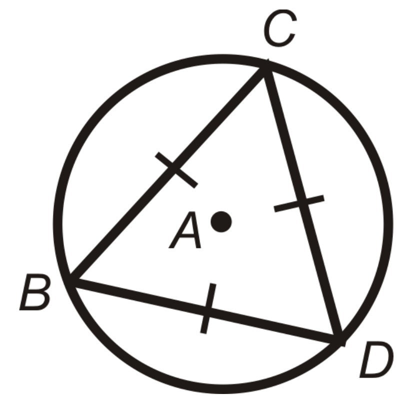
Lesson 3: Inscribed Angles

Standard(s): G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. Identify and describe relationships between central, inscribed, and circumscribed angles. Identify inscribed angles on a diameter as right angles. G.C.3 Construct the inscribed and circumscribed circles of a triangle. Prove properties of angles for a quadrilateral inscribed in a circle.

Essential Question: How do you use inscribed angles to solve problems?

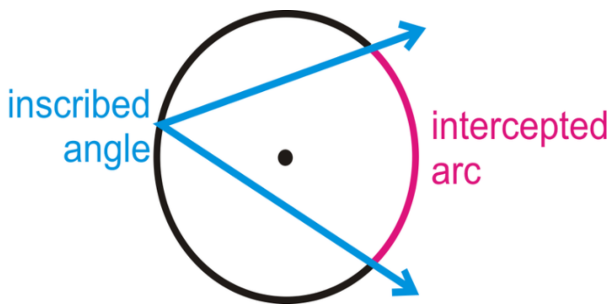
Warm up



1. What is the measure of each angle in the triangle? How do you know?

2. What do you know about the three arcs and what is the measure of each arc?

3. What is the relationship between the angles in the triangles and the measure of each arc?



We have discussed central angles so far in this chapter. We will now introduce another type of angle, the inscribed angle.

An i**nscribed angle** is an n angle with its vertex is the circle and its sides contain chords.

An **intercepted arc** is the arc that is on the interior of the inscribed angle and whose endpoints are on the angle.

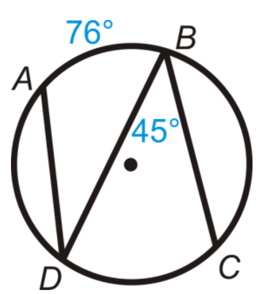
The vertex of an inscribed angle can be anywhere on the circle as long as its sides intersect the circle to form an intercepted arc.

From the warm up what did you discover about the relationship between an inscribed angle and its intercepted arc?

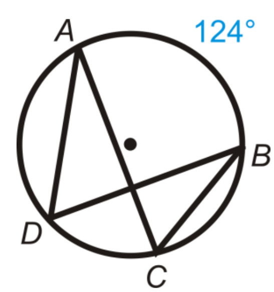
Inscribed Angle Theorem: If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.

Example:

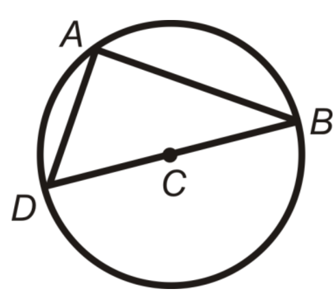
1. Find $m\widehat{DC}$ and $m\angle ADB$



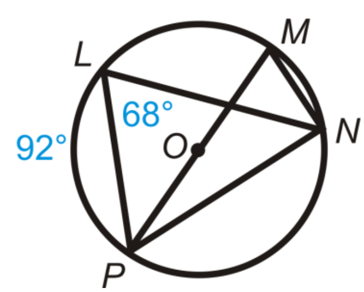
2. Find $m\angle ADB$ and $m\angle ACB$



3. Find $m\angle DAB$ in $\odot C$.

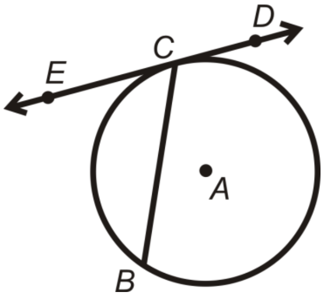


4. Find $m\angle PMN$, $m\widehat{PN}$, $m\angle MNP$, $m\angle LNP$, and $m\widehat{LN}$.



THEOREM: Inscribed angles that intercept the same arc are congruent.

