Name ____________________ Class ____________ Date ________

11.2 Solving Linear Systems by Substitution

Essential Question: How can you solve a system of linear equations by using substitution?

Resource Locker

Explore — Exploring the Substitution Method of Solving Linear Systems

Another method to solve a linear system is by using the substitution method.

In the system of linear equations shown, the value of y is given. Use this value of y to find the value of x and the solution of the system.

$$\begin{cases} y = 2 \\ x + y = 6 \end{cases}$$

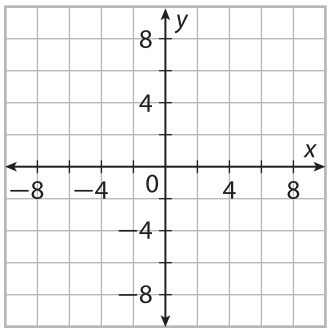
(A) Substitute the value of y in the second equation and solve for x.

$$x + y = 6$$
$$x + \square = 6$$
$$x = \square$$

(B) The values of x and y are known. What is the solution of the system?

Solution: $(\square, \square)$

(C) Graph the system of linear equations. How do your solutions compare?



(D) Use substitution to find the values of x and y in this system of linear equations. Substitute $4x$ for y in the second equation and solve for x. Once you find the value for x, substitute it into either original equation to find the value for y.

$$\begin{cases} y = 4x \\ 5x + 2y = 39 \end{cases}$$

Solution: $(\square, \square)$

Reflect

1. **Discussion** For the system in Step D, what equation did you get after substituting $4x$ for y in $5x + 2y = 39$ and simplifying?

____

2. **Discussion** How could you check your solution in part D?

____

____

Explain 1 Solving Consistent, Independent Linear Systems by Substitution

The **substitution method** is used to solve a system of equations by solving an equation for one variable and substituting the resulting expression into the other equation. The steps for the substitution method are as shown.

1. Solve one of the equations for one of its variables.
2. Substitute the expression from Step 1 into the other equation and solve for the other variable.
3. Substitute the value from Step 2 into either original equation and solve to find the value of the other variable.

Example 1 **Solve each system of linear equations by substitution.**

A $\begin{cases} 3x + y = -3 \\ -2x + y = 7 \end{cases}$

Solve an equation for one variable.

$3x + y = -3$	Select one of the equations.
$y = -3x - 3$	Solve for y. Isolate y on one side.

Substitute the expression for y in the other equation and solve.

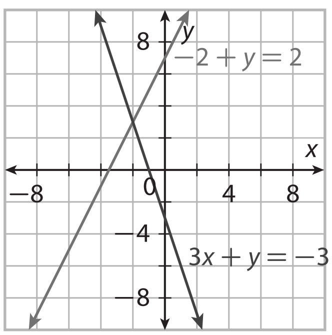
$-2x + (-3x - 3) = 7$	Substitute the expression for y.
$-5x - 3 = 7$	Combine like terms.
$-5x = 10$	Add 3 to both sides.
$x = -2$	Divide each side by -5.

Substitute the value for x into one of the equations and solve for y.

$3(-2) + y = -3$	Substitute the value of x into the first equation.
$-6 + y = -3$	Simplify.
$y = 3$	Add 6 to both sides.

So, $(-2, 3)$ is the solution of the system.

Check the solution by graphing.



$3x + y = -3$	$-2x + y = 7$
x-intercept: -1	x-intercept: $-\frac{7}{2}$
y-intercept: -3	y-intercept: 7

The point of intersection is $(-2, 3)$.

(B) $\begin{cases} x - 3y = 9 \\ x + 4y = 2 \end{cases}$

Solve an equation for one variable.

$x - 3y = 9$	Select one of the equations.
$x = \boxed{}$	Solve for x. Isolate x on one side.

Substitute the expression for ____ in the other equation and solve.

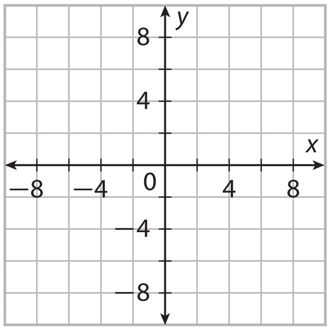
$\left(\boxed{}\right) + 4y = 2$	Substitute the expression for ____.
$\boxed{} = 2$	Combine like terms.
$\boxed{} = \boxed{}$	Subtract $\boxed{}$ from both sides.
$y = \boxed{}$	Divide each side by $\boxed{}$.

Substitute the value for y into one of the equations and solve for x.

$x - 3\left(\boxed{}\right) = 9$	Substitute the value of y into the first equation.
$\boxed{} = 9$	Simplify.
$x = \boxed{}$	Subtract $\boxed{}$ from both sides.

