Data Set 7.2 pt 1

1. The probability model below describes the number of repair calls that an appliance repair shop may receive during an hour.

<table>
<thead>
<tr>
<th>Repair Calls</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.1</td>
<td>0.3</td>
<td>0.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

\[ C \sim \text{df calls} \]

a. How many calls should the shop expect per hour?

\[ E(C) = 0(0.1) + 1(0.3) + 2(0.4) + 3(0.2) = 1.7 \text{ calls} \]

b. What is the standard deviation?

\[ \sigma_c^2 = (0-1.7)^2(0.1) + \ldots + (3-1.7)^2(0.2) \]

\[ \sigma_c = \sqrt{8} = 2.81 \text{ calls} \]

c. Find the mean and standard deviation of the number of repair calls the appliance shop should expect during an 8-hour day.

\[ \mu_T = 1.7 + 1.7 + 1.7 + 1.7 + 1.7 + 1.7 + 1.7 + 1.7 = 13.6 \text{ calls} \]

\[ \sigma_T^2 = (0.9)^2 8 = 6.48 \]

\[ \sigma_T = \sqrt{6.48} \approx 2.55 \text{ calls} \]

2. A commuter must pass through five traffic lights on her way to work and will have to stop at each one that is red. She estimates the probability model for the number of red lights she hits, as shown

\[ X = \text{# of red lights} \]

<table>
<thead>
<tr>
<th>X = # of red</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X=x)</td>
<td>0.05</td>
<td>0.25</td>
<td>0.35</td>
<td>0.15</td>
<td>0.15</td>
<td>0.05</td>
</tr>
</tbody>
</table>

a. How many red lights should she expect to hit each day?

\[ \mu_x = (0 \times 0.05) + \ldots + (5 \times 0.05) = 2.25 \text{ lights} \]

b. What's the standard deviation?

\[ \sigma_x^2 = (0-2.25)^2(0.05) + \ldots + (5-2.25)^2(0.05) \approx 1.5875 \]

\[ \sigma_x = \sqrt{1.5875} \approx 1.26 \text{ lights} \]

c. Find the mean and standard deviation of the number of red lights you can expect to hit during a 5-day work week.

\[ \mu_T = 2.25 + 2.25 + 2.25 + 2.25 + 2.25 = 11.25 \text{ lights} \]

\[ \sigma_T^2 = (1.26)^2(5) \approx 7.938 \]

\[ \sigma_T = 2.82 \]
3. Given the independent random variables with means and standard deviations as shown, find the mean and standard deviation of:
   a. \( 3X \)  
   \[ \mu_{3X} = 3(10) \]  
   \[ \sigma_{3X} = 3(2) = 6 \]  
   b. \( Y + 6 \)  
   \[ \mu_{Y+6} = 20 + 6 \]  
   \[ \sigma_{Y+6} = 5 \]  
   c. \( X + Y \)  
   \[ \mu_{X+Y} = 10 + 20 \]  
   \[ \sigma_{X+Y} = \sqrt{(2^2 + 5^2)} = 5.39 \]  
   d. \( X - Y \)  
   \[ \mu_{X-Y} = 10 - 20 \]  
   \[ \sigma_{X-Y} = \sqrt{(2^2 + 2^2)} = 2.83 \]  
   e. \( X_1 + X_2 \)  
   \[ \mu_{X+X} = 10 + 10 \]  
   \[ \sigma_{X+X} = \sqrt{\frac{(2^2 + 2^2)}{8}} = 2.83 \]  

