

Sequences of Transformations

Common Core Math Standards

The student is expected to:

COMMON CORE **G-CO.A.5**

... Specify a sequence of transformations that will carry a given figure onto another. Also G-CO.A.2, G-CO.B.6

Mathematical Practices

COMMON CORE **MP.5 Using Tools**

Language Objective

Explain to a partner why a transformation or sequence of transformations is rigid or nonrigid.

ENGAGE

Essential Question: What happens when you apply more than one transformation to a figure?

Possible answer: The transformations occur sequentially, and order matters. The result may be the same as a single transformation.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the photo and ask students to describe the snowflake in general terms, such as “It has six arms that look alike.” Then preview the Lesson Performance Task.

Name _____ Class _____ Date _____

3.1 Sequences of Transformations

Essential Question: What happens when you apply more than one transformation to a figure?



Resource Locker

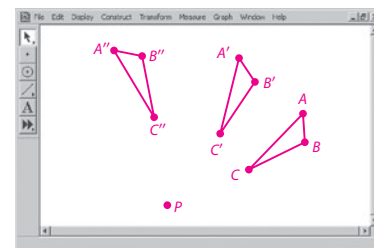
Explore Combining Rotations or Reflections

A transformation is a function that takes points on the plane and maps them to other points on the plane. Transformations can be applied one after the other in a sequence where you use the image of the first transformation as the preimage for the next transformation.

Find the image for each sequence of transformations.

- A** Using geometry software, draw a triangle and label the vertices A , B , and C . Then draw a point outside the triangle and label it P .

Rotate $\triangle ABC$ 30° around point P and label the image as $\triangle A'B'C'$. Then rotate $\triangle A'B'C'$ 45° around point P and label the image as $\triangle A''B''C''$. Sketch your result.

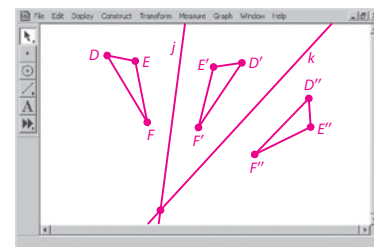


- B** Make a conjecture regarding a single rotation that will map $\triangle ABC$ to $\triangle A''B''C''$. Check your conjecture, and describe what you did.

A rotation of 75° (because $30 + 45 = 75$) should map $\triangle ABC$ to $\triangle A''B''C''$. By using the software to rotate $\triangle ABC$ 75° , I can see that this image coincides with $\triangle A''B''C''$.

- C** Using geometry software, draw a triangle and label the vertices D , E , and F . Then draw two intersecting lines and label them j and k .

Reflect $\triangle DEF$ across line j and label the image as $\triangle D'E'F'$. Then reflect $\triangle D'E'F'$ across line k and label the image as $\triangle D''E''F''$. Sketch your result.



- D** Consider the relationship between $\triangle DEF$ and $\triangle D''E''F''$. Describe the single transformation that maps $\triangle DEF$ to $\triangle D''E''F''$. How can you check that you are correct?

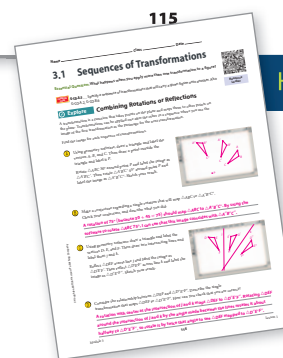
A rotation with center at the intersection of j and k maps $\triangle DEF$ to $\triangle D''E''F''$. Rotating $\triangle DEF$ around the intersection of j and k by the angle made between the lines rotates it about halfway to $\triangle D''E''F''$, so rotate it by twice that angle to see $\triangle DEF$ mapped to $\triangle D''E''F''$.

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Lesson 1



HARDCOVER PAGES 103–112

Turn to these pages to find this lesson in the hardcover student edition.

Reflect

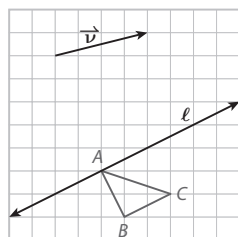
- Repeat Step A using other angle measures. Make a conjecture about what single transformation will describe a sequence of two rotations about the same center.
If a figure is rotated and then the image is rotated about the same center, a single rotation by the sum of the angles of rotation will have the same result.
- Make a conjecture about what single transformation will describe a sequence of three rotations about the same center.
A sequence of three rotations about the same center can be described by a single rotation by the sum of the angles of rotation.
- Discussion** Repeat Step C, but make lines j and k parallel instead of intersecting. Make a conjecture about what single transformation will now map $\triangle DEF$ to $\triangle D''E''F''$. Check your conjecture and describe what you did.
 $\triangle D''E''F''$ looks like a translation of $\triangle DEF$. I marked a vector from D to D'' and translated $\triangle DEF$ by it. The image coincides with $\triangle D''E''F''$, so two reflections in $\parallel$ lines result in a translation.

