Ⓐ Complete the table of values for the functions $f_1(x) = 1.2^x$ and $f_2(x) = 1.5^x$. Use a calculator to find the values and round to the nearest thousandth if necessary.

x	$f_1(x) = 1.2^x$	$f_2(x) = 1.5^x$
−2	0.694	**0.444**
−1	**0.833**	0.667
0	**1**	**1**
1	1.2	1.5
2	**1.44**	**2.25**

Ⓑ Select the option that makes the statement true.

($f_1(x)$/[✓ $f_2(x)$]) increases more quickly as x increases.

($f_1(x)$/[✓ $f_2(x)$]) approaches 0 more quickly as x decreases.

Ⓒ The y-intercept of $f_1(x)$ is **1**. The y-intercept of $f_2(x)$ is **1**.

Ⓓ Fill in the table of values for the functions $f_3(x) = 0.6^x$ and $f_4(x) = 0.9^x$. Round to the nearest thousandth again.

x	$f_3(x) = 0.6^x$	$f_4(x) = 0.9^x$
−2	2.778	**1.235**
−1	**1.667**	1.111
0	**1**	**1**
1	0.6	0.9
2	**0.36**	**0.81**

Ⓔ ([✓ $f_3(x)$]/$f_4(x)$) increases more quickly as x decreases.

([✓ $f_3(x)$]/$f_4(x)$) approaches 0 more quickly as x increases.

Ⓕ The y-intercept of $f_3(x)$ is **1**. The y-intercept of $f_4(x)$ is **1**.

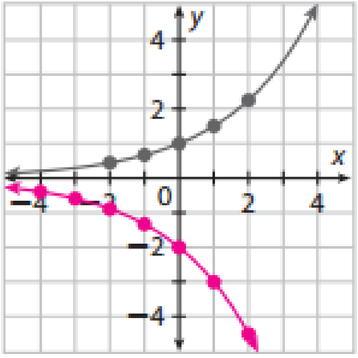
Reflect

1. Consider the function, $y = 1.3^x$. How will its graph compare with the graphs of $f_1(x)$ and $f_2(x)$? Discuss end behavior and the y-intercept.

All three graphs have the same y-intercept of 1. The graph of $y = 1.3^x$ falls between the other two graphs and increases more quickly than $f_1(x)$ but less quickly than $f_2(x)$ as x increases to the right of 0. The graph falls more quickly than that of $f_1(x)$ but less quickly than that of $f_2(x)$ as x decreases to the left of 0.

Ⓑ $f(x) = -2(1.5)^x$

x	$f(x) = -2(1.5)^x$
−4	−0.395
−3	−0.593
−2	−0.889
−1	−1.333
0	−2
1	−3
2	−4.5



End Behavior:

$f(x) \rightarrow \boxed{-\infty}$ as $x \rightarrow \infty$

$f(x) \rightarrow \boxed{0}$ as $x \rightarrow -\infty$

y-intercept: $\boxed{-2}$

Reflect

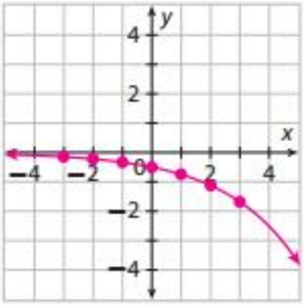
2. Discussion What can you say about the common behavior of graphs of the form $f(x) = ab^x$ with $b > 1$? What is different when a changes sign?

All graphs of the form $f(x) = ab^x$ with $b > 1$ approach 0 as x approaches $-\infty$ and have a y-intercept at $(0, a)$. The sign of a determines the end behavior as x approaches ∞; for $a > 0$ $f(x)$ increases toward infinity, and for $a < 0$, $f(x)$ decreases toward negative infinity.

Your Turn

Graph each function, and describe the end behavior and find the y-intercept of each graph.

3. $f(x) = -0.5(1.5)^x$



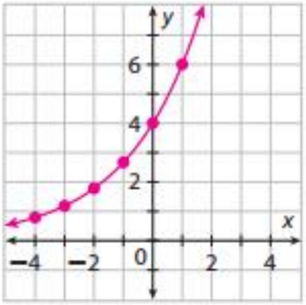
End behavior:

$f(x) \rightarrow -\infty$ as $x \rightarrow \infty$

$f(x) \rightarrow 0$ as $x \rightarrow -\infty$

y-intercept: −0.5

4. $f(x) = 4(1.5)^x$



End behavior:

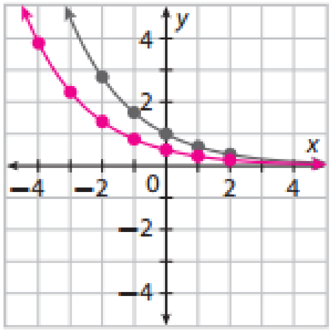
$f(x) \rightarrow \infty$ as $x \rightarrow \infty$