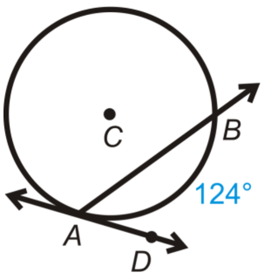
THEOREM: An angle that intercepts a semicircle is a right angle.



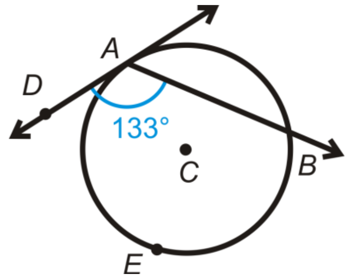
When an angle is on a circle, the vertex is on the circumference of the circle. One type of angle on a circle is the inscribed angle. Recall that an inscribed angle is formed by two chords and is half the measure of the intercepted arc. Another type of angle on a circle is one formed by a tangent and a chord (as in the diagram to the left). ***Whenever the vertex of the angle is on the circle the angle is half the measure of the intercepted arc.***

Example:

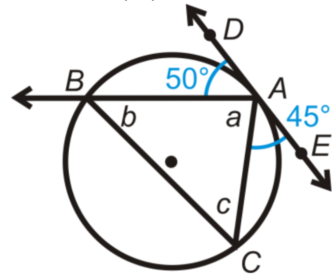
5. Find $m\angle BAD$



6. Find $m\widehat{AEB}$



7. Find a, b, c.



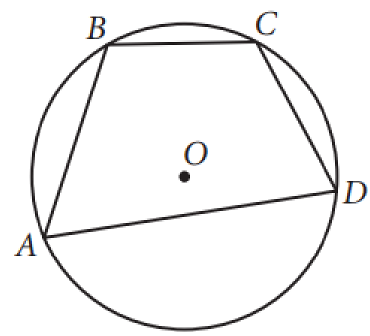
An **inscribed polygon** is a polygon where every vertex is on the circle. Note, that not every quadrilateral or polygon can be inscribed in a circle. Inscribed quadrilaterals are also called cyclic quadrilaterals. For these types of quadrilaterals, they must have one special property. **THEOREM: The opposite angles in an inscribed quadrilateral are supplementary**

Complete the following proof

Given: $ABCD$ is an inscribed quadrilateral in $\odot O$

Prove: $\angle A$ and $\angle C$ are supplementary; $\angle B$ and $\angle D$ are supplementary

Statements	Reasons
1. $ABCD$ is an inscribed quadrilateral in $\odot O$	**1.**
2. $m\angle A = \frac{1}{2}m\widehat{BD};\ m\angle C = \frac{1}{2}m\widehat{BAD}$	**2.**
3. $m\angle A + m\angle C = \frac{1}{2}m\widehat{BD} + \frac{1}{2}m\widehat{BAD}$	**3.**
4. $m\angle A + m\angle C = \frac{1}{2}(m\widehat{BD} + m\widehat{BAD})$	**4.**
5. $m\angle A + m\angle C = \frac{1}{2}$(arc measure of a circle)	**5.**
6. $m\angle A + m\angle C = \frac{1}{2}\ (360°) = 180°$	**6.**
7. $\angle A$ and $\angle C$ are supplementary	**7.**

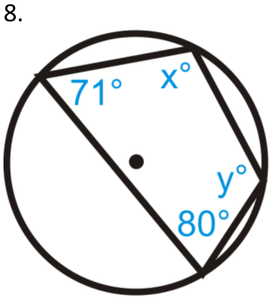


The steps above can be repeated for $\angle B$ and $\angle D$. Therefore, the opposite angles of $ABCD$ are supplementary.

Example:

Find the value of the missing variables.

8.



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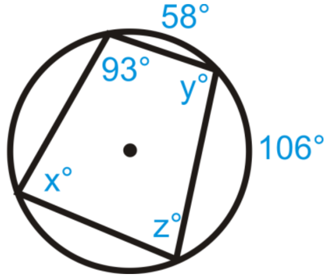

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