Explain 1 Combining Rigid Transformations

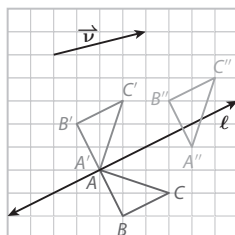
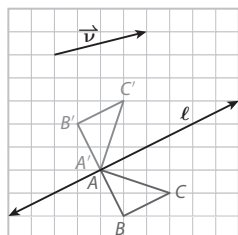
In the Explore, you saw that sometimes you can use a single transformation to describe the result of applying a sequence of two transformations. Now you will apply sequences of rigid transformations that cannot be described by a single transformation.

Example 1 Draw the image of $\triangle ABC$ after the given combination of transformations.

- A** Reflection over line ℓ then translation along $\vec{v}$



- Step 1** Draw the image of $\triangle ABC$ after a reflection across line ℓ . Label the image $\triangle A'B'C'$. **Step 2** Translate $\triangle A'B'C'$ along $\vec{v}$. Label this image $\triangle A''B''C''$.



EXPLORE

Combining Reflections

INTEGRATE TECHNOLOGY

Students have the option of completing the combining reflections activity either in the book or online.

QUESTIONING STRATEGIES

- ?** How can you use geometry software to check your transformations? **For reflections in parallel lines, use the measuring features to see if all points move the same distance in the same direction. For reflections in intersecting lines, rotate the preimage figure to see if the images are the same size and shape.**

EXPLAIN 1

Combining Rigid Transformations

AVOID COMMON ERRORS

Some students may transform the original figure twice instead of transforming the first image to get the second, and the second to get the third. Note that when performing two transformations with $A \rightarrow A'$ as the first transformation, A is the preimage and A' is the image. In the second transformation $A' \rightarrow A''$, A' is the preimage and A'' is the image.

PROFESSIONAL DEVELOPMENT

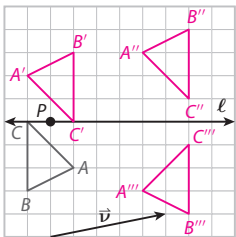
Math Background

Students have worked with individual transformations and should now be able to identify and describe translations, reflections, and rotations. In this lesson, they combine two or more of these transformations and may include sequences of nonrigid transformations. They must be able to visualize and predict the outcome of performing more than one transformation, as well as consider other transformations that produce the same final image. Throughout the lesson they must recall the properties of each transformation and the methods for drawing them.

QUESTIONING STRATEGIES

? After a rigid motion, an image has the same shape and size as the preimage. If you perform a sequence of rigid motions, will the final image have the same shape and size as the original? **Yes; each rigid motion preserves size and shape, so a sequence of rigid motions will also preserve size and shape.**

- B** 180° rotation around point P , then translation along $\vec{v}$, then reflection across line ℓ
- Apply the rotation. Label the image $\triangle A'B'C'$.
- Apply the translation to $\triangle A'B'C'$. Label the image $\triangle A''B''C''$.
- Apply the reflection to $\triangle A''B''C''$. Label the image $\triangle A'''B'''C'''$.



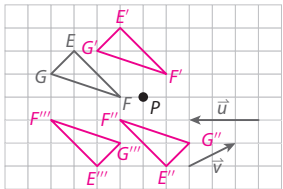
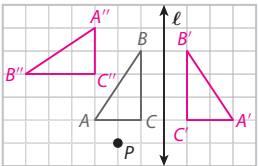
Reflect

- 4.** Are the images you drew for each example the same size and shape as the given preimage? In what ways do rigid transformations change the preimage?
Yes. Rigid transformations move the figure in the plane and may change the orientation, but they do not change the size or shape.
- 5.** Does the order in which you apply the transformations make a difference? Test your conjecture by performing the transformations in Part B in a different order.
Possible answer: Yes, if I reflect first, then rotate, and then translate, the final image is above line ℓ instead of below it.
- 6.** For Part B, describe a sequence of transformations that will take $\triangle A''B''C''$ back to the preimage.
Possible answer: In this case, reversing the order of the transformations will take the final image back to the preimage.

Your Turn

Draw the image of the triangle after the given combination of transformations.

- 7.** Reflection across ℓ then 90° rotation around point P
- 8.** Translation along $\vec{v}$ then 180° rotation around point P then translation along $\vec{u}$



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COLLABORATIVE LEARNING

Small Group Activity

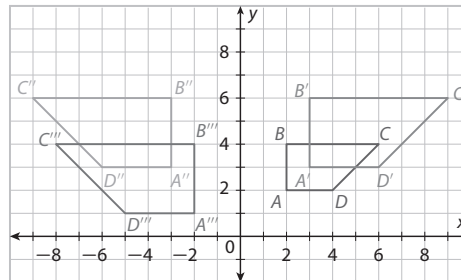
Geometry software allows students to focus on their predictions rather than on drawing multiple transformations. Give students the coordinates of a figure and a series of transformations. Instruct them to plot the points on graph paper and to sketch a prediction of the final image. Then have them use geometry software to perform the transformations and check the results against their predictions. After students have done this for several figures, ask them to brainstorm ways to make their predictions more accurate.