So, $\left(\boxed{}, \boxed{}\right)$ is the solution by graphing.

Check the solution by graphing.



$x - 3y = 9$	$x + 4y = 2$
x-intercept: $\boxed{}$	x-intercept: $\boxed{}$
y-intercept: $\boxed{}$	y-intercept: $\boxed{}$

The point of intersection is $\left(\boxed{}, \boxed{}\right)$.

Reflect

3. Explain how a system in which one of the equations if of the form $y = c$, where c is a constant is a special case of the substitution method.

4. Is it more efficient to solve $-2x + y = 7$ for x than for y? Explain.

Your Turn

5. Solve the system of linear equations by substitution.

$$\begin{cases} 3x + y = 14 \\ 2x - 6y = -24 \end{cases}$$

Explain 2 Solving Special Linear Systems by Substitution

You can use the substitution method for systems of linear equations that have infinitely many solutions and for systems that have no solutions.

Example 2 **Solve each system of linear equations by substitution.**

(A) $\begin{cases} x + y = 4 \\ -x - y = 6 \end{cases}$

Solve $x + y = 4$ for x.

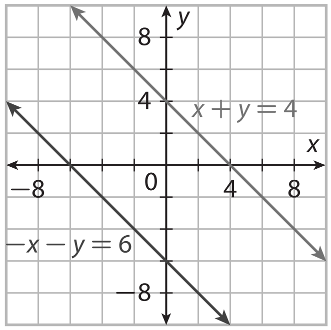
$$x = -y + 4$$

Substitute the resulting expression into the other equation and solve.

$-(-y + 4) - y = 6$ Substitute.

$-4 = 6$ Simplify.

The resulting equation is false, so the system has no solutions.



The graph shows that the lines are parallel and do not intersect.

(B) $\begin{cases} x - 3y = 6 \\ 4x - 12y = 24 \end{cases}$

Solve $x - 3y = 6$ for ______.

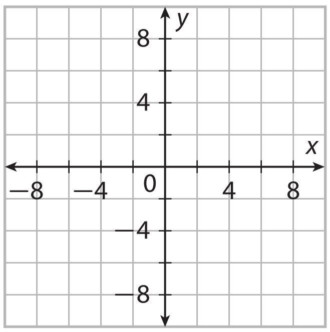
$x = \boxed{}$

Substitute the resulting expression into the other equation and solve.

$4\left(\boxed{}\right) - 12y = 24$ Substitute.

$\boxed{} = 24$ Simplify.

The resulting equation is ______, so the system has ____________.



The graphs are ____________,

so the system has ____________.

Reflect

6. Provide two possible solutions of the system in Example 2B. How are all the solutions of this system related to one another?

Your Turn

Solve each system of linear equations by substitution.

7. $\begin{cases} -2x + 14y = -28 \\ x - 7y = 14 \end{cases}$

8. $\begin{cases} -3x + y = 12 \\ 6x - 2y = 18 \end{cases}$

Explain 3 Solving Linear System Models by Substitution

You can use a system of linear equations to model real-world situations.

Example 3 **Solve each real-world situation by using the substitution method.**

(A) Fitness center A has a \$60 enrollment fee and costs \$35 per month. Fitness center B has no enrollment fee and costs \$45 per month. Let t represent the total cost in dollars and m represent the number of months. The system of equations $\begin{cases} t = 60 + 35m \\ t = 45m \end{cases}$ can be used to represent this situation. In how many months will both fitness centers cost the same? What will the cost be?

$60 + 35m = 45m$	Substitute $60 + 35m$ for t in the second equation.
$60 = 10m$	Subtract $35m$ from each side.
$6 = m$	Divide each side by 10.
$t = 45m$	Use one of the original equations.
$= 45(6) = 270$	Substitute 6 for m.
$(6, 270)$	Write the solution as an ordered pair.

Both fitness centers will cost \$270 after 6 months.

(B) High-speed Internet provider A has a \$100 setup fee and costs \$65 per month. High-speed internet provider B has a setup fee of \$30 and costs \$70 per month. Let t represent the total amount paid in dollars and m represent the number of months. The system of equations $\begin{cases} t = 100 + 65m \\ t = 30 + 70m \end{cases}$ can be used to represent this situation. In how many months will both providers cost the same? What will that cost be?

$\boxed{} = 30 + 70m$	Substitute $\boxed{}$ for t in the second equation.
$100 = \boxed{}$	Subtract $\boxed{}$ m from each side.
$\boxed{} = \boxed{}\, m$	Subtract $\boxed{}$ from each side.
$\boxed{} = m$	Divide each side by $\boxed{}$.

$t = 30 + 70m$ Use one of the original equations.

