pile. Students should try to match each drawing with its corresponding rigid motions.

23. If $ABCD$ and $WXYZ$ are congruent, then $ABCD$ can be mapped to $WXYZ$ using a rotation and a translation. Determine whether the statement is true or false. Then explain your reasoning.



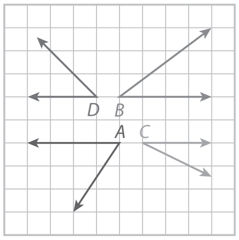
False. The figures do not have the same orientation, so the sequence of transformations must include a reflection.

24. Which segments are congruent? Which are not congruent? Explain.



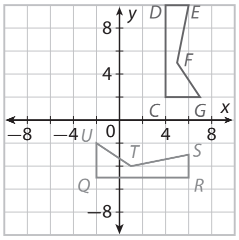
None are congruent. No rigid motions map one segment onto another.

25. Which angles are congruent? Which are not congruent? Explain.



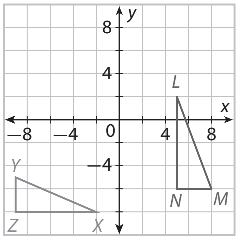
None are congruent. No rigid motions map one angle onto another.

26. The figures shown are congruent. Find a sequence of transformations that will map $CDEFG$ to $QRSTU$. Give coordinate notation for the transformations you use.



Rotate $CDEFG$ 90° clockwise about the origin: $(x, y) \rightarrow (y, -x)$. Then reflect across the x-axis: $(x, y) \rightarrow (x, -y)$. Then translate: $(x, y) \rightarrow (x - 4, y - 9)$.

27. The figures shown are congruent. Find a sequence of transformations that will map $\triangle LMN$ to $\triangle XYZ$. Give coordinate notation for the transformations you use.



Rotate $\triangle LMN$ 90° counter clockwise about the origin: $(x, y) \rightarrow (-y, x)$. Reflect across the y-axis: $(x, y) \rightarrow (-x, y)$. Translate: $(x, y) \rightarrow (x - 3, y - 13)$.

28. Which sequence of transformations does not map a figure onto a congruent figure? Explain.

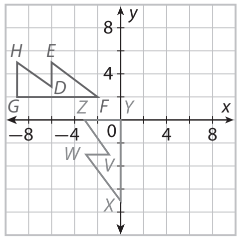
A. Rotation of 180° about the origin, reflection across the x-axis, horizontal translation $(x, y) \rightarrow (x + 4, y)$

B. Reflection across the y-axis, combined translation $(x, y) \rightarrow (x - 5, y + 2)$

C. Rotation of 180° about the origin, reflection across the y-axis, dilation $(x, y) \rightarrow (2x, 2y)$ **A dilation is not a rigid transformation.**

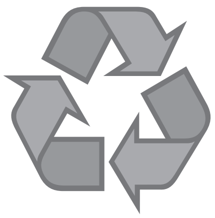
D. Counterclockwise rotation of 90° about the origin, reflection across the y-axis, combined translation $(x, y) \rightarrow (x - 11, y - 12)$

29. The figures shown are congruent. Find a sequence of transformations that will map *DEFGH* to *VWXYZ*. Give coordinate notation for the transformations you use.



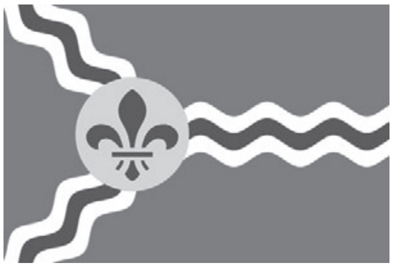
Map *DEFGH* to *VWXYZ* with a clockwise rotation of 90° around the origin, followed by a reflection across the *y*-axis, followed by a combined translation. rotation: $(x, y) \to (y, -x)$; reflection: $(x, y) \to (-x, y)$; translation: $(x, y) \to (x + 2, y - 9)$.

30. How can you prove that two arrows in the recycling symbol are congruent to each other?



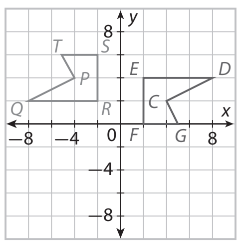
The arrows can each be mapped to each other by a rotation, which is a rigid transformation.

31. The city of St. Louis was settled by the French in the mid 1700s and joined the United States in 1803 as part of the Louisiana Purchase. The city flag reflects its French history by featuring the fleur-de-lis. How can you prove that the left and right petals are congruent to each other?



The petals can be mapped onto each other by a reflection, which is a rigid transformation.

32. **Draw Conclusions** Two students are trying to show that the two figures are congruent. The first student decides to map *CDEFG* to *PQRST* using a rotation of 180° around the origin, followed by the translation $(x, y) \to (x, y + 6)$. The second student believes the correct transformations are a reflection across the *y*-axis, followed by the vertical translation $(x, y) \to (x, y - 2)$. Are both students correct, is only one student correct, or is neither student correct?



Only the first student is correct. The two figures have the same orientation, so a sequence of transformations including a single reflection will change the orientation of the result.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Be sure that students' answers are as detailed and precise as possible. When stating that two figures in the coordinate plane are congruent, students should describe the specific rigid motion(s) that map one figure to the other. For instance, "a reflection across the x-axis" or "the reflection $(x, y) \to (x, -y)$" are detailed descriptions of a rigid motion in the coordinate plane, while "reflection" is not.