Explain 2 Combining Nonrigid Transformations

Example 2 Draw the image of the figure in the plane after the given combination of transformations.

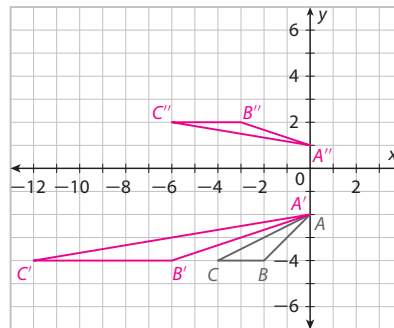
A $(x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right) \rightarrow (-x, y) \rightarrow (x + 1, y - 2)$

1. The first transformation is a dilation by a factor of $\frac{3}{2}$. Apply the dilation. Label the image $A'B'C'D'$.
2. Apply the reflection of $A'B'C'D'$ across the y -axis. Label this image $A''B''C''D''$.
3. Apply the translation of $A''B''C''D''$. Label this image $A'''B'''C'''D'''$.



B $(x, y) \rightarrow (3x, y) \rightarrow \left(\frac{1}{2}x, -\frac{1}{2}y\right)$

1. The first transformation is a [horizontal/vertical] stretch by a factor of 3.
Apply the stretch. Label the image $\triangle A'B'C'$.
2. The second transformation is a dilation by a factor of $\frac{1}{2}$ combined with a reflection.
Apply the transformation to $\triangle A'B'C'$. Label the image $\triangle A''B''C''$.



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Reflect

9. If you dilated a figure by a factor of 2, what transformation could you use to return the figure back to its preimage? If you dilated a figure by a factor of 2 and then translated it right 2 units, write a sequence of transformations to return the figure back to its preimage.
Dilate the figure by a factor of $\frac{1}{2}$. Possible answer: You could dilate the figure by a factor of $\frac{1}{2}$ then translate the figure left 2 units.
10. A student is asked to reflect a figure across the y -axis and then vertically stretch the figure by a factor of 2. Describe the effect on the coordinates. Then write one transformation using coordinate notation that combines these two transformations into one.
The x -coordinates change to their opposites. The y -coordinates are multiplied by a factor of 2. $(x, y) \rightarrow (-x, 2y)$

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Lesson 1

DIFFERENTIATE INSTRUCTION

Multiple Representations

Have students graph any three points on a coordinate plane and connect them to form a triangle. Ask students to perform two transformations on this triangle. Then instruct them to use the algebra rules to perform the same transformations. Students should compare the coordinates they found algebraically with those they found with the physical transformation. Then have them study the preimage and the final image to decide whether they could have used one transformation to obtain the same result. If so, ask them to use the algebraic rules to show that the single transformation is equivalent to the two original transformations.

EXPLAIN 2

Combining Nonrigid Transformations

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Relate *nonrigid transformation* to *rigid transformation* by comparing the size and shape of the original and image figures. Point out that a dilation preserves the shape but not the size of a figure, while a horizontal or vertical stretch does not preserve either the size or the shape of a figure.

QUESTIONING STRATEGIES

? How would you describe the image of a figure after a sequence of nonrigid transformations? **Either the size or the shape of the original figure changed, although it is possible that a subsequent transformation results in a figure of the original size and shape.**

? If you perform a sequence of nonrigid motions on a polygon, will the type of polygon change? Explain. **No. The polygon will have the same number of vertices, so it will be the same general polygon. If the original figure is regular, the nonrigid motions may give an image of a non-regular polygon. The image of a square may be a parallelogram, for example.**

CONNECT VOCABULARY **EL**

The word *rigid* derives from *rigidus*, the Latin word for *stiff*. Help students understand how *nonrigid transformation* is used to represent a type of transformation that gives an image that is a different size and/or shape of a preimage figure. Point out that the transformation can be in the plane or in the coordinate plane, and that a nonrigid transformation can be included in any sequence of combined transformations.

EXPLAIN 3

Predicting the Effect of Transformations

INTEGRATE MATHEMATICAL PRACTICES

Focus on Patterns

MP.8 Encourage students to predict the effect of transformations and then actually perform the transformations described in the example to verify their predictions. Have students repeat the same sequence of transformations using a different figure as the original figure. Ask whether the sequence of transformations affects the new figure in the same way.