$t = 30 + 70(\square)$ Substitute $\square$ for m.

$t = \square$

$(\square, \square)$ Write the sulotion as an ordered pair.

Both Internet providers will cost \$______ after ______ months.

Reflect

9. If the variables in a real-world situation represent the number of months and cost, why must the values of the variables be greater than or equal to zero?

Your Turn

10. A boat travels at a rate of 18 kilometers per hour from its port. A second boat is 34 kilometers behind the first boat when it starts traveling in the same direction at a rate of 22 kilometers per hour to the same port. Let d represent the distance the boats are from the port in kilometers and t represent the amount of time in hours. The system of equations $\begin{cases} d = 18t + 34 \\ d = 22t \end{cases}$ can be used to represent this situation. How many hours will it take for the second boat to catch up to the first boat? How far will the boats be from their port? Use the substitution method to solve this real-world application.

Elaborate

11. When given a system of linear equations, how do you decide which variable to solve for first?

12. How can you check a solution for a system of equations without graphing?

13. **Essential Question-Check-In** Explain how you can solve a system of linear equations by substitution.

Evaluate: Homework and Practice

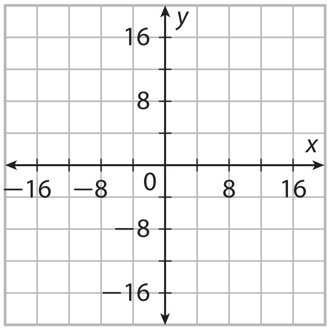
Personal Math Trainer
• Online Homework
• Hints and Help
• Extra Practice

1. In the system of linear equations shown, the value of y is given. Use this value of y to find the value of x and the solution of the system.

$$\begin{cases} y = 12 \\ 2x - y = 4 \end{cases}$$

a. What is the solution of the system?

b. Graph the system of linear equations. How do the solutions compare?



Solve each system of linear equations by substitution.

2. $\begin{cases} 5x + y = 8 \\ 2x + y = 5 \end{cases}$

3. $\begin{cases} x - 3y = 10 \\ x + 5y = -22 \end{cases}$

4. $\begin{cases} 5x - 3y = 22 \\ -4x + y = -19 \end{cases}$

5. $\begin{cases} x + 7y = -11 \\ -2x - 5y = 4 \end{cases}$

6. $\begin{cases} 2x + 6y = 16 \\ 3x - 5y = -18 \end{cases}$

7. $\begin{cases} 7x + 2y = 24 \\ -6x + 3y = 3 \end{cases}$

Solve each system of linear equations by substitution.

8. $\begin{cases} x + y = 3 \\ -4x - 4y = 12 \end{cases}$

9. $\begin{cases} 3x - 3y = -15 \\ -x + y = 5 \end{cases}$

10. $\begin{cases} x - 8y = 17 \\ -3x + 24y = -51 \end{cases}$

11. $\begin{cases} 5x - y = 18 \\ 10x - 2y = 32 \end{cases}$

12. $\begin{cases} -2x - 3y = 12 \\ -4x - 6y = 24 \end{cases}$

13. $\begin{cases} 3x + 4y = 36 \\ 6x + 8y = 48 \end{cases}$

Solve each real-world situation by using the substitution method.

14. The number of DVDs sold at a store in a month was 920 and the number of DVDs sold decreased by 12 per month. The number of Blu-ray discs sold in the same store in the same month was 502 and the number of Blu-ray discs sold increased by 26 per month. Let *d* represent the number of discs sold and *t* represent the time in months.

The system of equations $\begin{cases} d = 920 - 12t \\ d = 502 + 26t \end{cases}$ can be used to represent this situation. If this trend continues, in how many months will the number of DVDs sold equal the number of Blu-ray discs sold? How many of each is sold in that month?

15. One smartphone plan costs \$30 per month for talk and messaging and \$8 per gigabyte of data used each month. A second smartphone plan costs \$60 per month for talk and messaging and \$3 per gigabyte of data used each month. Let *c* represent the total cost in dollars and *d* represent the amount of data used in gigabytes. The system of equations $\begin{cases} c = 30 + 8d \\ c = 60 + 3d \end{cases}$ can be used to represent this situation. How many gigabytes would have to be used for the plans to cost the same? What would that cost be?

16. A movie theater sells popcorn and fountain drinks. Brett buys 1 popcorn bucket and 3 fountain drinks for his family, and pays a total of \$9.50. Sarah buys 3 popcorn buckets and 4 fountain drinks for her family, and pays a total of \$19.75. If p represents the number of popcorn buckets and d represents the number of drinks, then the system of equations $\begin{cases} 9.50 = p + 3d \\ 19.75 = 3p + 4d \end{cases}$ can be used to represent this situation. Find the cost of a popcorn bucket and the cost of a fountain drink.

