

Lesson 4: Interior and Exterior Angles of Circles

Standard(s): G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. Identify and describe relationships between central, inscribed, and circumscribed angles. Identify inscribed angles on a diameter as right angles. G.C.3 Construct the inscribed and circumscribed circles of a triangle. Prove properties of angles for a quadrilateral inscribed in a circle.

Essential Question: What are the relationships between measures of angles formed by intersecting chords and their intercepted arcs? What are the relationships between measures of angles formed by intersecting secants, tangents or secant/ tangents outside of a circle and their intercepted arcs?

An **interior (inside) angle** is an angle considered inside a circle when the vertex is somewhere inside the circle, but not on the center. All angles inside a circle are formed by two intersecting chords.

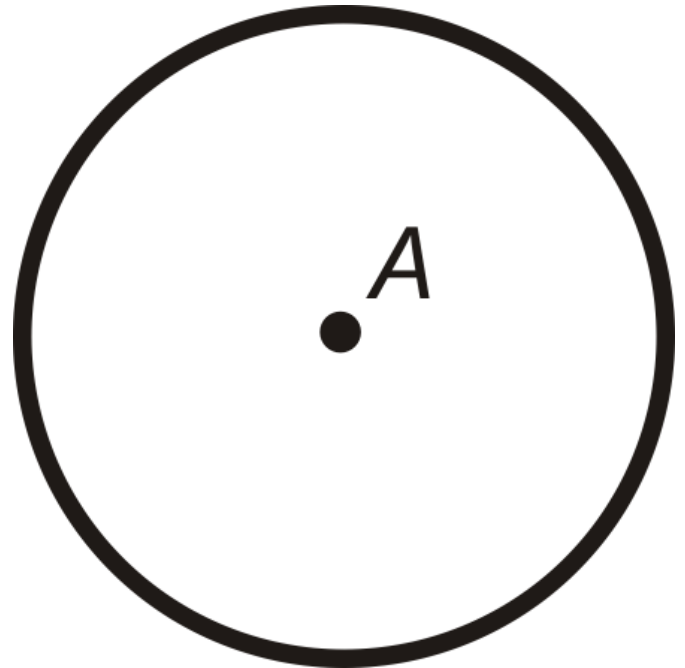
Construction Investigation:

Find the Measure of an Angle inside a Circle

Tools Needed: pencil, straightedge, protractor, colored pencils

In $\odot A$ perform the following constructions

- Draw a chord and label it \overline{BC} and \overline{DE} so that they do not intersect at A . Label the point of intersection P .
- Use color pencils to draw central angles $\angle DAB$ and $\angle CAE$.
- Use your protractor, to find the $m\angle DPB$, $m\angle DAB$, and $m\angle CAE$.
What is $m\widehat{DB}$ and $m\widehat{CE}$?



$$m\angle DPB = \quad m\angle DAB = \quad m\angle CAE =$$

$$m\widehat{DB} = \quad m\widehat{CE} =$$

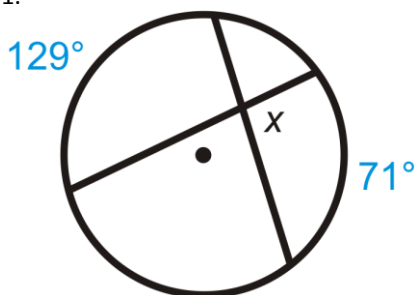
d. Find $\frac{m\widehat{DB} + m\widehat{CE}}{2} =$

- e. What did you notice

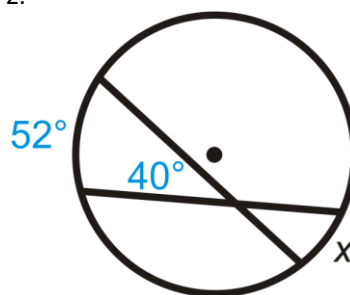
THEOREM: The measure of the angle formed by two chords that intersect inside a circle is the average of the measure of the intercepted arcs.

PRACTICE: Find x

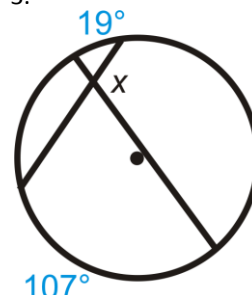
1.



2.



3.



An **exterior (outside) angle** is an angle is considered to be outside a circle if the vertex of the angle is outside the circle and the sides are tangents or secants. There are three types of angles that are outside a circle: an angle formed by two tangents, an angle formed by a tangent and a secant, and an angle formed by two secants. Just like an angle inside or on a circle, an angle outside a circle has a specific formula, involving the intercepted arcs.

Construction Investigation: Find the Measure of an Angle outside a Circle

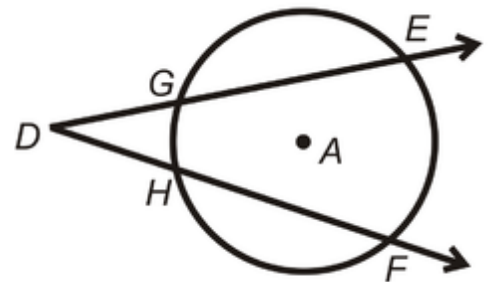
• In $\odot A$ perform the following constructions

- Use color pencils to draw central angles $\angle GAH$ and $\angle EAF$.
- Use your protractor, to find:

$$m\angle GAH = \quad m\angle EAF =$$

- Find $m\angle EDF =$

- Find $\frac{m\widehat{EF} - m\widehat{GH}}{2} =$



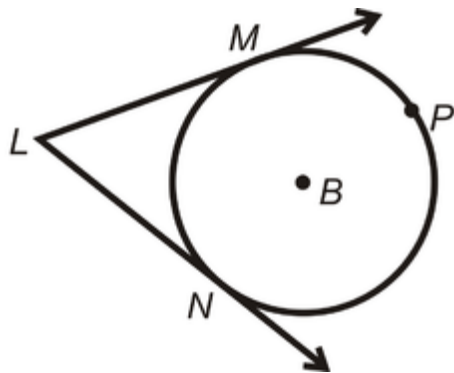
• In $\odot B$ perform the following constructions

- Use color pencils to draw central angles $\angle MBN$.
- Use your protractor, to find:

$$m\angle MBN =$$

- Find $m\angle MLN =$

- Find $\frac{m\widehat{MPN} - m\widehat{MN}}{2} =$



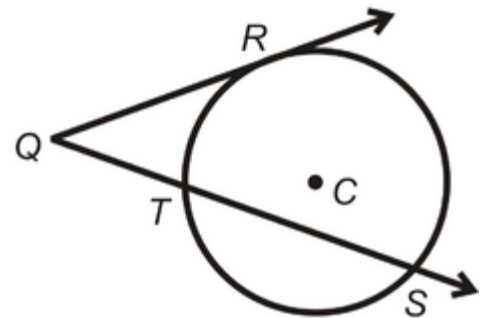
• In $\odot C$ perform the following constructions

- Use color pencils to draw central angles $\angle RCT$ and $\angle RCS$.
- Use your protractor, to find:

$$m\angle RCT = \quad m\angle RCS =$$

- Find $m\angle RQS =$

- Find $\frac{m\widehat{RS} - m\widehat{RT}}{2} =$



What did you notice for each circle?

THEOREM: The measure of an angle formed by two secants, two tangents, or a secant and a tangent drawn from a point outside the circle is equal to half the difference of the measures of the intercepted arcs.

PRACTICE: Find x