QUESTIONING STRATEGIES

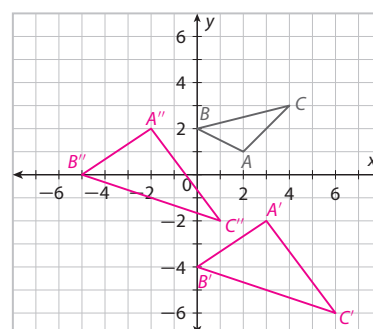
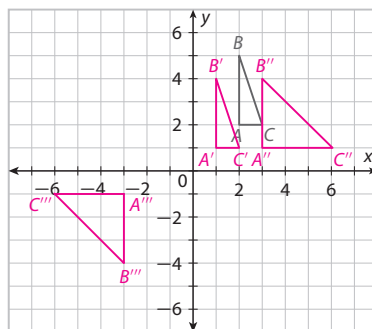


Why is it important to carefully label the vertices after each transformation? **Labeling the vertices will help distinguish the types of rigid and nonrigid transformations used in the sequence of transformations. Mislabeling a transformation in the sequence will likely result in an incorrect final image.**

Your Turn

Draw the image of the figure in the plane after the given combination of transformations.

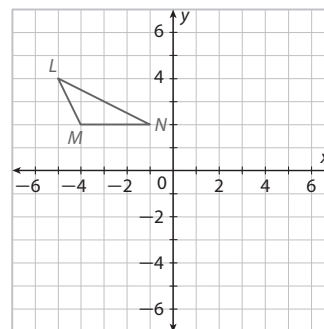
11. $(x, y) \rightarrow (x - 1, y - 1) \rightarrow (3x, y) \rightarrow (-x, -y)$ 12. $(x, y) \rightarrow \left(\frac{3}{2}x, -2y\right) \rightarrow (x - 5, y + 4)$



Explain 3 Predicting the Effect of Transformations

Example 3 Predict the result of applying the sequence of transformations to the given figure.

- A $\triangle LMN$ is translated along the vector $\langle -2, 3 \rangle$, reflected across the y -axis, and then reflected across the x -axis.



Predict the effect of the first transformation: A translation along the vector $\langle -2, 3 \rangle$ will move the figure left 2 units and up 3 units. Since the given triangle is in Quadrant II, the translation will move it further from the x - and y -axes. It will remain in Quadrant II.

Predict the effect of the second transformation: Since the triangle is in Quadrant II, a reflection across the y -axis will change the orientation and move the triangle into Quadrant I.

Predict the effect of the third transformation: A reflection across the x -axis will again change the orientation and move the triangle into Quadrant IV. The two reflections are the equivalent of rotating the figure 180° about the origin.

The final result will be a triangle the same shape and size as $\triangle LMN$ in Quadrant IV. It has been rotated 180° about the origin and is farther from the axes than the preimage.

LANGUAGE SUPPORT EL

Communicate Math

Have students work in pairs. Have the first student show the partner a graph of a preimage and transformed image and ask whether it is an example of a rigid or nonrigid transformation. The second student should describe the transformation and tell whether it is rigid or nonrigid, and why. The first student writes the explanation under the images. Students change roles and repeat the sequence with another set of images.

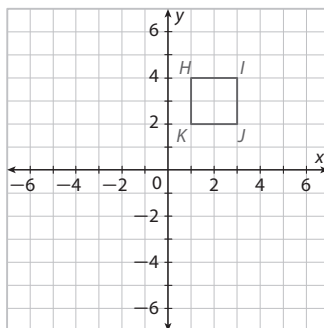
- B** Square $HIJK$ is rotated 90° clockwise about the origin and then dilated by a factor of 2, which maps $(x, y) \rightarrow (2x, 2y)$.

Predict the effect of the first transformation: **A 90° clockwise**

rotation will map it to Quadrant IV. Due to its symmetry, it will appear to have been translated, but will be closer to the x -axis than it is to the y -axis.

Predict the effect of the second transformation: **A dilation by a factor of 2 will double the side lengths of the square. It will also be further from the origin than the preimage.**

The final result will be **a square in Quadrant 4 with side lengths twice as long as the side lengths of the original. The image is further from the origin than the preimage.**

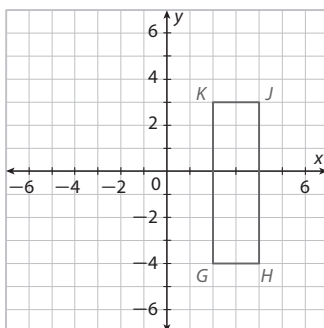


Your Turn

Predict the result of applying the sequence of transformations to the given figure.

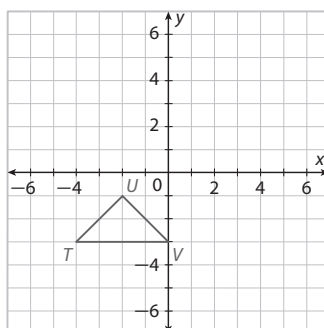
- 13.** Rectangle $GHJK$ is reflected across the y -axis and translated along the vector $\langle 5, 4 \rangle$.