17. Jen is riding her bicycle on a trail at the rate of 0.3 kilometer per minute. Michelle is 11.2 kilometers behind Jen when she starts traveling on the same trail at a rate of 0.44 kilometer per minute. Let d represent the distance in kilometers the bicyclists are from the start of the trail and t represent the time in minutes. The system of equations $\begin{cases} d = 0.3t + 11.2 \\ d = 0.44t \end{cases}$ can be used to represent this situation. How many minutes will it take Michelle to catch up to Jen? How far will they be from the start of the trail? Use the substitution method to solve this real-world application.

18. **Geometry** The length of a rectangular room is 5 feet more than its width. The perimeter of the room is 66 feet. Let L represent the length of the room and W represent the width in feet. The system of equations $\begin{cases} L = W + 5 \\ 66 = 2L + 2W \end{cases}$ can be used to represent this situation. What are the room's dimensions?

19. A cable television provider has a \$55 setup fee and charges \$82 per month, while a satellite television provider has a \$160 setup fee and charges \$67 per month. Let c represent the total cost in dollars and t represent the amount of time in months. The system of equations $\begin{cases} c = 55 + 82t \\ c = 160 + 67t \end{cases}$ can be used to represent this situation.

a. In how many months will both providers cost the same? What will that cost be?

b. If you plan to move in 12 months, which provider would be less expensive? Explain.

20. Determine whether each of the following systems of equations have one solution, infinitely many solutions, or no solution. Select the correct answer for each lettered part.

a. $\begin{cases} x + y = 5 \\ -6y - 6y = 30 \end{cases}$

b. $\begin{cases} x + y = 7 \\ 5x + 2y = 23 \end{cases}$

c. $\begin{cases} 3x + y = 5 \\ 6x + 2y = 12 \end{cases}$

d. $\begin{cases} 2x + 5y = -12 \\ x + 7y = -15 \end{cases}$

e. $\begin{cases} 3x + 5y = 17 \\ -6x - 10y = -34 \end{cases}$

21. Finance Adrienne invested a total of $1900 in two simple-interest money market accounts. Account A paid 3% annual interest and account B paid 5% annual interest. The total amount of interest she earned after one year was $83. If *a* represents the amount invested in dollars in account A and *b* represents the amount invested in dollars in account B, the system of equations $\begin{cases} a + b = 1900 \\ 0.03a + 0.05b = 83 \end{cases}$ can represent this situation. How much did Adrienne invest in each account?

H.O.T. Focus on Higher Order Thinking

22. Real-World Application The Sullivans are deciding between two landscaping companies. Evergreen charges a $79 startup fee and $39 per month. Eco Solutions charges a $25 startup fee and $45 per month. Let *c* represent the total cost in dollars and *t* represent the time in months. The system of equations $\begin{cases} c = 39t + 79 \\ c = 45t + 25 \end{cases}$ can be used to represent this situation.

a. In how many months will both landscaping services cost the same? What will that cost be?

b. Which landscaping service will be less expensive in the long term? Explain.

23. **Multiple Representations** For the first equation in the system of linear equations below, write an equivalent equation without denominators. Then solve the system.

$$\begin{cases} \dfrac{x}{5} + \dfrac{y}{3} = 6 \\ x - 2y = 8 \end{cases}$$

24. **Conjecture** Is it possible for a system of three linear equations to have one solution? If so, give an example.

25. **Conjecture** Is it possible to use substitution to solve a system of linear equations if one equation represents a horizontal line and the other equation represents a vertical line? Explain.

Lesson Performance Task

A company breaks even from the production and sale of a product if the total revenue equals the total cost. Suppose an electronics company is considering producing two types of smartphones. To produce smartphone A, the initial cost is \$20,000 and each phone costs \$150 to produce. The company will sell smartphone A at \$200. Let $C(a)$ represent the total cost in dollars of producing a units of smartphone A. Let $R(a)$ represent the total revenue, or money the company takes in due to selling a units of smartphone A. The system of equations $\begin{cases} C(a) = 20{,}000 + 150a \\ R(a) = 200a \end{cases}$ can be used to represent the situation for phone A.

To produce smartphone B, the initial cost is \$44,000 and each phone costs \$200 to produce. The company will sell smartphone B at \$280. Let $C(b)$ represent the total cost in dollars of producing b units of smartphone B and $R(b)$ represent the total revenue from selling b units of smartphone B. The system of equations $\begin{cases} C(b) = 44{,}000 + 200b \\ R(b) = 280b \end{cases}$ can be used to represent the situation for phone B.

Solve each system of equations and interpret the solutions. Then determine whether the company should invest in producing smartphone A or smartphone B. Justify your answer.