PEER-TO-PEER DISCUSSION

Have students discuss with a partner how to predict the sequence of rigid motions that may map a figure to a congruent figure. Then have them predict and test whether reversing the order of the sequence of rigid motions will produce the preimage figure.

CONNECT VOCABULARY EL

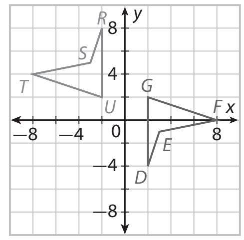
Have students relate the word *congruent* to the terms *equal* and *equivalent*. If a figure is congruent to another it is equal in shape and size. Show students similar figures that are equal in shape but not size, and discuss why they are not congruent.

JOURNAL

Have students write a journal entry in which they summarize what they know so far about congruence. Prompt students to include examples and non-examples of congruent figures and the methods of obtaining them and determining them.

33. Justify Reasoning Two students are trying to show that the two figures are congruent. The first student decides to map *DEFG* to *RSTU* using a rotation of 180° about the origin, followed by the vertical translation $(x, y) \rightarrow (x, y + 4)$. The second student uses a reflection across the *x*-axis, followed by the vertical translation $(x, y) \rightarrow (x, y + 4)$, followed by a reflection across the *y*-axis. Are both students correct, is only one student correct, or is neither student correct?

Both students are correct. Either of the sequences of transformation will map *DEFG* to *RSTU*. Recall that a rotation of 180° around the origin is the same as a reflection across both axes.



H.O.T. Focus on Higher Order Thinking

34. Look for a Pattern Assume the pattern of congruent squares shown in the figure continues forever.

Write rules for rigid motions that map square 0 onto square 1, square 0 onto square 2, and square 0 onto square 3.

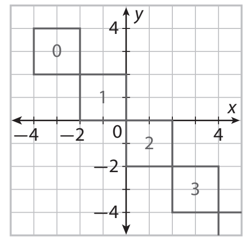
$(x, y) \rightarrow (x + 2, y - 2)$

$(x, y) \rightarrow (x + 4, y - 4)$

$(x, y) \rightarrow (x + 6, y - 6)$

Write a rule for a rigid motion that maps square 0 onto square *n*.

$(x, y) \rightarrow (x + 2n, y - 2n)$



35. Analyze Relationships Suppose you know that $\triangle ABC$ is congruent to $\triangle DEF$ and that $\triangle DEF$ is congruent to $\triangle GHJ$. Can you conclude that $\triangle ABC$ is congruent to $\triangle GHJ$? Explain.

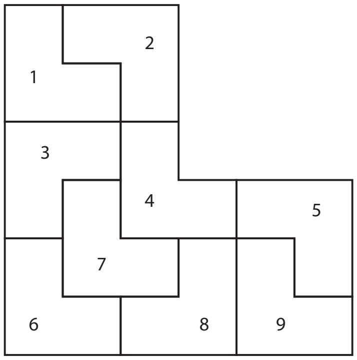
Yes; by the definition of congruence, there is a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle DEF$ and another that maps $\triangle DEF$ onto $\triangle GHJ$. The first sequence followed by the second sequence maps $\triangle ABC$ onto $\triangle GHJ$, so the triangles are congruent.

36. Communicate Mathematical Ideas Ella plotted the points $A(0, 0)$, $B(4, 0)$, and $C(0, 4)$. Then she drew $\overline{AB}$ and $\overline{AC}$. Give two different arguments to explain why the segments are congruent.

Both segments are 4 units long. Because the segments are the same length, they are congruent. A rotation of 90° maps $\overline{AB}$ onto $\overline{AC}$. Because there is a rigid motion that maps one segment onto the other, the segments are congruent.

Lesson Performance Task

The illustration shows how nine congruent shapes can be fitted together to form a larger shape. Each of the shapes can be formed from Shape #1 through a combination of translations, reflections, and/or rotations.



Describe how each of Shapes 2–9 can be formed from Shape #1 through a combination of translations, reflections, and/or rotations. Then design a figure like this one, using at least eight congruent shapes. Number the shapes. Then describe how each of them can be formed from Shape #1 through a combination of translations, reflections, and/or rotations.

Shape #2: Rotate Shape #1 180°.

Shape #3: Reflect Shape #1 vertically.

Shape #4: Translate Shape #1 down and right.

Shape #5: Rotate Shape #1 180° and then translate it down and right.

Shape #6: Translate Shape #1 down.

Shape #7: Translate Shape #1 down and right.

Shape #8: Translate Shape #1 down and then reflect it horizontally.

Shape #9: Translate Shape #1 down and right.

Answers will vary. Check students' answers.

EXTENSION ACTIVITY

The broken lines on the figure show how it can be divided into three congruent isosceles right triangles. Have students copy the figure and determine how it can be divided into eight congruent trapezoids.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 Ask students to show how each of the nine pieces in the Lesson Performance Task can be divided into two congruent shapes so that the entire shape can be constructed from 18 congruent shapes. **The shapes are trapezoids that form an L when placed together.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 An object viewed through certain types of lenses will appear to be flipped upside-down. Ask why the letters in the word STAR are flipped when seen through such a lens but the word CODE is not. **The letters C, O, D, and E are symmetric with respect to a line drawn horizontally through their centers, while the letters S, T, A, and R are not. The result is that CODE is indeed “flipped” by the lens, and the image through the lens appears exactly as it did before. The same is not true of the letters of STAR.**

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.