The reflection across the y -axis will move the rectangle from the right of the y -axis to the left of it. Due to the symmetry of the rectangle, it will appear to have been translated left 6 units. Then, translating along the vector $\langle 5, 4 \rangle$ will move the rectangle right 5 units and up 4 units. This will bring the rectangle fully into Quadrant I. The final result will be a rectangle that is the same shape and size as the preimage that has moved to sit on the x -axis in Quadrant I, closer to the y -axis than the preimage.



- 14.** $\triangle TUV$ is horizontally stretched by a factor of $\frac{3}{2}$, which maps $(x, y) \rightarrow (\frac{3}{2}x, y)$, and then translated along the vector $\langle 2, 1 \rangle$.

A horizontal stretch will pull points U and T away from the y -axis, making the triangle longer in the left-to-right direction. The translation along the vector $\langle 2, 1 \rangle$ will move the stretched triangle 2 units right and 1 unit up, which will move the triangle closer to the origin with one vertex on the x -axis and another across the y -axis. The final image will not be the same shape or size as the preimage.



ELABORATE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Patterns

MP.8 Discuss with students how to record and use algebraic patterns to represent a series of rigid and nonrigid motions in the coordinate plane. For example, a reflection in the x -axis is represented by $(x, y) \rightarrow (x, -y)$, while a reflection in the y -axis is represented by $(x, y) \rightarrow (-x, y)$. So, a reflection in the x -axis followed by a reflection in the y -axis is represented by $(x, y) \rightarrow (-x, -y)$.

QUESTIONING STRATEGIES

? Is a transformation from a sequence of rigid motions always rigid? **Yes. Each rigid motion preserves the size and shape of a figure, so the final image must have the same size and shape as the original figure.**

? What types of sequences of transformations can be undone? **A sequence of transformations using the same rigid motion can sometimes be undone by reversing the order of the sequence. It is possible that a sequence using different rigid motions or nonrigid motions cannot be undone directly, but students may be able to write a series of related transformations to undo the transformations.**

SUMMARIZE THE LESSON

? What features can you describe when predicting the result of more than one transformation? **Sample answer: You can predict which quadrant(s) the final image will be in, how far and in what direction it will be from the origin or from the original figure, its orientation, and whether the size or shape of the figure has changed.**

Elaborate

- 15. Discussion** How many different sequences of rigid transformations do you think you can find to take a preimage back onto itself? Explain your reasoning.

An infinite number. With rotations you just need to go 360° and you will be back where you started, and you can do that as many times as you want. You can always reflect back over a line. You can always go back left just as far as you went right, or up as many times as you went down in a translation, so you can take a preimage back onto itself in many ways. You can add extra transformations to find additional sequences.

- 16.** Is there a sequence of a rotation and a dilation that will result in an image that is the same size and position as the preimage? Explain your reasoning.

Yes, a rotation of 360° and a dilation of 1 will work.

- 17. Essential Question Check-In** In a sequence of transformations, the order of the transformations can affect the final image. Describe a sequence of transformations where the order does not matter. Describe a sequence of transformations where the order does matter.

Possible answer: A sequence of any number of rotations about the same point can be added together to make one rotation, even if they are a combination of clockwise and counterclockwise rotations. Any sequence of translations can also be done in any order. When a sequence includes a mix of different types of transformations, the order usually affects the final image, for example a rotation of 90° around a vertex followed by a dilation by a factor of 4 will have a different final image than the same figure dilated by a factor of 4 followed by a rotation of 90° .

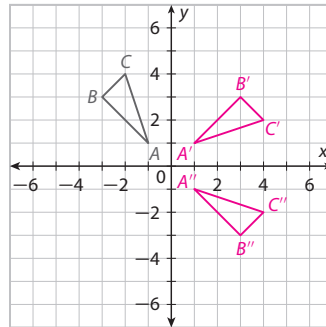
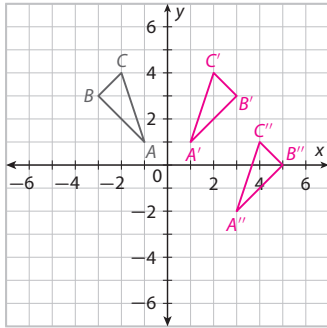
Evaluate: Homework and Practice



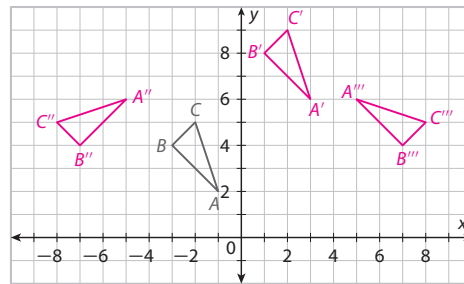
- Online Homework
- Hints and Help
- Extra Practice

Draw and label the final image of $\triangle ABC$ after the given sequence of transformations.