4. Given the independent random variables with means and standard deviations as shown, find the mean and standard deviation of:
   a. \( X - 20 \)  
   \[ \mu_{X-20} = 80 - 20 \]  
   \[ \sigma_{X-20} = 12 \]  
   b. \( 0.5Y \)  
   \[ \mu_{0.5Y} = 12(0.5) \]  
   \[ \sigma_{0.5Y} = 3(0.5) \]  
   c. \( X + Y \)  
   \[ \mu_{X+Y} = 80 + 12 \]  
   \[ \sigma_{X+Y} = \sqrt{12^2 + 3^2} = \sqrt{153} = 12.37 \]  
   d. \( X - Y \)  
   \[ \mu_{X-Y} = 80 - 12 \]  
   \[ \sigma_{X-Y} = \sqrt{12^2 + 3^2} = \sqrt{153} = 12.37 \]  
   e. \( Y_1 + Y_2 \)  
   \[ \mu_{Y+Y} = 12 + 12 \]  
   \[ \sigma_{Y+Y} = \sqrt{3^2 + 3^2} \]  
   \[ = \sqrt{18} = 4.24 \]
5. Given the independent random variables with means and standard deviations as shown, find the mean and standard deviation of:

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>120</td>
<td>12</td>
</tr>
<tr>
<td>Y</td>
<td>300</td>
<td>16</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{a. } & \quad \mu_{0.8Y} = 0.8(300) = 240 \\
\sigma_{0.8Y} &= 12(1.8) = 21.6
\end{align*}
\]

\[
\begin{align*}
\text{b. } & \quad \mu_{2X-100} = 2(120)-100 = 140 \\
\sigma_{2X-100} &= 2(2) = 4
\end{align*}
\]

\[
\begin{align*}
\text{c. } & \quad \mu_{X+2Y} = 120 + 2(300) = 720 \\
\sigma_{X+2Y} &= \sqrt{12^2 + (32)^2} = 34.18
\end{align*}
\]

\[
\begin{align*}
\text{d. } & \quad \mu_{3X-Y} = 3(120)-300 = 60 \\
\sigma_{3X-Y} &= \sqrt{(36)^2 + (16)^2} = 39.40
\end{align*}
\]

\[
\begin{align*}
\text{e. } & \quad \mu_{Y_1 + Y_2} = 300 + 300 = 600 \\
\sigma_{Y_1 + Y_2} &= \sqrt{(100)^2 + (100)^2} = 22.36
\end{align*}
\]

6. Given the independent random variables with means and standard deviations as shown, find the mean and standard deviation of:

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>80</td>
<td>12</td>
</tr>
<tr>
<td>Y</td>
<td>12</td>
<td>3</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{a. } & \quad \mu_{2Y+20} = 2(120)+20 = 260 \\
\sigma_{2Y+20} &= 2(3) = 6
\end{align*}
\]

\[
\begin{align*}
\text{b. } & \quad \mu_{3X} = 3(80) = 240 \\
\sigma_{3X} &= 3(12) = 36
\end{align*}
\]

\[
\begin{align*}
\text{c. } & \quad \mu_{0.25X+Y} = 0.25(80)+12 = 32 \\
\sigma_{0.25X+Y} &= \sqrt{(3)^2 + (3)^2} = 4.24
\end{align*}
\]

\[
\begin{align*}
\text{d. } & \quad \mu_{X-5Y} = 80-5(12) = 20 \\
\sigma_{X-5Y} &= \sqrt{(12)^2 + (15)^2} = 19.21
\end{align*}
\]

\[
\begin{align*}
\text{e. } & \quad \mu_{X_1 + X_2 + X_3} = 80 + 80 + 80 = 240 \\
\sigma_{X_1 + X_2 + X_3} &= \sqrt{(12)^2 + (3)^2} = 20.78
\end{align*}
\]
7. A grocery supplier believes that in a dozen eggs, the mean number of broken eggs is 0.6 with a standard deviation of 0.5 eggs. You buy three dozen eggs without checking them. \( X \) is the number of broken eggs in a dozen.

a. How many broken eggs do you expect to get?
\[
\mu_{x+x+x} = 3 \cdot \mu_x = 3 \cdot 0.6 = 1.8 \text{ eggs}
\]

b. What's the standard deviation?
\[
\sigma_{x+x+x} = \sqrt{(0.6)^2 \cdot 3} = 0.87 \text{ eggs}
\]

c. What assumptions did you have to make about the eggs in order to answer this question?
The dozens were independent of each other.