$f(x) \rightarrow 0$ as $x \rightarrow -\infty$

y-intercept: 4

Ⓑ $f(x) = 0.5\,(0.6)^x$

x	$f(x) = 0.5(0.6)^x$
−4	**3.858**
−3	**2.315**
−2	**1.389**
−1	**0.833**
0	**0.5**
1	**0.3**
2	**0.18**



End Behavior:

$f(x) \rightarrow$ **0** as $x \rightarrow \infty$

$f(x) \rightarrow$ **∞** as $x \rightarrow -\infty$

y-intercept: **0.5**

Reflect

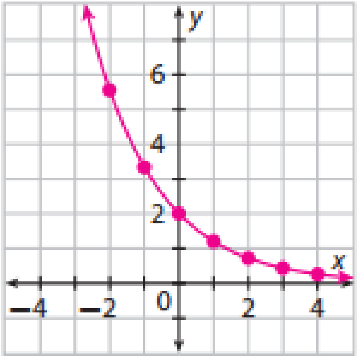
5. **Discussion** What can you say about the common behavior of graphs of the form $f(x) = ab^x$ with $0 < b < 1$? What is different when a changes sign?

All graphs of the form $f(x) = ab^x$ with $0 < b < 1$ approach 0 as x approaches ∞ and have a y-intercept at $(0, a)$. The sign of a determines the end behavior as x approaches $-\infty$; for $a > 0$, $f(x)$ approaches ∞ as x approaches $-\infty$, and for $a < 0$, $f(x)$ approaches $-\infty$ as x approaches $-\infty$.

Your Turn

Graph each function, and describe its end behavior and y-intercept.

6. $f(x) = 2(0.6)^x$



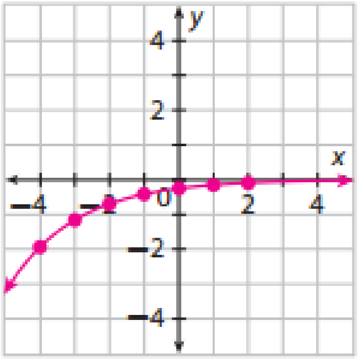
End behavior:

$f(x) \rightarrow 0$ as $x \rightarrow \infty$

$f(x) \rightarrow \infty$ as $x \rightarrow -\infty$

y-intercept: 2

7. $f(x) = -0.25(0.6)^x$



End behavior:

$f(x) \rightarrow 0$ as $x \rightarrow \infty$

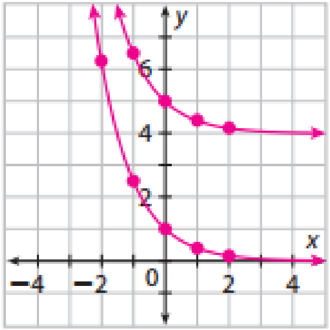
$f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$

y-intercept: −0.25

YourTurn

Graph the functions together on the same coordinate plane. Find the y-intercepts, and explain how they relate to the translation of the graph.

9. $f(x) = 0.4^x$ and $g(x) = 0.4^x + 4$

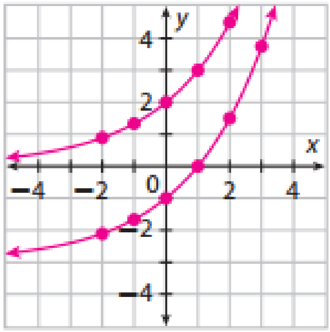


The y-intercept of $f(x)$ is 1.

The y-intercept of $g(x)$ is 5.

The y-intercept of $g(x)$ is 4 more than that of $f(x)$ because $g(x)$ is a vertical translation of $f(x)$ up by 4 units.

10. $f(x) = 2(1.5)^x$ and $g(x) = 2(1.5)^x - 3$



The y-intercept of $f(x)$ is 2.

The y-intercept of $g(x)$ is −1.

The y-intercept of $g(x)$ is 3 less than that of $f(x)$ because $g(x)$ is a vertical translation of $f(x)$ down by 3 units.

Elaborate

11. How do you determine the y-intercept of an exponential function $f(x) = ab^x + k$ that has been both stretched and translated?

The y-intercept of all the parent exponential functions $f(x) = b^x$ is 1. First multiply 1 by a to find the effect of the stretch, and then add k to find the effect of the translation.

12. Describe the end behavior of a translated exponential function $f(x) = b^x + k$ with $b > 1$ as x approaches $-\infty$.

Since all points are shifted by k, the function approaches k as x approaches $-\infty$.

13. **Essential Question Check-in** If a and b are positive real numbers and $b \neq 1$, how does the graph of $f(x) = ab^x$ change when b is changed?

If $b > 1$, increasing b makes the graph rise more quickly as x increases. If $0 < b < 1$, increasing b makes the graph fall more gradually as x increases.