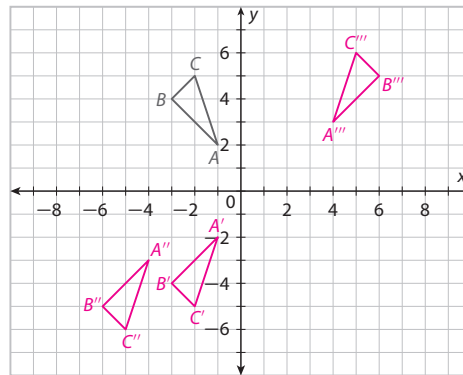
1. Reflect $\triangle ABC$ over the y -axis and then translate by $\langle 2, -3 \rangle$.
2. Rotate $\triangle ABC$ 90 degrees clockwise about the origin and then reflect over the x -axis.



3. Translate $\triangle ABC$ by $\langle 4, 4 \rangle$, rotate 90 degrees counterclockwise around A, and reflect over the y -axis.



4. Reflect $\triangle ABC$ over the x -axis, translate by $\langle -3, -1 \rangle$, and rotate 180 degrees around the origin.



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Lesson 1

EVALUATE



ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Combining Rotations or Reflections	
Example 1 Combining Rigid Transformations	Exercises 1–4
Example 2 Combining Nonrigid Transformations	Exercises 5–6
Example 3 Predicting the Effect of Transformations	Exercises 7–12

INTEGRATE MATHEMATICAL PRACTICES

Focus on Technology

MP.5 Students can use geometry software to do a sequence of transformations or to check a sequence of transformations. Remind students to use the measuring features to verify that a sequence of rigid motions preserves the size and shape of a figure.

Exercise Depth of Knowledge (D.O.K.) COMMON CORE Mathematical Practices

1–8	2 Skills/Concepts	MP.6 Precision
9–13	2 Skills/Concepts	MP.2 Reasoning
14	1 Recall of Information	MP.2 Reasoning
15–18	2 Skills/Concepts	MP.2 Reasoning
19	2 Skills/Concepts	MP.2 Reasoning

GRAPHIC ORGANIZERS

Suggest that students use a graphic organizer to keep track of the types of transformations and their properties in a sequence of transformations. This can help them remember to use the last image figure as they proceed with the sequence of transformations. For example:

Transformation 1	
Property 1	Property 2
↓	
Transformation 2	
Property 1	Property 2
↓	
Transformation 3	
Property 1	Property 2

AVOID COMMON ERRORS

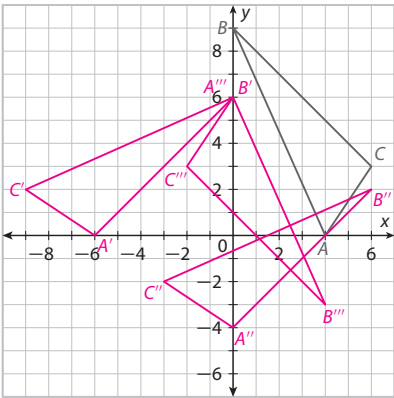
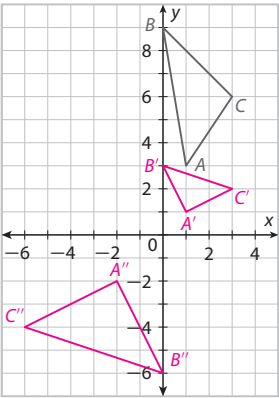
Some students may perform a combination of transformations in the wrong order. Emphasize the importance of doing the transformations in the correct order by asking them to rotate a triangle 90° in the plane and then reflect it in the *x*-axis. They will get a different result if the order is reversed.

SMALL GROUP ACTIVITY

Have students work in small groups to make a poster showing how to find a sequence of transformations in the coordinate plane using both rigid and nonrigid motions. Give each group a different sequence to transform. Then have each group present its poster to the rest of the class, explaining each step.

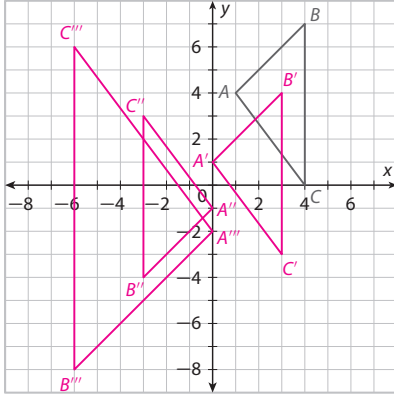
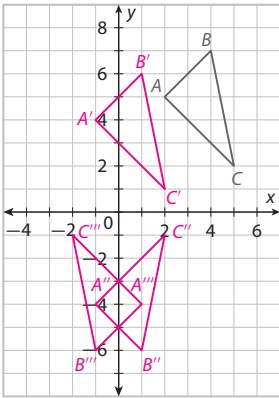
Draw and label the final image of $\triangle ABC$ after the given sequence of transformations.