8. A company selling vegetable seeds in packets of 20 estimates that the mean number of seeds that will grow is 18, with a standard deviation of 1.2 seeds. You buy 5 different seed packets. \( X \) is the seeds that don't grow.

a. How many bad seeds do you expect to get?
\[
\mu_{x+x+x+x+x} = 5 \cdot \mu_x = 5 \cdot 1.2 = 6 \text{ bad seeds}
\]

b. What's the standard deviation?
\[
\sigma_{x+x+x+x+x} = \sqrt{(1.2)^2 \cdot 5} = 2.68 \text{ bad seeds}
\]

c. What assumptions did you have to make about the seeds?
Packets were independent of each other.

9. A delivery company's trucks occasionally get parking tickets, and based on past experiences, the company plans that the trucks will average 1.3 tickets a month, with a standard deviation of 0.7 tickets.

a. If they have 18 trucks, what are the mean and standard deviation of the total number of parking tickets the company will have to pay this month?
\[
\mu_{x+x+x+x+...+x} = 18 \cdot 1.3 = 23.4 \text{ tickets}
\]
\[
\sigma_{x+x+x+x+...+x} = \sqrt{(0.7)^2 \cdot 18} = 2.97 \text{ tickets}
\]

b. What assumption did you make in answering?
The trucks are independent of each other.
10. Organizers of a televised fundraiser know from past experience that most people donate small amounts ($10-$25), some donate larger amounts ($50-$100), and a few people make very generous donations of $250, $500, or more. Historically, pledges average about $32 with a standard deviation of $54.
   a. If 120 people call in pledges, what are the mean and standard deviation of the total amount raised?
   \[ \mu_{x+x} = 620 \times 120 \quad \sigma_{x+x} = \sqrt{(54)^2 \times 120} \]
   \[ = 93840 \quad = 591.54 \]
   b. What assumption did you make in answering this question?
   The pledges are independent of each other.

11. An insurance company estimates that it should make an annual profit of $150 on each homeowner’s policy written, with a standard deviation of $6000.
   a. Why is the standard deviation so large? Because usually they have to pay out very little, but when they have a large payment, it is usually very large.
   b. If it writes only two of these policies, what are the mean and standard deviation of the annual profit?
   \[ \mu_{x+x} = 150 \times 2 \quad \sigma_{x+x} = \sqrt{(6000)^2 \times 2} \]
   \[ = 300 \quad = 8485.28 \]
   c. If it writes 10,000 of these policies, what are the mean and standard deviation of the annual profit?
   \[ \mu_{x+x} = (150 \times 10,000) \quad \sigma_{x+x} = \sqrt{(6000)^2 \times 10,000} \]
   \[ = 15,000,000 \quad = 600,000 \]
   d. Is the company likely to be profitable? Explain.
   The more policies they sell, the more likely they are to make a profit.
   e. What assumptions underlie your analysis?
   The policies are independent of each other.
12. A casino knows that people play the slot machine in hopes of hitting the jackpot but that most of them lose their dollar. Suppose a certain machine pays out an average of $0.92, with a standard deviation of $120.

   a. Why is the standard deviation so large? When there is a large payout, it is large.

   b. If you play 5 times, what are the mean and standard deviation of the casino's profit?

   \[ \mu_{x+x+x+x+x} = (0.08 \times 5) \]
   \[ \sigma_{x+x+x+x+x} = \sqrt{120^2 / (5)} \]
   \[ = 268.33 \]

   c. If the gamblers play the machine 1000 times in a day, what are the mean and standard deviation of the casino's profit?

   \[ \mu_{x+x+\ldots} = (0.08 \times 1000) \]
   \[ \sigma_{x+x+\ldots} = \sqrt{(120^2 \times 1000)} \]
   \[ = 3794.73 \]

   d. Is the casino likely to be profitable? Explain.

   The more people play, the more profitable.