5. $(x, y) \rightarrow \left(x, \frac{1}{3}y\right) \rightarrow (-2x, -2y)$
6. $(x, y) \rightarrow \left(-\frac{3}{2}x, \frac{2}{3}y\right) \rightarrow (x + 6, y - 4) \rightarrow \left(\frac{2}{3}x, -\frac{3}{2}y\right)$



Predict the result of applying the sequence of transformations to the given figure.

7. $\triangle ABC$ is translated along the vector $\langle -3, -1 \rangle$, reflected across the *x*-axis, and then reflected across the *y*-axis.
8. $\triangle ABC$ is translated along the vector $\langle -1, -3 \rangle$, rotated 180° about the origin, and then dilated by a factor of 2.



Possible answer: The translation moves the figure down one unit and left three units, mapping *A'* to the left of the *y*-axis and *C'* closer to the origin. The reflection first will map *A''B''C''* below the *x*-axis and change the orientation. The second reflection will map the figure mostly into Quadrant III, with *A'''* in Quadrant IV, and again change the orientation. The final image is the same size, shape, and orientation as the preimage.

Possible answer: The translation moves the figure down and to the left without changing the shape or orientation. The rotation about the origin moves the figure from Quadrants I and IV to Quadrants II and III without changing the orientation. The dilation doubles the side lengths. The final image is the same shape as the preimage but larger. It has the same orientation.

Module 3

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Lesson 1

Exercise Depth of Knowledge (D.O.K.) COMMON CORE Mathematical Practices

20 3 Strategic Thinking

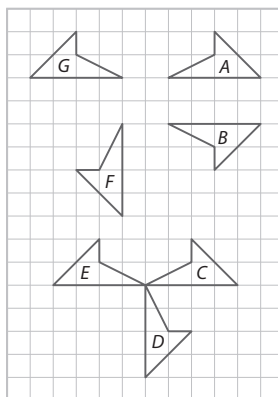
MP.6 Precision

21–23 2 Skills/Concepts

MP.2 Reasoning

In Exercises 9–12, use the diagram. Fill in the blank with the letter of the correct image described.

9. B is the result of the sequence: G reflected over a vertical line and then a horizontal line.
10. E is the result of the sequence: D rotated 90° clockwise around one of its vertices and then reflected over a horizontal line.
11. F is the result of the sequence: E translated and then rotated 90° counterclockwise.
12. A is the result of the sequence: D rotated 90° counterclockwise and then translated.

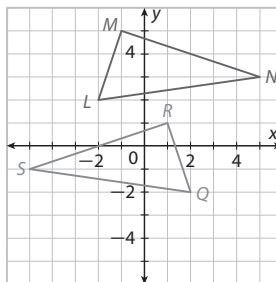


Choose the correct word to complete a true statement.

13. A combination of two rigid transformations on a preimage will always sometimes never produce the same image when taken in a different order.
14. A double rotation can always sometimes/never be written as a single rotation.
15. A sequence of a translation and a reflection always sometimes never has a point that does not change position.
16. A sequence of a reflection across the x -axis and then a reflection across the y -axis always sometimes/never results in a 180° rotation of the preimage.
17. A sequence of rigid transformations will always sometimes never result in an image that is the same size and orientation as the preimage.
18. A sequence of a rotation and a dilation will always sometimes never result in an image that is the same size and orientation as the preimage.

19. $\triangle QRS$ is the image of $\triangle LMN$ under a sequence of transformations. Can each of the following sequences be used to create the image, $\triangle QRS$, from the preimage, $\triangle LMN$? Select yes or no.

- a. Reflect across the y -axis and then translate along the vector $\langle 0, -4 \rangle$. ☒ Yes ☐ No
- b. Translate along the vector $\langle 0, -4 \rangle$ and then reflect across the y -axis. ☒ Yes ☐ No
- c. Rotate 90° clockwise about the origin, reflect across the x -axis, and then rotate 90° counterclockwise about the origin. ☐ Yes ☒ No
- d. Rotate 180° about the origin, reflect across the x -axis, and then translate along the vector $\langle 0, -4 \rangle$. ☒ Yes ☐ No



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INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 When writing the algebraic rules for the rigid motions and other transformations, review the quadrants and coordinates. Students should remember that a rotation through a positive angle is in the counterclockwise direction, and a rotation through a negative angle is in the clockwise direction.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Have students use the algebraic representation of a dilation in the coordinate plane as $(x, y) \rightarrow (kx, ky)$, where k is the scale factor. Ask students how they would represent the dilation that would “undo” this dilation. $(x, y) \rightarrow \left(\frac{1}{k}x, \frac{1}{k}y\right)$

PEER-TO-PEER DISCUSSION

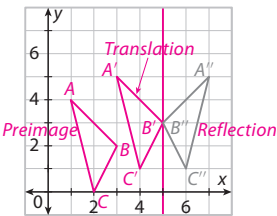
Ask students to discuss with a partner how to predict the final image for a combination of rigid transformations and nonrigid transformations. Then ask students to make a conjecture about the result of dilating a triangle whose vertices have coordinates (1, 1), (1, 4), (5, 1) with a scale factor of 2, followed by a reflection in the x -axis. **The image has vertices (2, -2), (2, -8), (10, -2).**

JOURNAL

Have students compare and contrast the methods they have learned for combining rigid transformations and nonrigid transformations in the coordinate plane.

20. A teacher gave students this puzzle: “I had a triangle with vertex A at (1, 4) and vertex B at (3, 2). After two rigid transformations, I had the image shown. Describe and show a sequence of transformations that will give this image from the preimage.”

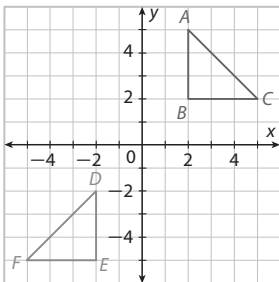
Possible answer: Translate by the vector (2,1) then reflect over the line $x = 5$.



H.O.T. Focus on Higher Order Thinking

21. **Analyze Relationships** What two transformations would you apply to $\triangle ABC$ to get $\triangle DEF$? How could you express these transformations with a single mapping rule in the form of $(x, y) \rightarrow (?, ?)$?

Possible answer: Reflect $\triangle ABC$ across the y -axis and then translate it down 7 units. A single mapping rule would be $(x, y) \rightarrow (-x, y - 7)$.



22. **Multi-Step** Muralists will often make a scale drawing of an art piece before creating the large finished version. A muralist has sketched an art piece on a sheet of paper that is 3 feet by 4 feet.

- a. If the final mural will be 39 feet by 52 feet, what is the scale factor for this dilation?

Scale factor: 13



- b. The owner of the wall has decided to only give permission to paint on the lower half of the wall. Can the muralist simply use the transformation $(x, y) \rightarrow (x, \frac{1}{2}y)$ in addition to the scale factor to alter the sketch for use in the allowed space? Explain.

Only if the artist wants the final version of the mural to be distorted. This mapping will shrink the height of the mural in half, but by keeping the original width, the shapes will change.

23. **Communicate Mathematical Ideas** As a graded class activity, your teacher asks your class to reflect a triangle across the y -axis and then across the x -axis. Your classmate gets upset because he reversed the order of these reflections and thinks he will have to start over. What can you say to your classmate to help him?

The order of these two reflections does not matter. The resulting image is the same for a reflection in the y -axis followed by a reflection in the x -axis as for a reflection in the x -axis followed by a reflection in the y -axis.

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Lesson Performance Task

The photograph shows an actual snowflake. Draw a detailed sketch of the “arm” of the snowflake located at the top left of the photo (10:00 on a clock face). Describe in as much detail as you can any translations, reflections, or rotations that you see.

Then describe how the entire snowflake is constructed, based on what you found in the design of one arm.

Check students' drawings.



In their descriptions, students should refer to specific features of their drawings. The line dividing the 10:00 arm in half is a line of reflection, with the portion of the flake on each side being (nearly) a reflection of the other side. There's a small imperfection in this description, with the large “ear” in the middle of the right side not quite having a mirror image where it should be. However, its almost-image on the other side can be created by reflecting the ear across the line of symmetry and then translating it slightly downward.

The entire flake can be created by rotating the arm through 60° , 120° , 180° , 240° , and 300° . For several of the new arms, the “ear” mentioned above appears in a slightly dilated form, or it appears several times as translations of one another.

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EXTENSION ACTIVITY

Have students research the claim that “all snowflakes are different.” Depending upon students’ interests, the claim may lead them to investigate how and where snowflakes form, how they change as they fall through the atmosphere, why they have a hexagonal structure, and the effects that temperature and humidity have upon their structure.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 A visual pattern can be described as a form or shape that repeats. Ask students to describe the snowflake in terms of patterns. **Sample answer: Each half of any one of the arms can be taken as a pattern that repeats approximately twelve times in the snowflake design, twice on each arm.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Ask students to write, say, or show sequences of rotations, reflections, and translations, using their hands. For example:

Reflection — both hands facing outward, thumbs nearly together.

Translation — one hand facing forward and one backward in the same orientation.

Rotation — one hand pointing left (thumb up) and one hand pointing right (thumb down), fingers facing each other.

CRITICAL THINKING

Ask students whether a snowflake is a two-dimensional object. Have them consider the effects of the third dimension on lines of symmetry, and how the snowflake appears from a side view rather than a top view.

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.